

# Supersingular Isogeny Key Encapsulation

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<https://sike.org>

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## Supersingular Isogeny **K**ey **E**ncapsulation (SIKE)

- ▶ IND-CCA2 KEM
- ▶ Based on **S**upersingular **I**sogeny **D**iffie-**H**ellman (SIDH)
- ▶ Uses Hofheinz et al. transformation (TCC 2017) on SIDH to achieve CCA security

The SIKE protocol specifies:

- ▶ Parameter sets
- ▶ Key/ciphertext formats
- ▶ Encapsulation/decapsulation mechanisms
- ▶ Choice of symmetric primitives (hash functions, etc.)

# Overview of SIDH

1. Public parameters: Supersingular elliptic curve  $E$  over  $\mathbb{F}_{p^2}$ .
2. Alice chooses a kernel  $A \subset E(\mathbb{F}_{p^2})$  and sends  $E/A$  to Bob.
3. Bob chooses a kernel  $B \subset E(\mathbb{F}_{p^2})$  and sends  $E/B$  to Alice.
4. The shared secret is

$$E/\langle A, B \rangle = (E/A)/\phi_A(B) = (E/B)/\phi_B(A).$$

Diffie-Hellman (DH)

$$\begin{array}{ccc} g & \xrightarrow{\quad} & g^x \\ \downarrow & & \downarrow \\ g^y & \xrightarrow{\quad} & g^{xy} \end{array}$$

SIDH

$$\begin{array}{ccc} E & \xrightarrow{\phi_A} & E/A \\ \phi_B \downarrow & & \downarrow \\ E/B & \xrightarrow{\quad} & E/\langle A, B \rangle \end{array}$$

# Changes for SIKE in second round

- ▶ New parameter sets: **SIKEp434**, SIKEp503, **SIKEp610**, SIKEp751, ~~SIKEp964~~
- ▶ New starting curve  $E : y^2 = x^3 + 6x^2 + x$
- ▶ Key compression:  $\approx 40\%$  smaller public keys and ciphertexts
- ▶ Updated security analysis

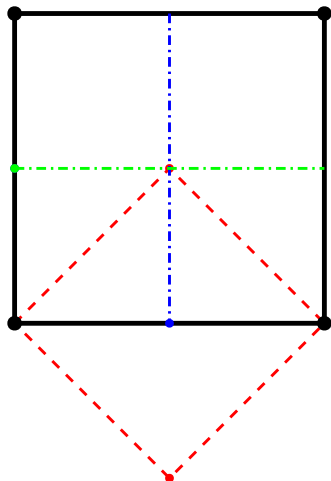
# Parameter sets

Scheme	prime $p$	$\log_2 p$	Security level
SIKEp434	$2^{216}3^{137} - 1$	433.14	NIST 1
SIKEp503	$2^{250}3^{159} - 1$	502.01	NIST 2
SIKEp610	$2^{305}3^{192} - 1$	609.31	NIST 3
SIKEp751	$2^{372}3^{239} - 1$	750.81	NIST 5

# New starting curve

The previous starting curve  $y^2 = x^3 + x$  has complex multiplication symmetries, reducing key entropy.

- ▶ **Red** kernel point yields curve isomorphic to starting curve.
- ▶ **Blue** and **green** kernel points yield curves isomorphic to each other.



# Key compression

Scheme	Public key	Decaps ( $\times 86\_64$ )
SIKEp434	330 bytes	$11.3 \times 10^6$ cc
SIKEp434_compressed	196 bytes	$18.9 \times 10^6$ cc
SIKEp503	378 bytes	$15.6 \times 10^6$ cc
SIKEp503_compressed	224 bytes	$25.5 \times 10^6$ cc
SIKEp610	462 bytes	$28.6 \times 10^6$ cc
SIKEp610_compressed	273 bytes	$45.5 \times 10^6$ cc
SIKEp751	564 bytes	$45.4 \times 10^6$ cc
SIKEp751_compressed	331 bytes	$72.8 \times 10^6$ cc

# Security analysis

Attack cost	SIKEp434			SIKEp610		
	$G$	$D$	$W$	$G$	$D$	$W$
Grover [1]	126	116	10	171	160	10
Tani (optimal $\#G$ ) [2]	124	114	25	169	159	25
Tani (optimal $D \times W$ ) [2]	131	122	10	177	166	10
Van Oorschot-Wiener [2]	132	14	128	177	14	173

1. *A framework for reducing the overhead of the quantum oracle for use with Grover's algorithm with applications to cryptanalysis of SIKE*, Benjamin I. Pring and Jean-François Biasse, MathCrypt 2019
2. *Quantum cryptanalysis in the RAM model: Claw-finding attacks on SIKE*, Sam Jaques and John Schanck, CRYPTO 2019



## Recent implementations

Decapsulation times, cc $\times 10^6$	SIKEp503	SIKEp751
ARM64 (NIST 2nd round)	47.4	159.5
ARM64 [1]	39.7	138.4
Cortex M4 [2]	183	491

1. *ARMv8 SIKE: Optimized Supersingular Isogeny Key Encapsulation on ARMv8 Processors*, Amir Jalali, Reza Azarderakhsh, Mehran Mozaffari Kermani, Matthew Campagna, and David Jao, IEEE TCAS, 10.1109/TCSE.2019.2920869. Code available at <https://github.com/amirjalali65/armv8-sike>
2. *SIKE Round 2 Speed Record on ARM Cortex-M4*, Hwajeong Seo, Amir Jalali, and Reza Azarderakhsh, 2019/535.

# Summary

## SIKE advantages:

- ▶ Smallest public key size
- ▶ Straightforward parameter selection
- ▶ No decryption error, Gaussians, rejection sampling, etc.
- ▶ Generic attacks are well understood
- ▶ Only KEM proposal not based on lattices / codes / LW[ER]

## SIKE disadvantages:

- ▶ Slow
- ▶ Future analysis may uncover non-generic attacks against SIKE (though none are known so far)

## Future work:

- ▶ Cryptanalysis and side-channel attacks