On the Security of COMET Authenticated Encryption Scheme

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Lightweight Authenticated Encryption Design

- Block cipher based.
- Rate-1.
- Small state size (close to \((n + \kappa)\)-bit).
- Simple design (simple operations like XOR, shifts and rotations).
Design Summary

- **Rate-1** and **Feedback**-based authenticated encryption mode.
- **Combined feedback** function:
  
  input is a function of current output and next plaintext block.

- **Nonce** and block counter-based **rekeying**.
- Parametrized by the block size, $n \in \{64, 128\}$. Tag size $t = n$.
- Two variants:
  - **COMET-128**: Here $n = 128$, key size $\kappa = 128$, nonce size $r = 128$.
  - **COMET-64**: Here $n = 64$, key size $\kappa = 128$, nonce size $r = 120$. 
Nonce-based Initial State Derivation

- For COMET-128:
  \[(Y_0, Z_0) := (K, IC_K(N))\]

- For COMET-64:
  \[(Y_0, Z_0) := (IC_K(0), K \oplus N||0^{32})\]

![Diagram of COMET-128 and COMET-64](image)
COMET: High-level Overview

Associated Data Processing

Here,
\[ 0 \leq i \leq a - 3 \]

\[ \text{ctrl}_{ad} = \begin{cases} 1 & \text{if } |A| > 0, \\ 0 & \text{otherwise} \end{cases} \]

\[ \text{ctrl}_{p, ad} = \begin{cases} 1 & \text{if } |A_{a-1}| < n, \\ 0 & \text{otherwise} \end{cases} \]
Ciphertext processing is symmetrically defined.
Tag Generation

Here,
\[ \ell = a + m \]

\[ \text{ctrl}_{tg} = \begin{cases} 
1 & \text{for tag generation,} \\
0 & \text{o.w.} 
\end{cases} \]
Design Features

- **Design simplicity**: Only requires shift and XOR operations apart from block cipher calls.
- **Small state size**: Possibility of close to $(n + \kappa)$-bit state size in area optimized implementation.
- **Inverse free**: No need for block cipher decryption.
- **Dynamic key updation**: No two blocks share the same key non-trivially.
- **Efficiency**: Single-pass scheme.
Submissions to NIST LwC Standardization Project

- COMET-128_AES-128/128 instantiated with AES-128/128. [Primary]
- COMET-64_Speck-64/128 instantiated with Speck-64/128.
- COMET-64_CHAM-64/128 instantiated with CHAM-64/128.
## COMET: Security Claims

<table>
<thead>
<tr>
<th>Submissions</th>
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<tbody>
<tr>
<td></td>
<td>Time</td>
<td>Data (in bytes)</td>
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<tr>
<td>COMET-128_AES-128/128</td>
<td>$2^{119}$</td>
<td>$2^{64}$</td>
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<td>COMET-128_CHAM-128/128</td>
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We focus on the **security of COMET-128**.
COMET-128: Security Model

AEAD Security Game

- Indistinguishability game between the ideal system $O_0$ and real system $O_1$, where

$$
O_0 := (\$, \bot, IC^\pm) \quad O_1 := (\text{COMET-128}.E_K, \text{COMET-128}.E_K, IC^\pm).
$$

- Advantage of any adversary $A$ against COMET-128 is defined as:

$$
\text{Adv}_{\text{COMET-128}}^{\text{aead}}(A) := \left| \Pr[A^{O_1} = 1] - \Pr[A^{O_0} = 1] \right|.
$$

- $A$ is computationally unbounded, but bounded in number of queries to its oracle.

- $A$ operates under two restrictions:
  - Nonce-respecting: No two encryption query share the same nonce.
  - Non-trivial forger: An encryption query $(N, A, M)$ yields $(C, T)$, a decryption query $(N, A, C, T)$ is not allowed.
COMET-128: Security Result

Theorem

For $\sigma_e, \sigma_d < 2^{127}$, $q_p < 2^{127}$, and $(q_e, q_d, \sigma_e, \sigma_d, q_p)$-adversary $\mathcal{A}$ we have

$$\text{Adv}_{\text{COMET-128}}^{\text{aead}}(\mathcal{A}) \leq \frac{4\sigma_c^2}{2^{256}} + \frac{14\sigma_c q_p}{2^{249}} + \frac{3\sigma_c^2}{2^{128}} + \frac{3.01 q_p}{2^{121}} + \frac{4\sigma_c}{2^{128}} + \frac{q_c}{2^{64}} + \frac{6 q_p \sigma_d}{2^{188.5}}$$

- $q_e$ and $q_d$ denote the number of queries to COMET-128.E$_K$ and COMET-128.D$_K$, respectively.
- $\sigma_e$ and $\sigma_d$ denote the sum of input (associated data and message) lengths across all encryption and decryption queries, respectively; $q_c = q_e + q_d$ and $\sigma_c = \sigma_e + \sigma_d$.
- $q_p$ denotes the number of direct queries to the block cipher.
Proof tool: Coefficient-H Technique

- Concentrates on the query-response tuple, called the transcript, generated by $A$’s interaction with the oracle at hand.
- Let $\Theta_1$: transcript random variable corresponding to $O_1$.
- Let $\Theta_0$: transcript random variable corresponding to $O_0$.
- Identify a set of bad transcripts, $\Omega_{\text{bad}}$.
- Compute $\Pr[\Theta_0 \in \Omega_{\text{bad}}] \leq \epsilon_{\text{bad}}$.
- Show that $\frac{\Pr[\Theta_1 = \omega]}{\Pr[\Theta_0 = \omega]} \geq (1 - \epsilon_{\text{ratio}})$ for all $\omega \notin \Omega_{\text{bad}}$.
- Then, $\text{Adv}^{\text{aead}}_{\text{COMET-128}}(A) \leq \epsilon_{\text{bad}} + \epsilon_{\text{ratio}}$. 
COMET-128 : Security Proof Sketch

Notational Conventions

- Variables in encryption queries are defined as per the figures.
- Variables in decryption queries are defined analogously, topped with a bar.
- Variables in primitive queries are defined analogously, topped with a hat.

Oracle description

- Real oracle: Faithfully responds to encryption, decryption and primitive queries.
- Ideal oracle:
  
  For the encryption query: samples $X_1, \ldots, X_\ell, T \leftarrow \{0, 1\}^n$, and sets $(Y_j, C_j) = \mathcal{O}(X_{a+j+1}, M_j)$ for all $0 \leq j \leq m$. Sets $Y_j = X_j \oplus A_j$ for $1 \leq j \leq a$. Returns $(C, T)$.
  
  For decryption query: Returns $\perp$ symbol.
  
  For primitive query: Responds faithfully using $IC^\pm$.

- After the query phase, both the oracles release all encryption query internal variables and the secret key.
Identifying bad events

- **Kcoll (key guessing/recovery):**
  
  B1: \( \exists i \in [q_e], j \in [m^i], \) such that \( Z^i_j = K. \)
  
  B2: \( \exists i \in [q_d], j \in [\tilde{m}^i], \) such that \( \tilde{Z}^i_j = K. \)
  
  B3: \( \exists i \in [q_p], \) such that \( \tilde{Z}^i = K. \)
  
  B4: \( \exists i \in [q_e], \) such that \( Z_0^i = * || 0^{n/2}. \)
  
  B5: \( \exists i \in [q_d], \) such that \( \tilde{Z}_0^i = * || 0^{n/2}. \)
  
  B6: \( \exists (i, j) \in [q_e] \times [m^i], (i', j') \in [q_d] \times [\tilde{m}^{i'}], \) such that \( N^i \neq \tilde{N}^{i'} \) and \( Z^i_j = \tilde{Z}^{i'}_{j'}. \)

- **EEmatch (encryption-encryption state matching):**
  
  B7: \( \exists (i, j) \in [q_e] \times [m^i], (i', j') \in [q_e] \times [m^{i'}], \) such that \( (Z^i_j, Y^i_j) = (Z^{i'}_{j'}, Y^{i'}_{j'}). \)
  
  B7: \( \exists (i, j) \in [q_e] \times [m^i], (i', j') \in [q_e] \times [m^{i'}], \) such that \( (Z^i_j, X^i_j) = (Z^{i'}_{j'}, X^{i'}_{j'}). \)
Identifying bad events

- **EP\text{match}** (encryption-primitive state matching):
  
  B9: \( \exists (i, j) \in [q_e] \times [m'] \) and \( i' \in [q_p] \), such that \((Z_j^i, Y_j^i) = (\hat{Z}_{i'}, \hat{Y}_{i'})\).
  
  B10: \( \exists (i, j) \in [q_e] \times [m'] \) and \( i' \in [q_p] \), such that \((Z_j^i, X_j^i) = (\hat{Z}_{i'}, \hat{X}_{i'})\).

- **EP\text{Kcoll}** (technical requirement: key exhaustion via primitive query):
  
  B11: \( \exists (i, j) \in [q_e] \times [m'] \) such that \(|\{j \in [q_p] : \hat{Z}_j = Z_i^i\}| \geq 2^{n-1}\).
Identifying bad events

- Chain (valid forgery via primitive (and encryption) queries):
  Let \(\text{domain}(\omega_p) := \{(\hat{Z}_i, \hat{Y}_i)\}_{i \in [q_p]}\) and \(\text{range}(\omega_p) := \{(\hat{Z}_i, \hat{X}_i)\}_{i \in [q_p]}\).

\[
\delta_i := \begin{cases} 
\max_{\bar{c}_0 \ldots k_1 - c_{0 \ldots k_1}} (\bar{a}_i + k) & \text{if } \bar{A}_i = A_i \land (\bar{A}_i, \bar{C}_i) \neq (A_i, C_i) \\
\max_{\bar{A}_0 \ldots k_1 - A_{0 \ldots k_1}} (k) & \text{otherwise.}
\end{cases}
\]

\[
\delta'_i := \begin{cases} 
\max_{\bar{x}_{i+1} \ldots j} (j) & \text{if } \bar{X}_i^{j+1} \in \text{range}(\omega_p) \\
\delta_i & \text{otherwise.}
\end{cases}
\]

B12: chain using primitive queries
\[\exists i \in [q_d] \text{ such that } \delta_i \geq 0, \delta'_i = \bar{\ell}^i \text{ and } \bar{X}^{i+1}_{\bar{\ell}_i} = \bar{T}^i.\]

B13: partial chain using primitive queries followed by encryption query
\[\exists i \in [q_d], (i', j') \in [q_e] \times [m''] \text{ such that } 0 \leq \delta_i < \delta'_i < \bar{\ell}^i \text{ and } (\bar{Z}^{i'}_{\delta'_i}, \bar{Y}^{i'}_{\delta'_i}) = (Z^i_{j'}, Y^i_{j'}).\]
Bounding $\Pr [\Theta_0 \in \Omega_{bad}]$

- $\Pr [K_{col1}]$: using the fact that $K \leftarrow \{0, 1\}^\kappa$
  \[
  \Pr [B1] \leq \frac{\sigma_e}{2^\kappa}; \quad \Pr [B2] \leq \frac{\sigma_d}{2^\kappa}; \quad \Pr [B3] \leq \frac{q_p}{2^\kappa}.
  \]
  \[
  \Pr [B4 | \neg B3] \leq \frac{q_e}{2^{n/2}}; \quad \Pr [B5 | \neg B3] \leq \frac{q_d}{2^{n/2}}; \quad \Pr [B6] \leq \frac{\sigma_e \sigma_d}{2^\kappa}.
  \]

- $\Pr [E_{E\text{match}} | \neg K_{col1}]$: using the fact that $K \leftarrow \{0, 1\}^\kappa$ and $X_j, X'_j \leftarrow \{0, 1\}^n$.

  \[
  \Pr [B7] \leq \frac{\sigma_e^2}{2^{n+\kappa}}; \quad \Pr [B8] \leq \frac{\sigma_e^2}{2^{n+\kappa}}.
  \]
Bounding \( \Pr[\Theta_0 \in \Omega_{\text{bad}}] \)

- \( \Pr[\text{EPmatch}|\neg K_{\text{coll}}] \):
  
  - Primitive query occurs before encryption query:
    
    \[
    \Pr[\text{EPmatch}|\neg K_{\text{coll}}] \leq 2q_p\sigma_e/2^{n+\kappa}.
    \]

  - Primitive query after encryption query:
    
    Let, \( m_{\text{coll}}(x) := |\{X_j^i = x : (i,j) \in [q_e] \times [m']\}| \) and \( M_{\text{coll}} \) denote the event \( \max_x m_{\text{coll}}(x) \geq n \). Then,
    
    \[
    \Pr[\text{EPmatch}|\neg K_{\text{coll}}] \leq \Pr[M_{\text{coll}}] + \Pr[\text{EPmatch}|(K_{\text{coll}} \lor M_{\text{coll}})]
    \leq \frac{\sigma_e}{2^{n-1}} + \frac{2nq_p}{2^\kappa}.
    \]

- \( \Pr[\text{EPK}_{\text{coll}}] \): using the fact that the number of keys which are repeated in primitive queries at least \( 2^{n-1} \) times is at most \( q_p/2^{n-1} \).

  \[
  \Pr[\text{EPK}_{\text{coll}}] \leq \frac{2\sigma_e q_p}{2^{n+\kappa}}.
  \]
Bounding $\Pr[\Theta_0 \in \Omega_{\text{bad}}]$:

- $\Pr[\text{Chain} | \neg(K_{\text{coll}} \lor E_{\text{Ematch}} \lor E_{\text{Pmatch}})]$:

  Using graph-based analysis (similar to Beetle).

  Let $G_{\omega_p} = (V, E)$ be an edge-labeled graph where $V = \text{domain}(\omega_p)$ and $((\hat{Z}_j, \hat{Y}_i), (\hat{Z}_j, \hat{Y}_j), C^*) \in E$ if and only if

  $$(\hat{Z}_j, \hat{Y}_j) = (IC_{\hat{Z}_i}(\hat{Y}_i), IC_{\hat{Z}_i}(\hat{Y}_i) \oplus C^*)$$

  A walk $\mathcal{W}$ from vertex $W_0$ to $W_k$ with label $C = (C_1, \ldots, C_k)$, denoted $W_0 \xrightarrow{C} W_k$, is

  $$W_0 \xrightarrow{C_1} W_1 \cdots W_{k-1} \xrightarrow{C_k} W_k.$$
**Bounding** \( \Pr [\Theta_0 \in \Omega_{bad}] \)

- \( \Pr [\text{Chain} \mid \neg (K_{\text{coll}} \lor E_{\text{Ematch}} \lor E_{\text{Pmatch}})] \):

  A multi-chain with label \( C = (C_1, \ldots, C_k) \), denoted \( C_C \), is a set of labeled walks \( \{\mathcal{W}_1, \ldots, \mathcal{W}_s\} \) such that for all \( 1 \leq i \leq s \),

  \[
  \mathcal{W}_i : (\hat{Z}_0^i, \hat{Y}_0^i) \xrightarrow{C} (\hat{Z}_k^i, \hat{Y}_k^i) \land \hat{Y}_0^1 = \cdots = \hat{Y}_0^s \land \hat{X}_{k+1}^1 = \cdots = \hat{X}_{k+1}^s.
  \]

\[
\begin{align*}
\mathcal{W}_1 & : (\hat{Z}_0^1, \hat{Y}_0^1) \xrightarrow{C_1} (\hat{Z}_1^1, \hat{Y}_1^1) \xrightarrow{C_2} (\hat{Z}_2^1, \hat{Y}_2^1) \xrightarrow{C_3} (\hat{Z}_3^1, \hat{Y}_3^1) \xrightarrow{C_4} (\hat{Z}_4^1, \hat{Y}_4^1) \xrightarrow{\text{IC}} \hat{X}_5^1 \\
\mathcal{W}_2 & : (\hat{Z}_0^2, \hat{Y}_0^2) \xrightarrow{C_1} (\hat{Z}_1^2, \hat{Y}_1^2) \xrightarrow{C_2} (\hat{Z}_2^2, \hat{Y}_2^2) \xrightarrow{C_3} (\hat{Z}_3^2, \hat{Y}_3^2) \xrightarrow{C_4} (\hat{Z}_4^2, \hat{Y}_4^2) \xrightarrow{\text{IC}} \hat{X}_5^2 \\
& \vdots \\
\mathcal{W}_s & : (\hat{Z}_0^s, \hat{Y}_0^s) \xrightarrow{C_1} (\hat{Z}_1^s, \hat{Y}_1^s) \xrightarrow{C_2} (\hat{Z}_2^s, \hat{Y}_2^s) \xrightarrow{C_3} (\hat{Z}_3^s, \hat{Y}_3^s) \xrightarrow{C_4} (\hat{Z}_4^s, \hat{Y}_4^s) \xrightarrow{\text{IC}} \hat{X}_5^s
\end{align*}
\]

\[
\Pr [B_{11} \mid \neg (K_{\text{coll}} \lor E_{\text{Ematch}} \lor E_{\text{Pmatch}})] \leq \sum_{i \in [q]} \Pr \left[ |C_{\delta_i} \bar{m}_i| \geq \lambda_i \right] + \frac{\lambda_i}{2^{\kappa}}.
\]
COMET-128 : Security Proof Sketch

Bound on $\Pr \left[ |C_{\delta_{i}\ldots m_{i}}| \geq \lambda_{i} \right]$ and $\lambda_{i}$

- Three ways to construct a multi-chain structure:
  - Forward-only: all queries of the form $(\hat{Z}_{i}, \hat{Y}_{i})$.
    \[
    \Pr \left[ C_{\text{fwd}} \geq n \left\lfloor \frac{q_{P}}{2^{n}} \right\rfloor \right] \leq \frac{1}{2^{n}},
    \]
    (by bounding the multicollisions on $\hat{X}_{j}$)
  - Backward-only: all queries of the form $(\hat{Z}_{i}, \hat{X}_{i})$.
    \[
    \Pr \left[ C_{\text{bck}} \geq n \left\lfloor \frac{q_{P}}{2^{n}} \right\rfloor \right] \leq \frac{1}{2^{n}}.
    \]
    (by bounding the multicollisions on $\hat{Y}_{j}$)
  - Both forward and backward type queries: reduced to multicollision event at some index $1 \leq i \leq \bar{i}$ (using Pigeonhole-principle).
    \[
    \Pr \left[ C_{\text{fwd-bck}} \geq \bar{i} \frac{2\sqrt{n}q_{P}}{2^{n/2}} + \frac{2q_{P}}{2^{n}} \right] \leq \frac{1}{2^{n}}.
    \]

- $\Pr \left[ |C_{\delta_{i}\ldots m_{i}}| \geq \bar{i} \frac{2\sqrt{n}q_{P}}{2^{n/2}} + 2n \left\lfloor \frac{q_{P}}{2^{n}} \right\rfloor + \frac{2q_{P}}{2^{n}} \right] \leq \frac{3}{2^{n}}$. 

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COMET-128 : Security Proof Sketch

- $\Pr [B_{11}|\neg(K\text{coll} \lor E\text{Ematch} \lor E\text{Pmatch})] \leq \frac{2\sqrt{n}\sigma_d q_p}{2^{\kappa+n/2}} + \frac{2nq_d}{2^\kappa} \left[ \frac{q_p}{2^n} \right] + \frac{2q_d q_p}{2^{n+\kappa}} + \frac{3q_d}{2^n}.$

- $\Pr [B_{12}|\neg(K\text{coll} \lor E\text{Ematch} \lor E\text{Pmatch})]$ can be bounded in a similar fashion.

$$\Pr [B_{12}|\neg(K\text{coll} \lor E\text{Ematch} \lor E\text{Pmatch} \lor B_{11})] \leq \frac{2\sqrt{n}\sigma_d q_p}{2^{\kappa+n/2}} + \frac{2nq_d}{2^\kappa} \left[ \frac{q_p}{2^n} \right] + \frac{2q_d q_p}{2^{n+\kappa}} + \frac{3q_d}{2^n}.$$

Finally, $\Pr [\text{Chain}|\neg(K\text{coll} \lor E\text{Ematch} \lor E\text{Pmatch})] \leq \frac{6\sqrt{n}\sigma_d q_p}{2^{\kappa+n/2}} + \frac{6nq_d}{2^\kappa} \left[ \frac{q_p}{2^n} \right] + \frac{4q_d q_p}{2^{n+\kappa}} + \frac{6q_d}{2^n}.$
Good transcript analysis

Given any good transcript $\omega$:

$$\frac{\Pr[\Theta_1 = \omega]}{\Pr[\Theta_0 = \omega]} \geq \left( 1 - \frac{2\sigma_d(\sigma_e + q_p)}{2^{\kappa+n}} - \frac{2q_d}{2^n} \right).$$

- First term bounds the probability that for some decryption query $i$ an intermediate input $(\hat{Z}_j, \hat{Y}_j)$ collides with some encryption/primitive input, for $j > \delta_i$.
- The second term bounds the probability that some decryption forgery succeeds given that all intermediate inputs are fresh.

This completes the proof.
Thank you. Questions...
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