FrodoKEM
practical quantum-secure key encapsulation
from generic lattices

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NXP  Microsoft Research  McMaster University
Google  University of Michigan  CWI
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FrodoPKE (IND-CPA) \[\text{[FujisakiOkamoto’99,HHK’17]}\] (generic transform) FrodoKEM (IND-CCA)
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\begin{itemize}
\item \textbf{Concrete Instantiations}
\begin{enumerate}
\item FrodoKEM-640: targets Level 1 security ($\geq$ AES-128).
\item FrodoKEM-976: targets Level 3 security ($\geq$ AES-192).
\item Other parameterizations are easy, by changing compile-time constants.
\end{enumerate}
\end{itemize}
Pedigree

Learning With Errors (LWE) [Regev’05]

- Lineage of [Ajtai’96,AjtaiDwork’97]: worst-case/average-case reductions:
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Public-Key Encryption/Key Exchange

- Many schemes with tight (CPA-)security reductions from LWE:
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- FrodoPKE [this work]: wider error distributions, new parameters, . . .
Assumption: for uniformly random matrix $A$ over $\mathbb{Z}_q$ and $S$ from $\chi$, $[A, B] \approx SA \approx c \equiv \text{uniform over } \mathbb{Z}_q$.

LWE and FrodoPKE

Learning With Errors

- Dimension $n$, modulus $q$, error distribution $\chi$ on ‘small’ integers.
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Bounded-distance decoding on a random ‘$q$-ary’ lattice defined by $A$:
LWE and FrodoPKE

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$$S \leftarrow \chi^{k \times n} \quad pk = \text{seed}_A, \ B \approx SA$$

$$\quad (A = \text{expand(}\text{seed}_A\text{)} \in \mathbb{Z}_q^{n \times n})$$

(Images courtesy xkcd.org)
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(A, B, C, C') $\overset{c}{\equiv} \text{unif}$

(Images courtesy xkcd.org)
Distinctive Features of FrodoPKE/KEM

1. Generic, algebraically unstructured lattices: plain LWE.

2. ‘Semi-wide’ errors conforming to a worst-case/average-case reduction from a previously studied lattice problem: BDD with DGS.

3. Simple design and constant-time implementation:
   - power-of-2 modulus $q$ for cheap & easy modular arithmetic
   - straightforward error sampling
   - no ‘reconciliation’ or error-correcting codes for removing noise
   - x64 implementation: 256 lines of plain C code (+ preexisting symmetric primitives)
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Risk Category 1: Geometric & Algebraic Structure

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Our Foundation: Plain LWE on Unstructured Lattices

- LWE is bounded-distance decoding on a lattice defined by the uniformly random, unstructured matrix \( \mathbf{A} \).
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Our Foundation: Plain LWE on Unstructured Lattices

- LWE is bounded-distance decoding on a lattice defined by the uniformly random, unstructured matrix $A$.
- No algebraic or ‘planted’ geometric structure in the lattice.
Semi-Wide Errors

Choosing an Error Distribution

- Narrower errors $\Rightarrow$ smaller parameters $q, n \Rightarrow$ better efficiency.

Risk Category 2: Narrow Errors

1. LWE with $O(1)$-bounded error is poly($n$)-time solvable [AG’11,ACFP’14] given large-poly($n$)-many samples. (PKEs don’t reveal this many!)

2. Worst-case-hardness theorems need Gaussian error of $\sigma > \sqrt{n/(2\pi)}$.

- Or narrower error, but only for few LWE samples. (PKEs reveal more!)

= Sizeable gap between known-vulnerable and worst-case-hard params.

New Worst-Case Hardness

- A latent reduction from [R’05,PRS’17] works for our $\sigma \approx \eta(Z)$.

- Works for a bounded poly($n$) number of LWE samples: covers PKEs!
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Worst-Case Problem: **BDD with DGS** [AR’04, R’05, LLM’06, DRS’14]

- Given $N$ samples from discrete Gaussian $D_{\mathcal{L}^*}$, decode $\mathcal{L}$ to distance $d$.
- State of the art is limited to distance $d < \sqrt{\ln(N)/(2\pi)}$. 

Theorem (extracted from [R’05, PRS’17])

Solving LWE for Gaussian error $\sigma \geq \eta(Z)$ with $m = \text{poly}(n)$ samples

$\Rightarrow$ solving BDD at distance $d = \sigma \sqrt{2\pi}$ with $N = m \cdot \text{poly}(n)$ DGS samples.
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\[ \Downarrow \]

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**Interpretation**

- Theoretical support & more confidence for semi-wide Gaussian error with limited number of samples.
- Reduction is non-tight; for concrete security we use cryptanalysis.

(Tightening the time & sample overhead is a good research direction.)
Concrete Parameters

- Use ‘core-SVP’ methodology [ADPS’16] to lower-bound the first-order exponential time (and space) of SVP in appropriate dimension.
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\begin{table}[h]
\centering
\begin{tabular}{l|cccrr}
\hline
 & $n$ & $q$ & $\sigma$ & Bits of Security & \\
 & & & & $C \geq$ & $Q \geq$ \\
\hline
FrodoKEM-640 & 640 & $2^{15}$ & 2.75 & 143 & 103 \\
FrodoKEM-976 & 976 & $2^{16}$ & 2.3 & 209 & 150 \\
\hline
\end{tabular}
\end{table}
## Performance

- **Speed** (in kilocycles, 3.4GHz Intel Core i7-6700 Skylake, AES-NI):

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- **Sizes** (in bytes):

<table>
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https://FrodoKEM.org

Thanks!