Duality

Building blocks for Threshold FHE

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MPTS 2023: NIST Workshop on Multi-Party Threshold Schemes 28 Sep 2023

Quick overview of Fully Homomorphic Encryption (FHE)

- Correctness: Dec(Enc(x)) =(≈) x ; Dec(Eval(f, Enc(x))) =(≈) f(x)
- Complex algorithms: (functional) bootstrapping
- Compactness + efficiency: key/modulus switching, RNS representation, relinearization
- Lattice-based schemes [Gen09, Bra12, FV12, GHS13, BGV14, DM15, CGGI16, CKKS17]

- Typical single-server use case
  - Trusted client generates the keys, encrypts and decrypts
  - Server might or might not operate with secrets

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Quick overview of Threshold FHE (ThFHE)

- Multiple types of multi-party FHE
  - Multi-key FHE, Threshold FHE, Hybrid
- Goal: multiple parties emulate both the client and the server
- [BD10, AJL+12, LATV12, BGG+18, CCS19, SPT+21, MBH23, CCK23]

- Typical threshold FHE use case: each party has some local secret input
  - Parties **jointly** generate the secret and public keys
  - Each party evaluates the function (can be both **non-interactive** and **interactive**)
  - Parties **jointly** decrypt the result

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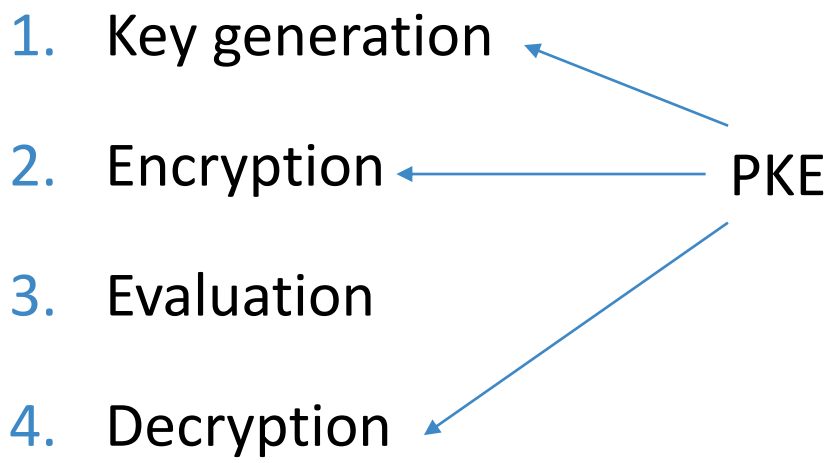
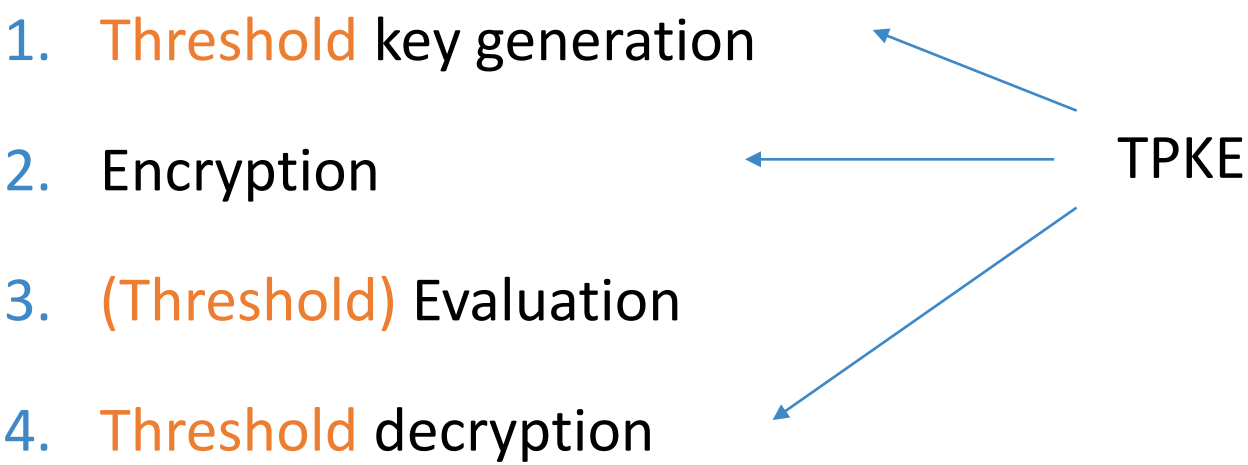
FHE -> Threshold FHE





Static vs Adaptive Corruptions

Trusted vs Untrusted Setup

Honest vs Dishonest Majority

Passive vs Active Security

Game- vs Simulation-based Definition

Synchronous vs Asynchronous Communication

Pre-Q vs PQ resilience

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Background: Security notions for PKE

- IND-CPA: encryption oracle access
- IND-CCA1: a priori decryption oracle access or “lunchtime attack”
- IND-CCA2: decryption oracle access

Security notions for TPKE

- The adversary has access to partial decryptions
- IND-TCPA, IND-TCCA [FPS01]: encryption + partial decryption oracle access, + decryption oracle access
- Smudging/Noise flooding/Sanitization [DS16, MW16]
- Threshold PKE: threshold key gen + threshold decryption
  - **Not trivial** to thresholdize existing lattice-based PKE schemes [BGGK17, BGG+18, CS19, KLO+19, DLN+21, CCMS21, CHI+21, BTT22, ASY22, GKS23]
  - Issues: secret sharing of key, growth of parameters, complex algorithms, transforms, etc.

Is TPKE a building block for ThFHE?

Yes… and no.
Evaluation brings new challenges!

Background: “Passive” security notions for FHE

- IND-CPA’: encryption + evaluation oracle access **(careful in the modular approach)**
- IND-CPA^D [LM21, LMSS22]: oracle access to “encrypt-evaluate-decrypt”
- Circuit privacy [Gen09, OPCPC14, BdPMW16, KS23]: all ciphertexts have the same distribution
- funcCPA [AV21, AGHV22]: oracle access for “decrypt-evaluate-encrypt”

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Standardizing passive ThFHE

- Security of threshold PKE does not automatically imply security of threshold FHE
- IND-TCPA’ [BS23, KS23]: encryption + evaluation + partial decryption oracle access
- Real/Ideal functionality [DPSZ12]

Gadgets & building blocks

- Passive security definitions for ThFHE: IND-TCPA’, passive MPC, (thresholdized?) funcCPA
- Noise flooding/sanitization/smudging
- Cryptographic closeness: statistical distance [AJL+12], bit-security distance [MW18, LMSS22], Rényi divergence [BLRL+18, BS23, CSS+23]

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More protocol-gadgets for passive ThFHE

- Key generation
  - Secret sharing schemes
- Interactive masked decryption
  - => Interactive bootstrapping/scheme switching [CLO+13, SPT+21, MBH23, GGP+23]
  - Useful to make evaluation more efficient and to translate to other cryptographic schemes

What about active security?

Slower, but on its way?

Background: Active security notions for (Th)FHE

- IND-CCA1: a priori decryption oracle access
  - Practical FHE schemes require encryptions (of functions) of the secret key -> incompatible with the decryption oracle; other constructions in [CRRV17]
- IND-CCA2: decryption oracle access
  - No HE can satisfy this
- Targeted malleability [PR08, BSW11]: allow decryptions of some function families
- IND-CVA [LMNV10, CGG16, CCCM22, CCCM23]: oracle access for plaintext validity
- Malicious security [VKH23]: correctness, completeness, soundness
- IND-TCPA + threshold verifiability + decryption simulatability [BGG+18, ABGS22]
- Real/ideal functionality with active security [DDE+23, Sma23]

Standardizing active ThFHE

- Threshold additively/somewhat HE, FHE with active security [DKL+13, KPR18, RST+22, ABGS22, CMS+23, DDE+23]

Gadgets & building blocks

- Active security definition for ThFHE
- Honest majority to help with verifiable computation
- Non interactive zero-knowledge proofs (PQ) / Homomorphic signatures

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Thank you!

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