NTRU

Cong Chen, Oussama Danba, Jeffrey Hoffstein,
Andreas Hülsing, Joost Rijneveld, Tsunekazu Saito,
John M. Schanck, Peter Schwabe, William Whyte,
Keita Xagawa, Takashi Yamakawa, Zhenfei Zhang

Third round update
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No changes

- No changes to the scheme.
- No new parameter sets.

What might change?

- Revised security analysis.
- Maybe a small tweak to the KDF.

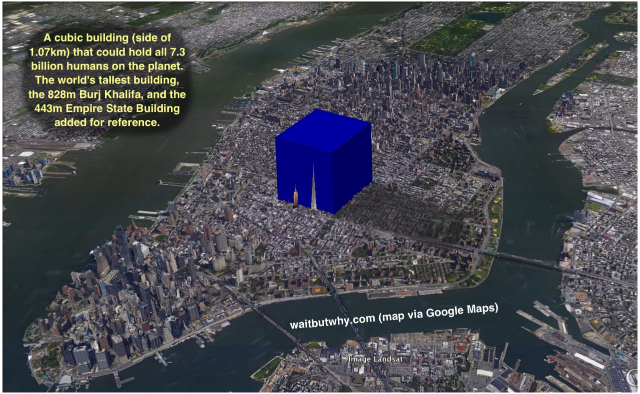
Security updates

- **Ducas** (EUROCRYPT 2018):
  "Dimensions for free".
- **Dachman-Soled–Ducas–Gong–Rossi** (CRYPTO 2020):
  More refined analysis of the blocksize needed in primal attack.
  New "leaky-LWE-estimator" scripts.
- **Albrecht–Gheorghiu–Postlethwaite–Schanck** (ASIACRYPT 2020):
  Heuristic non-asymptotic costs for one subroutine of sieving-based attacks.
- **May**, "How to meet ternary LWE keys" (CRYPTO 2021):
  Improved combinatorial attacks.
- **Nguyen**, "Boosting the hybrid attack on NTRU" (later today!).

"Beyond Core-SVP" estimate

Defined in Section 5.2 of Round 3 Kyber document.
Accounts for **dimensions-for-free** and **progressive sieving**.
Gives a circuit model **gate count**. Ignores data locality.

	ntruhrss701	ntruhps2048677
lattice attack dim. d	1328	1228
BKZ-blocksize	466	496
d_{4f}	41	43
sieving dimension	425	453
$\log_2$(gates)	**168**	**176**
$\log_2$(memory in bits)	**105**	**111**



Volume of $\approx 2^{52}$ microSD cards.

Assume **1 petabyte** per card, then 2^{52} cards hold roughly the amount of memory used in the 2^{168} operation attack on ntruhrss701.

Reaction attacks / Key reuse attacks

Background:

- **Hall–Goldberg–Schneier** (ICICS, 1999): Reaction attacks on McEliece, Hwang–Rao, Ajtai–Dwork, Goldreich–Goldwasser–Halevi. Suggestion that all "closest-point cryptosystems" systems are vulnerable.
- **Hoffstein–Silverman** (NTRU Tech Report 15, 1999): Reaction attacks on NTRU.
- **Jaulmes–Joux** (CRYPTO 2000): Reaction attacks on NTRU with a certain message padding mechanism.

Very well understood. Eliminated by CCA protections, but see fault injection literature.

Reaction attacks / Key reuse attacks

Recent papers, e.g. ePrint 2019/1022, ePrint 2021/168, ...

- Target a "CPA version" of NTRU that is claimed to be interesting for efficiency reasons.
- No data to support the claim that "CPA versions" are significantly more efficient.

Are “CPA versions” efficient?

- ntruhrss701 (CCA-secure)
  Decaps: **58219** cycles
- ntruhrss701 + {improper key} (CCA-secure)
  Decaps: **45580** cycles (-21.7%)
- ntruhrss701 + {improper key, explicit rejection} (CCA-secure)
  Decaps: **34101** cycles (-41.4%)
- ntruhrss701 + {improper key, explicit rejection, no message check} (CPA-secure)
  Decaps: **33891** cycles (-41.8%)

Dropping CCA protections does not significantly improve performance.

Symmetric primitive negotiation

It's annoying that "ntruhrss701" implies the use of SHA3-256 for key derivation.

- Not everyone likes this choice! Google/Cloudflare experiment used SHA256.
- Risk of incompatibility / proliferation of parameter sets. (Magnified because the other KEMs have the same problem.)
- The implicit rejection step is fragile and hard to test.

Symmetric primitive negotiation

Suggestion:

- Unify KDFs for the remaining KEMs.
- Avoid proliferation of parameter sets by defining

  $$\mathrm{PQKem}(\textsf{ntruhrss701}, \textsf{HKDF-SHA256})$$

  like DHKem from HPKE[1] (+ implicit rejection).
- Do this **now**, before picking a winner.

[1] Section 4.1 of https://www.ietf.org/archive/id/draft-irtf-cfrg-hpke-09.html

Alternative key derivation

Current construction:

1. (ok, msg) = Decaps(c, sk1)
2. k1 = H(msg)
3. k2 = H(sk2 ++ c)
4. output = k1 if ok else k2

Alternative:

1. (ok, msg) = Decaps(c, sk1)
2. ikm = msg `if` ok `else` sk2
3. k1 = KDF(ok ++ ikm, kem_context)

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- Requires length('sk2') = length('msg').
- kem_context includes c.
- Implementation should follow Sec. 15 of Bernstein–Persichetti 2018.

Which parameter set should I use?

Use ntruhrss701 or ntruhps2048677.

- Default to ntruhrss701.
- Consider ntruhps2048677 if
  1. you understand the cost of constant-time sorting, and
  2. you need the 208 byte savings in key and ciphertext sizes $(1138 \to 930)$.

Why NTRU?

- The performance is excellent.
- There's no performance benefit to skipping CCA protections.
- The choice of symmetric primitives is limited to the choice of KDF.
- The size/security tradeoffs are very good.
- You don't have to think about decryption failures.
- You don't have to think about patents.

Supplemental material

“Beyond Core-SVP” estimate

	ntruhps2048509	ntruhps4096821
lattice attack dim. d	955	1526
BKZ-blocksize	364	615
d_{4f}	35	50
sieving dimension	329	565
$\log_2$(gates)	**139**	**209**
$\log_2$(memory in bits)	**84**	**134**