

# Threshold Cryptography: Ready for Prime Time?

Hugo Krawczyk, IBM

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(with thanks to many colleagues and collaborators -  
see references at the end)

# Threshold Cryptography: Ready for Prime Time?

- I would say YES! (except that I said the same 20 years ago...) 
- Huge increase in quantity and sensitivity of stored data
- Cloud storage and computing as the prevalent paradigm
- Huge key management operations (see Campagna's AWS talk at RWC'19)
- Privacy concerns, awareness, regulations
- Awareness of dangers of centralization: Facebook, Google, Amazon, *WeChat*
- *Distributed trust* becoming a more "familiar" notion via blockchain
- Advances in cryptography: MPC, homomorphic techniques, ZK, ...
- **New applications!** **THIS WORKSHOP**

# With great opportunities come great challenges



- The distributed trust notion (a hard one to reason about)
- True distribution → *diversity*: O/S, h/w, geography, tooling, admin, policy and authorization, credential management,... → **fault independence**
- Wide Area Networks, asynchronous networks, *going beyond  $n/3$  !*
- Role of secure h/w (enclaves, HSMs), virtualization, side channel sec.
- Distributed key generation, share recovery, proactive, *identifying cheaters*
- Integration with MPC, blockchains, ZK proofs, ...
- Large scale TC (10's/100's/1000's/millions parties?)
- Post quantum techniques, including symmetric crypto and inf. Theoretic (NIST competition: Prioritize schemes with threshold implementations)

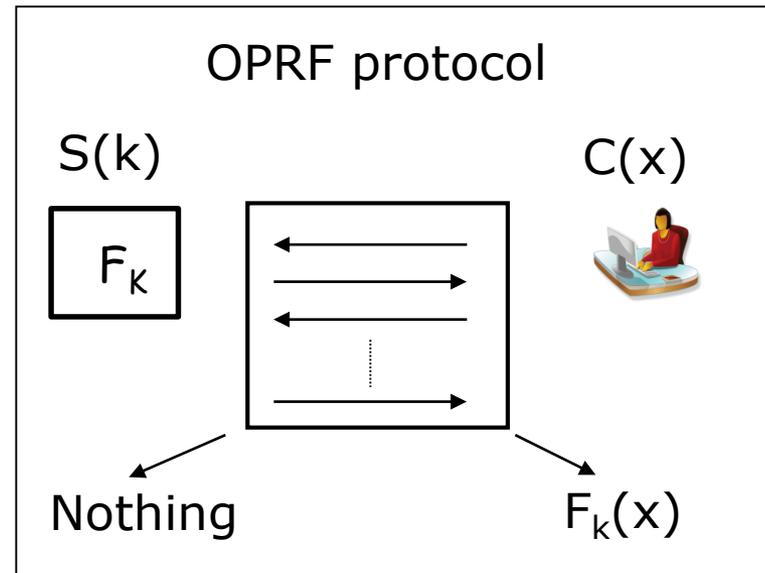
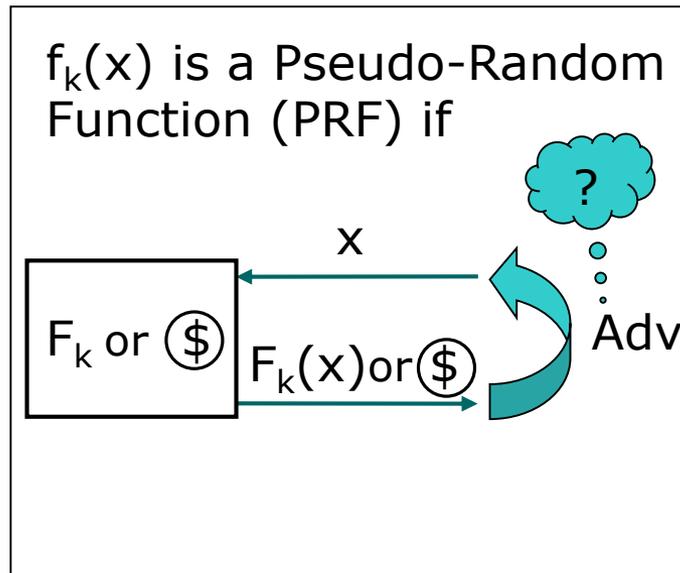


Your favorite challenge here

Mine: Build a serious open source  
platform for threshold cryptography

Demonstrate new *practical* applications!

# Oblivious PRF (OPRF)



- ❑ OPRF: An interactive PRF “service” that returns PRF results *without learning the input or output of the function*
- ❑ *A POWERFUL primitive*

$$H'(x, H(x)^k)$$

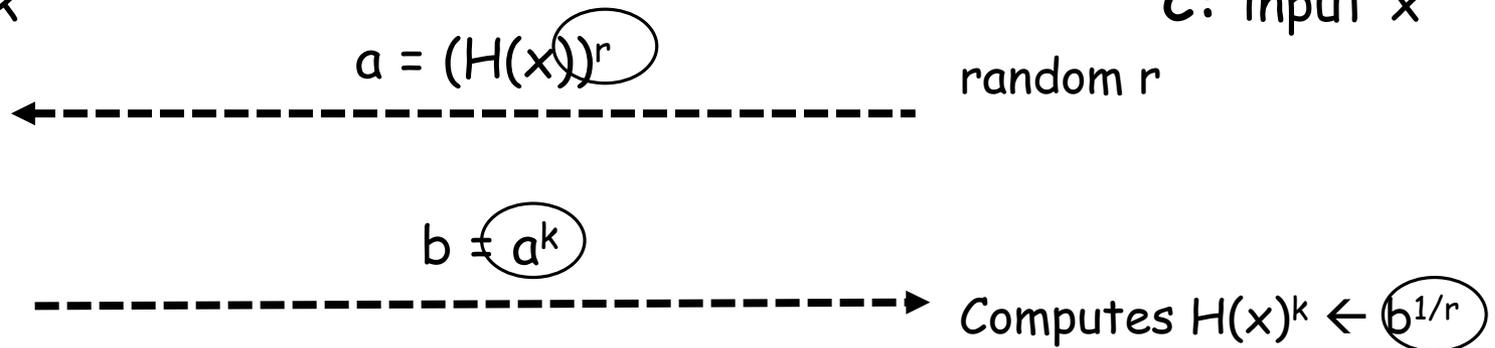
# DH-OPRF

[CP93...NPR99...FIPR05  
...JL10...JKK14...]

- PRF:  $F_k(x) = H(x)^k$ ; input  $x$ , key  $k$  in  $\mathbb{Z}_q$ ;  $H = \text{RO}$  onto  $G$  (of order  $q$ )
- Oblivious computation via Blind DH Computation ( $S$  has  $k$ ,  $C$  has  $x$ )

$S$ : key  $k$

$C$ : input  $x$



- The blinding factor  $r$  works as a one-time encryption key:  
*hides*  $H(x)$ ,  $x$  *and*  $F_k(x)$  *perfectly from*  $S$  (and from any observer)
- Computational cost: one round, 2 exponentiations for  $C$ , one for  $S$ 
  - Variant: fixed base exponentiation for  $C$  (even faster)

# Threshold DH-OPRF

- Single server solution:  $F_k(x) = (H(x))^k$
- Multi-server solution: server  $S_i$  initialized with  $(t,n)$ -share  $k_i$
- Shamir in the exponent (polynomial interpolation)
  - $F_k(x) = (H(x))^{\lambda_{i1}k_{i1}} \cdot (H(x))^{\lambda_{i2}k_{i2}} \cdot \dots \cdot (H(x))^{\lambda_{i,t+1}k_{i,t+1}}$
  - C sends same  $a = (H(x))^r$  to  $t + 1$  servers;
  - $S_{ij}$  raises  $a^{\lambda_{ij}k_{ij}}$  and sends back to U who deblinds and multiplies \*
- Efficiency (!): 2 exp's for client (indep of  $t, n$ ), 1 per server, 1 round

\* If responders among servers not known a-priori, interpolation done by U  
. (one multi-exponentiation; can be further optimized [Patel-Yung])

# Threshold DH-OPRF (more features)

- Threshold operation *transparent to client*
  - Client sends one and same msg to all servers and aggregation of  $a^{k_{ij}}$  to  $a^k$  can be done by a single server (proxy)
- Distributed key generation (key never exists in one physical place)
- Share recovery, Proactive security (fundamental for long-lived keys)
- Verifiability: With  $g^k$ , C can verify that  $H(x)^k$  computed correctly
  - Preserves client transparency using interactive verification (2x cost)
  - Can also use BLS for “built-in verifiability”

# Proving Threshold DH-OPRF [JKKX'17]

- UC Definition of Threshold OPRF: Extends the (single) OPRF UC formulation of JKKX'16
- Ticketing mechanism: increases when threshold of servers responds; decreases when client reconstructs an output
  - Avoids extraction and other proof elements that degrade performance
- Proof of Threshold DH-OPRF based on Gap-OMDH assumption in ROM, and on Gap-TOMDH to achieve a stronger flavor
  - OMDH: “Q interactions with  $(\cdot)^k \rightarrow$  No more than  $g_1^k, \dots, g_Q^k$  on random  $g_i$ ”
  - T-OMDH: require  $t+1$  online attempts for each  $g_i^k$ .



# PPSS: Password Protected Secret Sharing

(password-protected distributed storage)

# How to protect a secret with a password

- Goal: protect secrecy and availability with a single password
  - Single server = Single point of compromise for secrecy (offline dict attacks) and for availability (server gone, secret gone) → multi-server solution
- Crypto solution: keep the secret encrypted in multiple locations; secret share the encryption key in multiple servers (t-out-of-n)
  - Availability insured if t+1 available, secrecy if t or less corrupted
- But how do you authenticate to each server for share retrieval?
  - A strong independent password with each server? Not realistic
  - Same (or slight-variant) password for each server? Not good

→ Each server is a single point of compromise!

# How to protect a secret with a password

- Password-Protected Secret Sharing (PPSS) guarantees
  - Breaking into  $t$  servers leaks nothing about secret or password (assumes all server info lost: shares, long-term keys, password file, etc.)
  - Only adversary option: Guess the password, try it in an online attack.
- Definition [BJSL'12, CLLN'14, JKKX'16]
  - Only *unavoidable online* attacks allowed: Attacker needs at least  $t+1$  *online* interactions to validate a single guessed password
  - Offline attacks are not possible, except if  $t+1$  servers compromised
  - Subtlety: User needs a way to verify the reconstructed secret is correct (w/o that information allowing offline attacks) **Important: No PKI**

# TOPPSS: PPSS via Threshold OPRF [JKKX'17]

- Idea: Define the retrieved secret as  $s = \text{OPRF}_k(\text{pwd})$  and implement the computation as a Threshold OPRF
  - U: send  $a = H(\text{pwd})^r$ , get  $a^{k_i}$ , reconstruct  $s = H(\text{pwd})^k$  + mechanism to test  $s$
- Definitions and analysis tricky but protocol very simple
  - Crucial detail: Must be able to verify the correct secret reconstruction
  - Note: No PKI reliance (except for initialization)
- PPSS performance: same as Threshold DH-OPRF
  - Single round, total 2 exp for client, 1 exp for each server, client transparent
- Proactive security and other goodies (as in underlying T-OPRF)

# From $(t,n)$ -PPSS to $(t,n)$ -threshold PAKE

- $(t,n)$ -TPAKE [MSJ'02]: Single-password PAKE b/w U and any subset of  $n$  servers - *secure* as long as at most  $t$  servers are corrupted
  - Addresses the main threat to passwords today, namely, leakage via server compromise (even  $t$  adversarial servers learn nothing about password)
- *Generic composition theorem*: PPSS + KE  $\rightarrow$  T-PAKE [JKK14]
  - $\rightarrow$  *First single-round T-PAKE and best computational performance*  
(2 exp user, 1 exp server)
    - Best previous work required 10 msgs plus  $14t$  exponentiations for client and  $7t$  for each server (even a dedicated 2-out-of-2 sol'n required 5 msgs)

# More Password Applications from (T-)OPRF

- OPAQUE = “an asymmetric (1,1)-PAKE” (hopefully integration w/TLS 1.3)
  - Much more secure than “password-over-TLS” (pwd never exposed)
    - First client-server PKI-free PAKE secure against pre-computation attacks!
  - Server can be implemented as **threshold OPRF**: Best protection against server compromise and offline attacks (the way most passwords are stolen)
- SPHINX: Server-based online password manager
  - User only remembers master password, interacts with SPHINX server(s) to create *random independent* passwords for each of its accounts
  - Magic property: Breaking into the server leaks *nothing* on the user’s master password or on the random account passwords
    - after breaking the server an online guessing attack is still required

# OPRF-based Key Management

- Ciphertexts and keys need to be stored separately. How? Client stores ciphertexts, outsources the key to a key management server (KMS)
- Today: All encryption keys exposed to the KMS and to channel between KMS and client (e.g., tls failures, certificates, termination points, CDN,...)
- Using OPRF: KMS learns *nothing* about key or object being encrypted, and neither do observers of client-KMS channel (unconditional security)
- Plus: If client assigns unpredictable identifiers to objects  
→ forward security (keys remain secure upon full compromise of KMS)
- It gets better: **Threshold and proactive security!!**

(Oh. And non-interactive key rotation.)

# Threshold Decryption

- General use case: Data encrypted under a service public key; decryption possible only upon collaboration of  $t$  servers
  - Data protected up to the compromise of  $t$  servers
- Examples
  - Long-term data storage: sensitive and valuable information, e.g. personal information, legal and financial documents, cryptographic keys, etc.
  - Computation on encrypted data, only decrypt results (e.g., voting, FHE)
  - Specialized cases of computation on encrypted data
    - Next: Two such examples

# Operations on Encrypted Sets

- Set representation using polynomials [FNP'04, KS'05]
  - Set of elements  $a_1, \dots, a_n$  represented by n-degree polynomial  $(x-a_1) \cdots (x-a_n)$
  - Membership test:  $a$  is in the set iff  $P(a)=0$
  - Adding an element: If  $P(x)$  represents  $S$ ,  $P'(x)=P(x)(x-a)$  represents  $S' = S \cup \{a\}$
- Privacy preserving operations: Encode coefficients using linear homomorphic encoding (via Elgamal encryption)
  - Define Elgamal encoding  $E(v) = EG_h(g^v) = (g^k, h^k g^v)$  under PK  $h$ .
  - $P(x) = \sum_{i=0}^n P_i x^i$  represented as encoding of coefficients
$$E(P_i) = EG(g^{P_i}) = (g^{k_i}, h^{k_i} \cdot g^{P_i})$$

# Operations on Encrypted Sets

- Element addition:  $P'(x) = P(x)(x - a)$

$$P'_i = P_i - aP_{i-1}; \quad E(P'_i) = EG(g^{P_i}) \cdot (EG(g^{P_{i-1}}))^{-a}$$

- Membership test:  $a \in S$  iff  $P(a) = 0$

$$E(P(a)) = E\left(\sum_{i=0}^n P_i a^i\right) = EG\left(g^{\sum_{i=0}^n P_i a^i}\right) = \prod_{i=0}^n (EG(g^{P_i}))^{a^i}$$

– Given  $E(P_0), \dots, E(P_n)$  and  $a$ , compute  $C = \prod_{i=0}^n (EG(g^{P_i}))^{a^i}$

– **Decrypt  $C$  and conclude  $a \in S$  iff result is 1.**

- In our applications, test uses **threshold decryption**

Note: nothing learned about the values of  $P_i$ , only whether  $P(a) = 0$

(also note that coefficient decryption is not possible)

# Digital Asset Transfer in Blockchain

- Example: Know Your Customer (KYC)
- Bank A performs KYC for customer U while opening account
  - U and A *own* the KYC file; A is willing to share it with other parties, e.g. bank B, upon *U's request* and upon *payment* by the receiving party
  - U does not want A and B to know each other's identity (for *privacy*)
  - U wants to remain *anonymous* to any party other than A and B and wants repeated uses of KYC to remain *unlinkable*.
  - A does not want another entity (e.g., bank B) to sell U's KYC – doing so represents *counterfeit* by Bank B (even if done in collaboration with U)
- Blockchain solution helps to enforce all the above properties (pseudonyms, commitments, payment, recording, ZK proofs, ...)

# Counterfeit/Duplicate Prevention

- Set of submitted values  $a$  (asset hashes) is recorded in blockchain via an encoded polynomial (i.e., encoded coefficients committed to  $b/c$ )
- Submitter of asset hash  $a$ , computes ciphertext  $C$  (encoding of  $P(a)$ ) and an encoding of  $P(x)(x-a)$  (using public homomorphic operations)
  - Submitter proves in ZK correct computation with respect to a committed (and hidden) value  $a$
- Blockchain peers **threshold decrypt**  $C$  (after randomizing it)
  - If result is 1, they reject value  $a$  (as already recorded)
  - Otherwise, they update the encoded-coefficients in blockchain to those submitted for  $P(x)(x-a)$

# Cryptography for #MeToo

- Most sexual assault is perpetrated by repeat offenders
- Goal: Identify survivors of same perpetrator while protecting anonymity of accusers and accused *except if #accusations > quorum*
- Ideal functionality: Accusers submit a (accuser-id, perpetrator-id) accusation to a trusted third party who matches perpetrators, and contact survivors if count for an accused goes over the quorum.
- We show a solution that achieves such functionality with full privacy

# Solution via Threshold Decryption

- Use encoded/encrypted polynomials to **encode a multi-set**
- Accusation against accused A recorded as encoding of  $P(x)(x-a)$
- Accused A reaches Q accusations when  $(x-A)^Q/P(x)$
- Testing  $(x-A)^Q/P(x)$  via randomized (Q-1)-th derivative of  $P(x)$
- Reduces to checking  $P^{(Q-1)}(A) = 0$  (derivative is “homomorphic”)
  - implemented via public operations on encoded polynomials and a single membership test using decryption (as in BC example but more involved)
- Test performed via **threshold decryption** by a set of dedicated servers

# Concluding Remarks

- We live in exciting times
- The world cries for distributed cryptography (even if they don't know it)
- Threshold cryptography is one of the most useful and practical branches of MPC - great applications!
- Varied, interesting, timely, practical, ready-to-deploy solutions
- Many challenges ahead, a lot to invent...
- ... and to implement and deploy (open source, please)
- Important role for NIST: Credibility, visibility, motivation
  - Standards and **best practices** as inputs to regulations

# Works Mentioned and Colleagues

- TOPPSS: S. Jarecki, A. Kiayas, H. Krawczyk and J. Xu, [eprint.iacr.org/2017/363](http://eprint.iacr.org/2017/363)
- T-PAKE: S. Jarecki, A. Kiayas, and H. Krawczyk, [eprint.iacr.org/2014/650](http://eprint.iacr.org/2014/650)
- OPAQUE: S. Jarecki, H. Krawczyk and J. Xu, [eprint.iacr.org/2018/163](http://eprint.iacr.org/2018/163)
- SPHINX: M. Shirvanian, S. Jarecki, H. Krawczyk, N. Saxena, [eprint/2018/695](http://eprint.iacr.org/2018/695)
- KMS: S. Jarecki, H. Krawczyk and J. Resch, [eprint.iacr.org/2018/733](http://eprint.iacr.org/2018/733) (preliminary)
- Asset transfer: H. Gunasinghe, A. Kundu, E. Bertino, H. Krawczyk, K. Singh, S. Chari, D. Song, The Web Conference, WWW'2019.
- MeToo: B. Kuykendall, H. Krawczyk, and T. Rabin, PETS 2019.
- Beyond  $n/3$ : C. Cachin, H. Krawczyk, T. Rabin, J. Resch, C. Stathakopoulou, "Tunable Protocols for Threshold and Proactive Cryptography", coming soon.
- **A. Davidson, I. Goldberg, N. Sullivan, G. Tankersley, F. Valsorda: Privacy Pass: Bypassing Internet Challenges Anonymously, PETS'18** *Only example of deployed OPRF?*



**Thanks!**

# Define “Threshold Cryptography”

- Threshold Cryptography: A special case of secure multi-party computation (MPC).
- TC characterized by the *distribution of a centralized service* for protecting both secrecy and availability.  
Clients can think of the service as one unit.
  - TC as an “implementation issue”, behind the scenes (the less the client is aware of it, the better – client transparency – communication via gateway)
  - The MPC happens at the servers, clients do not run an MPC or talk among themselves (though they may talk to a set of servers – e.g., for decryption)
- TC as one of the most useful and easier to motivate MPC flavors