

# Falcon - An Update

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→ Falcon stands for:

Fast Fourier lattice-based compact signatures over NTRU

→ Falcon is a:

- Signature scheme
- Based on the GPV framework [GPV08]
- Relying on NTRU lattices [HHP<sup>+</sup>03]

→ The main design principle:

**Compactness:** to minimize  $|pk| + |sig|$

## What remained the same?

- Almost everything
- Specification for NIST levels I and V
- Security estimates

## What changed?

- We removed the parameter set for NIST level III
  - Specification becomes much simpler
  - Algorithm count: **22** → **14**
  - Now only one modulus ( $q = 12289$ ), one type of ring ( $\mathbb{Z}[x]/(x^n + 1)$ )
- New portable and constant-time implementations

Thanks to the community [[OSHG19](#), [ZSS18](#), [KRVV19](#), [LAZ19](#)] for helping to improve Falcon.

We work over the cyclotomic ring  $\mathcal{R} = \mathbb{Z}_q[x]/(x^n + 1)$ .

## → Keygen()

- 1 Gen. matrices **A**, **B** with coefficients in  $\mathcal{R}$  such that:
  - >  $\mathbf{BA} = 0$
  - > **B** has small coefficients
- 2  $\text{pk} \leftarrow \mathbf{A}$
- 3  $\text{sk} \leftarrow \mathbf{B}$

## → Sign( $m, \text{sk}$ )

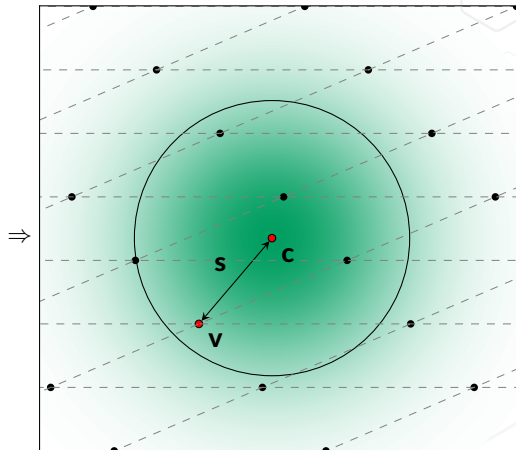
- 1 Compute **c** such that  $\mathbf{cA} = H(m)$
- 2  $\mathbf{v} \leftarrow$  “a vector in the lattice  $\Lambda(\mathbf{B})$ , close to **c**”
- 3  $\mathbf{s} \leftarrow \mathbf{c} - \mathbf{v}$

The signature  $\text{sig}$  is  $\mathbf{s} = (s_1, s_2)$

## → Verify( $m, \text{pk}, \text{sig}$ )

Accept iff:

- 1 **s** is short
- 2  $\mathbf{sA} = H(m)$



**On the theory side**, Falcon instantiates the GPV framework:

- Tight security proof in the ROM [GPV08]
- Tight security proof in the QROM [BDF<sup>+</sup>11]

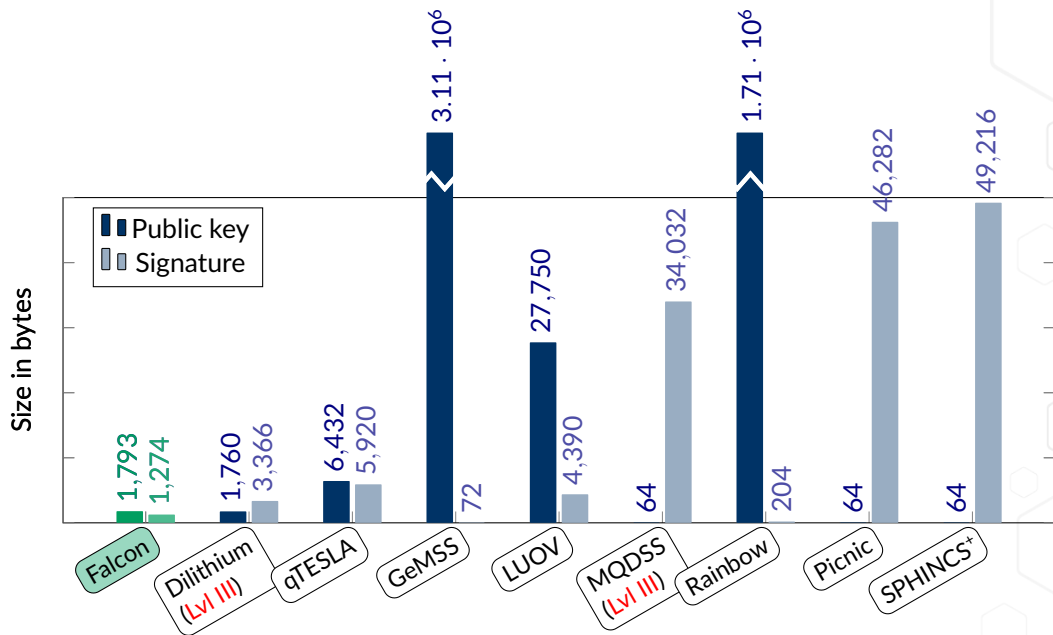
**On the practical side**, we consider the following lines of attack:

- **Lattice reduction** ⇒ The most effective [MW16]
- **Learning attacks** [GJSS01, GS02, NR06, DN12, YD18] ⇒ Impervious by design
- **“Overstretched NTRU”** [ABD16, CJL16, KF17] ⇒ Immune by parameters
- **Combinatorial** [How07, BKW00] ⇒ Immune by parameters
- **Algebraic** [CDPR16, CDW17, DPW19] ⇒ Not a threat as far as we know

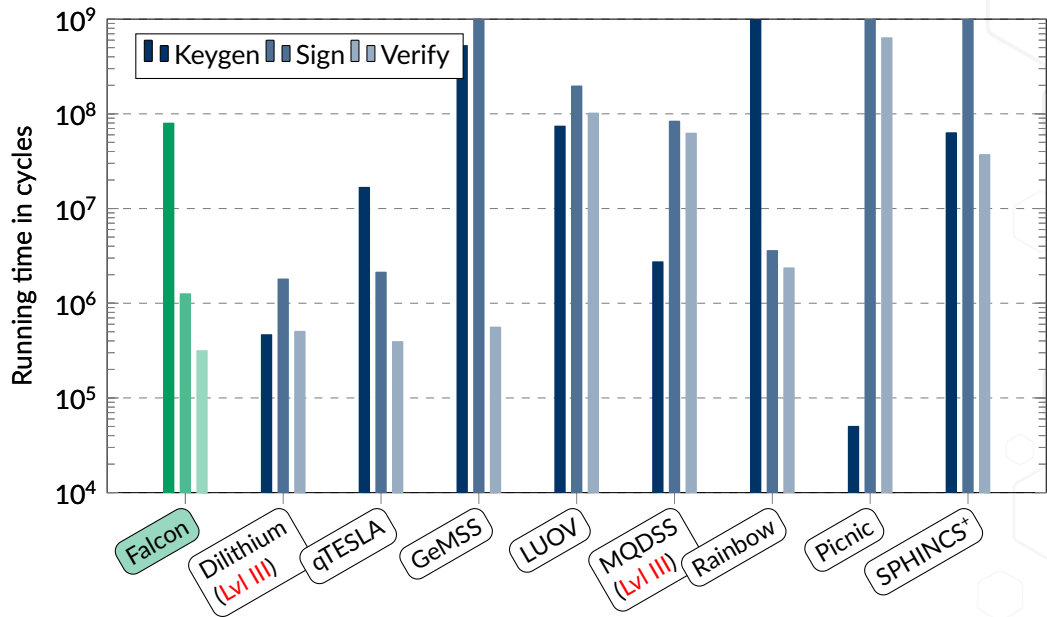
**NTRU lattices:**

- Extensively studied [HPS98, CS97, May99, MS01, HHPW05, GHN06, How07, Flu15]
- “Large” secrets  $f, g$  makes Falcon immune against many attacks

# Communication Costs at NIST Level V (Spec.)



# Computation Costs at NIST Level V (Spec.)



## → Portable:

- If no FPU available, FP arithmetic is software emulated
  - Performance hit of emulation ⇒ About one order of magnitude
  - No infinities, NaNs or subnormals
- Tested on x86/PowerPC/ARM, in 32- and 64-bit
  - Max stack < 3kB
  - Max RAM < 80 kB

## → Fully constant-time:

- New Gaussian sampler over the integers
  - Simple, fast, portable and constant-time
  - See Mélissa's talk this afternoon [[PRR19](#)]
- Variable-time operations eliminated from signing procedure
- Memory accesses only at non-secret addresses



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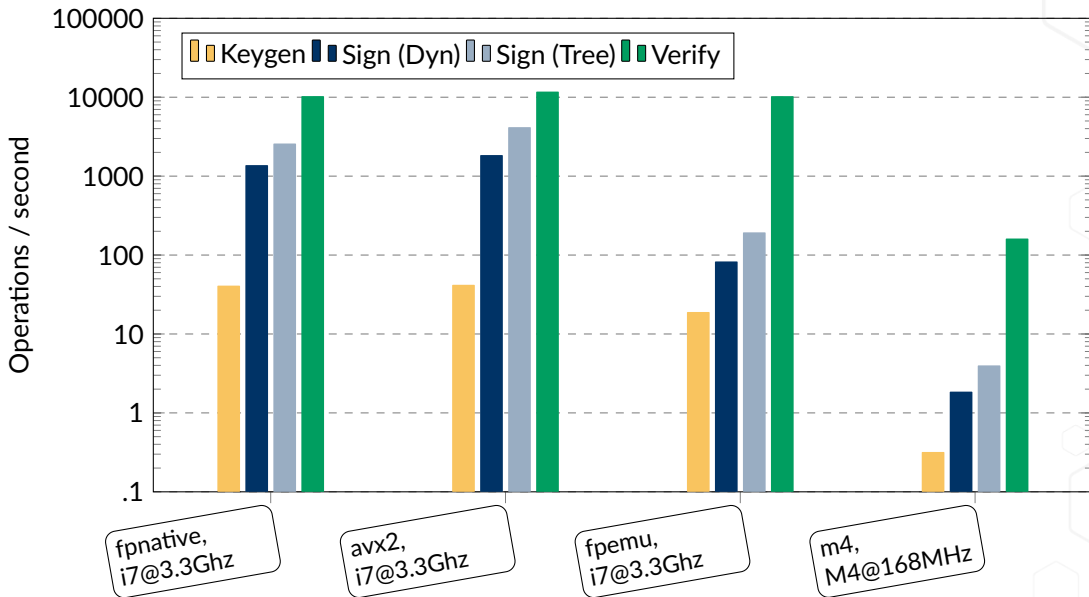
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→ **Integrated** to **PQClean**, **pqm4** and **SUPERCOP**.

→ The **code** and associated **note** are both on Falcon's website.

# New Implementations - NIST Level V



3 modes of operation (sizes in bytes, NIST level V):

→ <b>Classical:</b>	$ pk  = 1793$	$ sig  = 1273$	Total = 2996
→ <b>Message-recovery [dLP16]:</b>	$ pk  = 1793$	$ sig  = 768^*$	Total = 2561
→ <b>Key-recovery [PFH<sup>+</sup>19]:</b>	$ pk  = 64$	$ sig  = 2506$	Total = 2570

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Falcon can be turned into an IBE (identity-based encryption) scheme:

**Falcon + New Hope = IBE**

- See [GPV08, DLP14, MSO17] for details
- Orders of magnitude faster than pairing-based IBEs

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Falcon can also be turned into a ring signature scheme (variation of [RST01], [LAZ19]).

## Falcon is still:

- Secure
- Compact
- Fast
- Modular (3 modes, IBE, etc.)

## Falcon is now:

- Simpler
- Portable
- Constant-time

## Use cases:

- Certificate authorities
- Blockchain
- Firmware update
- IBE
- Ring signatures

## The future:

- New, unique functionalities
- Sanity check: statistical test suite

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# Thank you!


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