Trapdoor simulation of quantum algorithms

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Joint work with:
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“What is your algorithm?”
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Critical question for ECC security:

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Quantum-algorithm design is moving beyond textbook stage into algorithms without proofs.

Example: subset-sum exponent $\approx 2^{41}$ from 2013 Bernstein–Jeffery–Lange–Meurer.

Don’t expect proofs or provability for the best quantum algorithms to attack post-quantum crypto.

How do we obtain confidence in analysis of these algorithms?
Quantum experiments are hard.

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Analogy:
Public hasn’t carried out
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Vastly larger extrapolation for the quantum situation. Imagine attacker performing $2^{80}$ operations on $2^{40}$ qubits; compare to today’s challenges of $2^1, 2^2, 2^3, 2^4, 2^5, 2^6$ qubits.
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Theorem statement is easier.
Steps in proof are easier.
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The standard structure of an algorithm simulation:
Compute $s_0, s_1, s_2, \ldots$ and $t_0, t_1, t_2, \ldots$ such that $s_i$ represents the algorithm state at time $t_i$.
Prove that the computation matches the original algorithm.

Special case: experiment.
The computation is the original algorithm plus printouts of state.
Particularly easy proof.
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Algorithm simulation is a computer-assisted proof of the algorithm's performance for a particular input.

Compared to traditional proofs:
- Theorem statement is easier.
- Steps in proof are easier.
- We need to generalize beyond a single input.
- Provability is guaranteed.
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Compute $s_0, s_1, s_2, \ldots$ and $t_0, t_1, t_2, \ldots$ such that $s_i$ represents algorithm state at time $t_i$.

Prove that the computation matches the original algorithm.

Special case: experiment.

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Trapdoor simulation

Input to simulation doesn’t have to be input to original algorithm.

Simulation can use extra input that makes simulation much faster than original.

Typical examples:

• Algorithm input: $f(x)$.
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This is still useful: can try many choices of $x$, understand algorithm for $f(x)$.
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Childs: Yes. Typo, already fixed in 2005 journal version.