From: Alperin-Sheriff, Jacob (Fed)
Sent: Friday, January 12, 2018 12:51 PM
To: pqc-comments
Cc: pqc-forum@list.nist.gov
Subject: OFFICIAL COMMENT: LAC

LAC Team:

Your submission seems to incorrectly claim a connection to worst-case hardness problems for ideal lattices. The connection only applies to error distributions (statistically close enough to) the distributions for alpha described in Section 3 of LPR10 (https://eprint.iacr.org/2012/230.pdf), and that requires alpha * q >= omega(\sqrt{log n).

The {-1,0,1} distributions for error you used are not statistically close to any such distribution.

—Jacob Alperin-Sheriff

From: Jacob Alperin-Sheriff <jacobmas@gmail.com>
Sent: Sunday, January 14, 2018 11:38 AM
To: pqc-comments
Cc: pqc-forum
Subject: OFFICIAL COMMENT: LAC

LAC Team:

I believe there is an attack that reduces the security of your scheme by (close to) a factor of 2 in the exponent from the estimates given in your submission.

The key point is that for n=2^k, k>=2

x^{n}+1 = (x^(n/2) + 91*x^(n/4) - 1)(x^(n/2) - 91*x^(n/4) - 1) mod 251
(note that x^n+1 is reducible modulo any prime p when k>=2, and for some choices it reduces to even smaller polynomials that may be more devastating).

The aggressive choice of error distribution {-1,0,1} used in LAC appears to then yield an attack that essentially combines the observations/framework in Section 3.2 of "How (Not) to Instantiate Ring-LWE" by Peikert https://eprint.iacr.org/2016/351.pdf with algorithms for solving SVP.

Let g=x^(n/2)+91*x^(n/4)-1, h=x^(n/2)-91*x^(n/4)-1, and let Id_g, Id_h be the associated ideals generated by g and 251 (respectively, h and 251)

Then for e a degree at most n polynomial with coefficients in (-1,0,1), the coefficients of e reduced modulo the ideal generated by g and 251

(i.e. e mod Id_g)

will all be of the form

+-{0,1,2} +- {0,91}

(and similarly for h)

Let (a,b=s*a+e (mod R_q) be the public key for LAC.

Now, let a_g = a mod Id_g, b_g = b mod Id_g
a_h = a mod Id_h, b_h = b mod Id_h

Then I believe (I haven't checked yet with easily feasible n (like n=64), will hopefully do next week) that the shortest vector x=(x_1,x_2,x_3,x_4)

solution to the (ring)-I-SIS problem

[a_g, 91*a_g, 1, 91]x = b_g

should be such that x_1+91*x_2 = s (mod Id_g), x_3+91*x_4=e (mod Id_g), and similarly for the case of h.

Assuming this is true, recovering s and e reduces to solving SVP for 2 instances of (n/2) dimensional lattices and then Chinese remainder theorem-ing to recover s and e.

I will try to implement this some time this week in Sage for an easily feasible n to see if I'm correct about the form of the shortest vector.

--
-Jacob Alperin-Sheriff

From: Jacob Alperin-Sheriff <jacobmas@gmail.com>
Sent: Sunday, January 14, 2018 2:40 PM
To: pqc-comments
Cc: pqc-forum
Subject: Re: OFFICIAL COMMENT: LAC

Hi everybody,

Vadim Lyubashevsky pointed out that I made a stupid mistake here about the dimensions of the resulting lattice so disregard everything above except (if it helps) the form of the error modulo the ideals.

On Jan 14, 2018 11:38 AM, "Jacob Alperin-Sheriff" <jacobmas@gmail.com> wrote:

> LAC Team:
>
> I believe there is an attack that reduces the security of your scheme by (close to) a factor of 2 in the exponent from the estimates given in your submission.
>
> The key point is that for n=2^k, k>=2
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>
> Then I believe (I haven't checked yet with easily feasible n (like n=64), will hopefully do next week) that the shortest vector x=(x_1,x_2,x_3,x_4)

From: Alperin-Sheriff, Jacob (Fed)
Sent: Thursday, January 18, 2018 10:59 AM
To: pqc-comments
Cc: pqc-forum@list.nist.gov
Subject: OFFICIAL COMMENT: LAC

Hi all,

New idea on LAC.

Remember, the ring is x^512+1, q=251, error distribution is +- with probability 1/4, 0 with probability 1/2 in LAC.

I noticed (well I had the general idea and then used Sage to check all the possible multipliers) that if you multiply s and e by 11 (so the error distribution becomes \pm 11 each with probability 1/4, 0 with probability 1/2) then each of the error coefficients e_i will be at most +- 25 after reducing modulo g=x^(n/2)+91*x^(n/4)-1, specifically in the distribution

[0, 3, 8, 11, 14, 19, 22, 25, 226, 229, 232, 237, 240, 243, 248], and similarly for s
Moreover, the smaller values are more likely, for each individual coefficient they are distributed with probability {0: 0.111, 3: 0.111, 8: 0.074, 11: 0.074, 14: 0.074, 19: 0.037, 22: 0.037, 25: 0.037, 226: 0.037, 229: 0.037, 232: 0.037, 237: 0.074, 240: 0.074, 243: 0.074, 248: 0.111}

I haven't attempted to prove anything yet for the values of n that are of interest, but exhaustively computing for w=8 reveals that with probability at least .5, the Euclidean norm of the coefficients of e mod g [e being the error], will be at most 21,

For 5000 random samples in the w=512 case, the Euclidean norm
of the coefficient vector of ((11*e) mod g) was 207 or less with probability .5.

So, assuming I didn't make another stupid mistake again, the question is then whether this is going to be short enough that z=(11*s (mod g) ,11*e (mod g),-1)
will be the shortest solution of

Az=0

where A= [(a)mod g | 1 | (11*b) mod g] is a 256 x 513

By the above bound on the coefficient vector (11*e), we can get the (11*s (mod g), 11*e (mod g), -1) should be at most 292.75 with probability at least .25.

Lemma 5.2 in Micciancio-Regev's "Worst-case to Average-case Reductions based on Gaussian Measures" https://pdfs.semanticscholar.org/9935/ac933507b63a2d6eac41c87aa4ab4835be52.pdf gives that a solution **has** to exist of norm at least 356.9 in this kind of a SIS instance, but I'm not so up on how tight that bound is and if we can hope that (11*s (mod g), 11 * e(mod g), -1) will be the shortest vector.

I did try something in Sage for smaller values of n, but the results were inconclusive (BKZ didn't find the desired vector, but the desired vector was also of smaller norm than anything in the BKZ-reduced basis, suggesting that I'm doing something wrong). Code is included below; help would be appreciated.

```
#!/usr/bin/env sage

import sys
from sage.all import *

def sample_single_error():
    t=randint(0,3)
    if t==0:
        return -1
    elif t<3:
        return 0
    return 1

def sample_noise(S):
    n=S.degree()
    ret_lst=[sample_single_error() for i in range(n)]
    return S(ret_lst)

def my_gen_lattice2(n=4, q=11, seed=None,
                quotient=None, dual=False, ntl=False, lattice=False,t=5):
    """
    This is a modification of the code for the gen_lattice function from Sage

    Randomness can be set either with ``seed``, or by using
    :func:`sage.misc.randstate.set_random_seed`.

    INPUT:

    - ``type`` -- one of the following strings
        - ``'cyclotomic'`` -- Special case of ideal. Allows for
          efficient processing proposed by [LM2006]_.
    - ``n`` -- Determinant size, primal:`det(L) = q^n`, dual:`det(L) = q^{m-n}`.
      For ideal lattices this is also the degree of the quotient polynomial.
    - ``m`` -- Lattice dimension, `L \subseteq Z^m`.
    - ``q`` -- Coefficient size, `q-Z^m \subseteq L`.
    - ``t`` -- BKZ Block Size
    - ``seed`` -- Randomness seed.
    - ``quotient`` -- For the type ideal, this determines the quotient
      polynomial. Ignored for all other types.
    - ``dual`` -- Set this flag if you want a basis for `q-dual(L)`, for example
      for Regev's LWE bases [Reg2005]_.
    - ``ntl`` -- Set this flag if you want the lattice basis in NTL readable
      format.
    - ``lattice`` -- Set this flag if you want a
```

```
  :class:`FreeModule_submodule_with_basis_integer` object instead
  of an integer matrix representing the basis.

OUTPUT: ``B`` a unique size-reduced triangular (primal: lower_left,
  dual: lower_right) basis of row vectors for the lattice in question.

EXAMPLES:



Cyclotomic bases with n=2^k are SWIFFT bases::

    sage: sage.crypto.gen_lattice(type='cyclotomic', seed=42)
    [11  0  0  0  0  0  0  0]
    [ 0 11  0  0  0  0  0  0]
    [ 0  0 11  0  0  0  0  0]
    [ 0  0  0 11  0  0  0  0]
    [ 4 -2 -3 -3  1  0  0  0]
    [ 3  4 -2 -3  0  1  0  0]
    [ 3  3  4 -2  0  0  1  0]
    [ 2  3  3  4  0  0  0  1]

Dual modular bases are related to Regev's famous public-key
encryption [Reg2005]_::

    sage: sage.crypto.gen_lattice(type='modular', m=10, seed=42, dual=True)
    [ 0  0  0  0  0  0  0  0  0 11]
    [ 0  0  0  0  0  0  0  0 11  0]
    [ 0  0  0  0  0  0  0 11  0  0]
    [ 0  0  0  0  0  0 11  0  0  0]
    [ 0  0  0  0  0 11  0  0  0  0]
    [ 0  0  0  0 11  0  0  0  0  0]
    [ 0  0  0  1 -5 -2 -1  1 -3  5]
    [ 0  0  1  0 -3  4  1  4 -3 -2]
    [ 0  1  0  0 -4  5 -3  3  5  3]
    [ 1  0  0  0 -2 -1  4  2  5  4]


"""
from sage.rings.finite_rings.integer_mod_ring import IntegerModRing
from sage.matrix.constructor import identity_matrix, block_matrix
from sage.matrix.matrix_space import MatrixSpace
from sage.rings.integer_ring import IntegerRing
if seed is not None:
    from sage.misc.randstate import set_random_seed
    set_random_seed(seed)



ZZ = IntegerRing()
ZZ_q = IntegerModRing(q)

A = identity_matrix(ZZ_q, n/2)


from sage.arith.all import euler_phi
from sage.misc.functional import cyclotomic_polynomial

# we assume that n+1 <= min( euler_phi^{-1}(n) ) <= 2*n
found = False
for k in range(2*n,n,-1):
    if euler_phi(k) == n:
```

```
            found = True
            break
        if not found:
            raise ValueError("cyclotomic bases require that n "
                                 "is an image of Euler's totient function")
    R = ZZ_q['x'].quotient(cyclotomic_polynomial(k, 'x'), 'x')
    g=x**(n/2)+91*x**(n/4)-1
    T=ZZ_q['x'].quotient(x**(n/2)+91*x**(n/4)-1)


    a_pol=R.random_element()

#     for i in range(m//n):
#         print("i={0}".format(i))
#         A = A.stack(R.random_element().matrix())
    s_pol=sample_noise(R)
    e_pol=sample_noise(R)

    s_pol2=T((11*s_pol).list())
    e_pol2=T((11*e_pol).list())

    Z_mat=s_pol2.matrix().augment(e_pol2.matrix())
    Z_mattop=Z_mat[0:1].augment(matrix(1,1,ZZ.one()*-1))


    b_pol=(a_pol*s_pol+e_pol)*11
    print("s_pol={0}\ne_pol={1}".format((s_pol*11).list(),(e_pol*11).list()))

    a_pol2 = T(a_pol.list())# % S(g.list())
    b_pol2 = T(b_pol.list())# % S(g.list())
#     print("a={0}\nb={1}".format(a_pol,b_pol))

    A=a_pol2.matrix().stack(A)
    A=A.stack(b_pol2.matrix()[0:1])

    print("{0}\n".format(A))
    # switch from representatives 0,...,(q-1) to (1-q)/2,....,(q-1)/2
    def minrepnegative(a):
        if abs(a-q) < abs(a): return (a-q)*-1
        else: return a*-1
    def minrep(a):
        if abs(a-q) < abs(a): return (a-q)
        else: return a
    A_neg = A[0:(n/2)].lift().apply_map(minrepnegative)
    b_neg= A[(n):(n+1)].lift().apply_map(minrepnegative)
    Z_fixed=Z_mattop.lift().apply_map(minrep)
    print("Z_fixed={0}\n||Z_fixed||={1}".format(Z_fixed,float(Z_fixed[0].norm())))
    print('Z_fixed*A={0}\n\n'.format(Z_fixed*A))

    #    print("{0}\n".format(A_neg))
    B=block_matrix([[ZZ(q), ZZ.zero(),ZZ.zero()],[ZZ.one(),A_neg,ZZ.zero()
],[ZZ.zero(),b_neg,ZZ.one()]],
                         subdivide=False)
    #print("B=\n{0}".format(B))
    #print("B*A=\n{0}\n\n".format(B*A))

    B_BKZ=B.BKZ(block_size=t,proof=True)

    print("B_BKZ[0]=\n{0}\n".format(B_BKZ[0]))

    for i in range(0,n+1):
        print('{0}: {1}'.format(i,1.*float(B_BKZ[i].norm())))
```

```
if ntl and lattice:
    raise ValueError("Cannot specify ntl=True and lattice=True ")
if ntl:
    return B._ntl_()
elif lattice:
    from sage.modules.free_module_integer import IntegerLattice
    return IntegerLattice(B)
else:
    return B
```

From:	sd lattice
To:	pqc-comments
Cc:	pqc-forum@list.nist.gov
Subject:	OFFICIAL COMMENT: LAC
Date:	Tuesday, January 23, 2018 10:32:58 AM
Attachments:	norms.pdf testpolynomialring.py

Dear Alperin-Sheriff, Jacob

Thank you very much for your comments on LAC.

1: About the connection to worst-case hardness problems: In the design of LAC we considered the requirement that alpha*q>=omega(\sqrt{log n}). The parameters of LAC do not satisfy this requirement directly. However, we notice that, according to the modulus reduction result proposed in [1], varying the dimension n and the modulus q individually while keeping nlogq fixed essentially preserves the hardness of LWE. We estimate the concrete hardness of the RLWE problem by using the method given in [2]. The result shows that the modulus reduction result also works for the hardness of RLWE. Concretely, for the parameter of LAC-128, where q=251,n=512, sigma=1/sqrt(2), the hardness is:

primal attack:
classical cost= 148
quantum cost= 135
dual attack:
classical cost= 147
quantum cost= 133

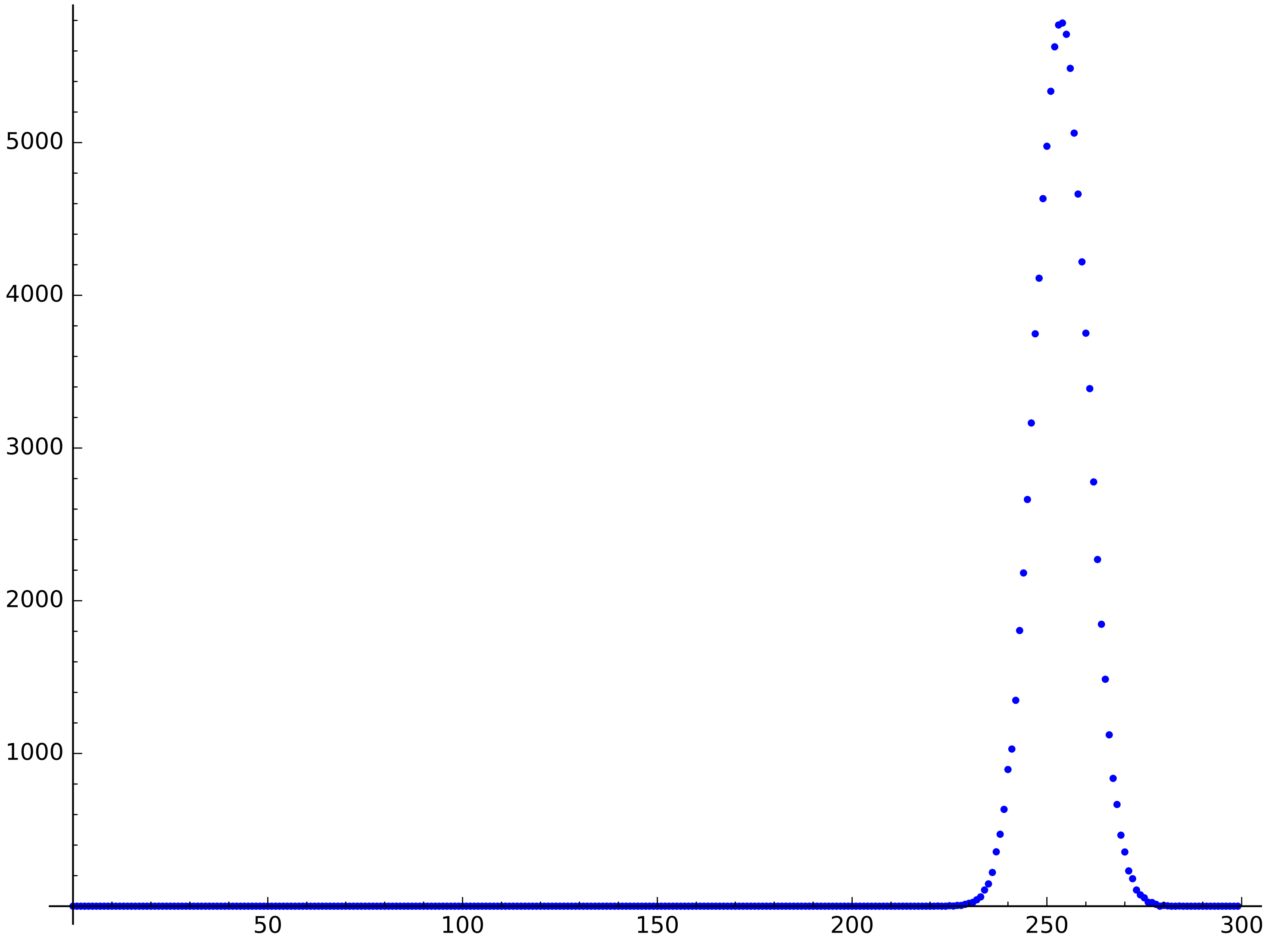
when we set q=251*251, n=256, sigma= 1/sqrt(2) * 251, the hardness is:
primal attack:
classical cost= 156
quantum cost= 141
dual attack:
classical cost= 154
quantum cost= 139
Note that, during the varying of n and q, we need to keep alpha=sigma*sqrt(2*pi)/q fixed, so we set sigma= 1/sqrt(2) * 251. In this case, alpha*q=1/sqrt(2) * 251>omega(\sqrt{log 256}).

2: About the modulo attack. This is really an interesting observation. We used Sage to check this idea, for 100000 random samples, our result shows that the length of z=[11*s mod g, 11*e mod g, -1] is an Gaussian distribution with mean 253.59 and standard deviation 6.9. The sage code and the figure of the distribution can be found in the attachment. According to Micciancio and Regev's "Lattice Based Cryptography",:
lambda_1(\Lambda_q^{\perp})=sqrt(513/(2*pi*e))*251^{256/513}=86.36. It is clear that, the length of vector z is much longer than lambda_1, and the BKZ algorithm cannot be used to find z.

Note that, the bound of Lemma 5.2 in Micciancio-Regev's "Worst-case to Average-case Reductions based on Gaussian Measures" is not tight enough. Concretely, the bound in this lemma is sqrt(513)*251^{256/513}=356.9, which is much larger than 86.36.

[1]Zvika Brakerski, Adeline Langlois, Chris Peikert, Oded Regev, Damien Stehlé: **Classical hardness of learning with errors.** STOC 2013: 575-584
[2]Erdem Alkim, Léo Ducas, Thomas Pöppelmann, Peter Schwabe: **Post-quantum Key Exchange - A New Hope.** USENIX Security Symposium 2016: 327-343
[3] https://cims.nyu.edu/~regev/papers/pqc.pdf

Best Wishes.
LAC Team



5000
4000
3000
2000
1000
50
100
150
200
250
300

From: Alperin-Sheriff, Jacob (Fed)
To: sd lattice; pqc-comments
Cc: pqc-forum@list.nist.gov
Subject: Re: OFFICIAL COMMENT: LAC
Date: Tuesday, January 23, 2018 10:38:34 AM

1. [1] doesn't work for ring-LWE.

2. You're right that my idea definitely doesn't work as written, I've since confirmed that the shortest vectors modulo the ideal are several times shorter than the induced (z,e,-1) vector (it may simply never be helpful in any ring, I'm still investigating that)

Thanks for the response.

From: sd lattice <luxianhui@outlook.com>
Date: Tuesday, January 23, 2018 at 10:33 AM
To: pqc-comments <pqc-comments@nist.gov>
Cc: "pqc-forum@list.nist.gov" <pqc-forum@list.nist.gov>
Subject: OFFICIAL COMMENT: LAC

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From: Christopher J Peikert <cpeikert@alum.mit.edu>
Sent: Tuesday, January 23, 2018 11:03 AM
To: Alperin-Sheriff, Jacob (Fed)
Cc: pqc-comments; pqc-forum@list.nist.gov; sd lattice
Subject: Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

On Tue, Jan 23, 2018 at 10:38 AM Alperin-Sheriff, Jacob (Fed) <jacob.alperin-sheriff@nist.gov> wrote:

> 1. [1] doesn't work for ring-LWE.

To be clear, modulus switching does work for both LWE and Ring-LWE (among other problems), as claimed below. But the results from [1] do not say anything about (Ring-)LWE with *binary errors*, which is apparently what LAC uses. (Gaussians are the only error distributions considered in [1].) It does not appear that LAC is supported asymptotically by any known worst-case hardness theorem.

Sincerely yours in cryptography, Chris

> **From:** sd lattice <luxianhui@outlook.com>
> **Date:** Tuesday, January 23, 2018 at 10:33 AM
> **To:** pqc-comments <pqc-comments@nist.gov>
> **Cc:** "pqc-forum@list.nist.gov" <pqc-forum@list.nist.gov>
> **Subject:** OFFICIAL COMMENT: LAC
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>
> primal attack:
>
> classical cost= 148

From: Alperin-Sheriff, Jacob (Fed)
Sent: Tuesday, January 23, 2018 11:06 AM
To: Christopher J Peikert
Cc: pqc-comments; pqc-forum@list.nist.gov; sd lattice
Subject: Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

LAC uses +-1 with probability 1/4 each, 0 with probability 1/2 so technically not binary. You're right that a better answer would have been things like "Hardness of SIS and LWE with Small Parameters
" https://eprint.iacr.org/2013/069.pdf don't apply to ring-LWE (although it could easily be adapted for the LAC error distribution for the general LWE case).

From: Christopher J Peikert <cpeikert@alum.mit.edu>
Date: Tuesday, January 23, 2018 at 11:02 AM
To: "Alperin-Sheriff, Jacob (Fed)" <jacob.alperin-sheriff@nist.gov>
Cc: pqc-comments <pqc-comments@nist.gov>, "pqc-forum@list.nist.gov" <pqc-forum@list.nist.gov>, sd lattice <luxianhui@outlook.com>
Subject: Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

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Sincerely yours in cryptography, Chris

> ---
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> **From:** sd lattice <luxianhui@outlook.com>
> **Date:** Tuesday, January 23, 2018 at 10:33 AM
> **To:** pqc-comments <pqc-comments@nist.gov>
> **Cc:** "pqc-forum@list.nist.gov" <pqc-forum@list.nist.gov>
> **Subject:** OFFICIAL COMMENT: LAC
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> Dear Alperin-Sheriff, Jacob
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> Thank you very much for your comments on LAC.

From: Christopher J Peikert <cpeikert@alum.mit.edu>
Sent: Tuesday, January 23, 2018 11:16 AM
To: Alperin-Sheriff, Jacob (Fed)
Cc: pqc-comments; pqc-forum@list.nist.gov; sd lattice
Subject: Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

On Tue, Jan 23, 2018 at 11:05 AM Alperin-Sheriff, Jacob (Fed) <jacob.alperin-sheriff@nist.gov> wrote:

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> " https://eprint.iacr.org/2013/069.pdf don't apply to ring-LWE (although it could easily be adapted for the LAC error distribution for the general LWE case).

Yes, that paper gets closer to the mark for plain LWE with binary (or ternary) errors. But to prevent any confusion: it would not support the plain-LWE analogue of LAC either, because the number of samples is limited to smaller than what LAC reveals to the attacker.

Sincerely yours in cryptography, Chris

> **From:** Christopher J Peikert <cpeikert@alum.mit.edu>
> **Date:** Tuesday, January 23, 2018 at 11:02 AM
> **To:** "Alperin-Sheriff, Jacob (Fed)" <jacob.alperin-sheriff@nist.gov>
> **Cc:** pqc-comments <pqc-comments@nist.gov>, "pqc-forum@list.nist.gov" <pqc-forum@list.nist.gov>, sd lattice <luxianhui@outlook.com>
> **Subject:** Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC
>
> On Tue, Jan 23, 2018 at 10:38 AM Alperin-Sheriff, Jacob (Fed) <jacob.alperin-sheriff@nist.gov> wrote:
>
> > 1. [1] doesn't work for ring-LWE.
>
> To be clear, modulus switching does work for both LWE and Ring-LWE (among other problems), as claimed below. But the results from [1] do not say anything about (Ring-)LWE with *binary errors*, which is apparently what LAC uses. (Gaussians are the only error distributions considered in [1].) It does not appear that LAC is supported asymptotically by any known worst-case hardness theorem.
>
> Sincerely yours in cryptography, Chris

From: Alperin-Sheriff, Jacob (Fed)
Sent: Thursday, February 15, 2018 2:23 PM
To: Christopher J Peikert
Cc: pqc-comments; pqc-forum@list.nist.gov; sd lattice
Subject: Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

Coming back to this point "Yes, [Hardness of SIS and LWE with Small Parameters] gets closer to the mark for plain LWE with binary (or ternary) errors. But to prevent any confusion: it would not support the plain-LWE analogue of LAC either, because the number of samples is limited to smaller than what LAC reveals to the attacker."

If I'm understanding everything properly, the paper doesn't say anything at all about the plain LWE analogue of LAC and more generally, the public key in any ring-LWE (or plain LWE) scheme using the Lindner-Peikert formulation of primal Regev, because that paper requires m >= n + omega(log n) in order to use the Micciancio-Mol SIS equivalence, and we have m=n for LAC. Is this correct?

From: Christopher J Peikert <cpeikert@alum.mit.edu>
Date: Tuesday, January 23, 2018 at 11:16 AM
To: "Alperin-Sheriff, Jacob (Fed)" <jacob.alperin-sheriff@nist.gov>
Cc: pqc-comments <pqc-comments@nist.gov>, "pqc-forum@list.nist.gov" <pqc-forum@list.nist.gov>, sd lattice <luxianhui@outlook.com>
Subject: Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

On Tue, Jan 23, 2018 at 11:05 AM Alperin-Sheriff, Jacob (Fed) <jacob.alperin-sheriff@nist.gov> wrote:

> LAC uses +-1 with probability 1/4 each, 0 with probability 1/2 so technically not binary. You're right that a better answer would have been things like "Hardness of SIS and LWE with Small Parameters
>
> " https://eprint.iacr.org/2013/069.pdf don't apply to ring-LWE (although it could easily be adapted for the LAC error distribution for the general LWE case).

Yes, that paper gets closer to the mark for plain LWE with binary (or ternary) errors. But to prevent any confusion: it would not support the plain-LWE analogue of LAC either, because the number of samples is limited to smaller than what LAC reveals to the attacker.

Sincerely yours in cryptography, Chris

> ---
>
> **From:** Christopher J Peikert <cpeikert@alum.mit.edu>
> **Date:** Tuesday, January 23, 2018 at 11:02 AM
> **To:** "Alperin-Sheriff, Jacob (Fed)" <jacob.alperin-sheriff@nist.gov>
> **Cc:** pqc-comments <pqc-comments@nist.gov>, "pqc-forum@list.nist.gov" <pqc-forum@list.nist.gov>, sd lattice <luxianhui@outlook.com>
> **Subject:** Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

From: Mike Hamburg <mike@shiftleft.org>
Sent: Thursday, February 15, 2018 8:39 PM
To: pqc-comments
Cc: pqc-forum@list.nist.gov
Subject: OFFICIAL COMMENT: LAC

Hello LAC authors,

I noticed that the LAC reference and optimized code submissions take variable time, at least for the BCH decoding step.

How difficult is it to fix this to run in constant time? Do you plan to implement a constant-time decoding algorithm so that we can test its performance?

Thanks,
— Mike Hamburg

From: Alperin-Sheriff, Jacob (Fed)
Sent: Friday, February 16, 2018 8:54 AM
To: Mike Hamburg; pqc-comments
Cc: pqc-forum@list.nist.gov
Subject: Re: OFFICIAL COMMENT: LAC

Mike,

I had also noticed that the BCH decoding algorithms are written in a variable time manner, but I ended up forgetting to/putting off mention it here after I did some testing of the times/cycles taken by the BCH decoding step and it didn't seem to vary depending on the number of errors it had to decode.

This doesn't mean I or NIST consider the code or the algorithm "okay" (meaning of no potential concern), just that I wasn't able to find a way that it leaked any useful information and so it slipped my mind.

Did you find something different?

On 2/15/18, 8:39 PM, "Mike Hamburg" <mike@shiftleft.org> wrote:

Hello LAC authors,

I noticed that the LAC reference and optimized code submissions take variable time, at least for the BCH decoding step.

How difficult is it to fix this to run in constant time? Do you plan to implement a constant-time decoding algorithm so that we can test its performance?

Thanks,
— Mike Hamburg

From: Alperin-Sheriff, Jacob (Fed)
Sent: Friday, February 16, 2018 10:07 AM
To: pqc-comments
Cc: pqc-forum@list.nist.gov
Subject: OFFICIAL COMMENT: LAC

LAC authors:

I believe LAC256 falls significantly short of 256 bits of security under the CCA attack model (that allows up to 2^64 chosen ciphertext queries [see pg 15 of the CFP https://csrc.nist.gov/CSRC/media/Projects/Post-Quantum-Cryptography/documents/call-for-proposals-final-dec-2016.pdf]).

A simple calculation of binom(2048,1024+310)/(2**2048) gives that (modeling the hash function as a random oracle) for a message chosen uniformly at random, the vector (r,e_1) in LAC.CPA.Enc (which in the LAC.CCA.Enc gets its "randomness" from the message) will have Hamming weight of at least 1024+310 with probability at least 2^{-143.3}, so with high probability, 2^{-210} or so hash function evaluations should yield at least 2^{64} messages giving such high Hamming weights for (r,e_1) in LAC.CPA.Enc.

After doing this 2^{210}*cost(hash) precomputation (which presumably costs signficantly less than 256 bits), we should be able to recover almost any key with 2^{64} chosen ciphertext queries.

For messages that yield Hamming weights of (r,e_1) of 1024+310, testing gives that each bit fails to be correct with probability 2^{-6.0496}, so a simple Python script that enough bits (56 or more) to cause failure on the entire message will occur with probability ~ 2^{-50.51}. As a result, over 2^{64} message queries we should expect with high probability to get far more than the necessary number of failures to recover s based on a rough application of the analysis in https://eprint.iacr.org/2016/085.pdf

—Jacob Alperin-Sheriff

From: Martin Tomlinson <mt@post-quantum.com>
Sent: Friday, February 16, 2018 10:40 PM
To: pqc-comments
Cc: pqc-forum@list.nist.gov
Subject: OFFICIAL COMMENT: LAC
Attachments: signature.asc

Dear LAC authors,

Although you decided to use BCH codes as the error correcting code in LAC, a better choice is to use binary Goppa codes since the maximum code length of these codes is 2^m and not 2^m-1. There is an additional information bit that can be corrected and no messy padding. A BCH decoder can be used straightforwardly to implement the error correction decoder for Goppa codes as first pointed out by Charles Retter in his 1975 paper: "Decoding Goppa codes with a BCH decoder.", IEEE Transactions on Information Theory, 21(1):112⊥112.

One significant advantage is that you can benefit from the extensive research in the last decade that has been applied to the McEliece system to avoid side channel information leakage in the syndrome calculation, Berlekamp-Massey and root finding algorithms. For example the reference implementations of the NTS-KEM submission or the Classic McEliece submission could be used in LAC with only minor modifications to integrate within the procedures the variability of the parameter t.

Best regards

Martin

From: cpeikert@gmail.com on behalf of Christopher J Peikert <cpeikert@alum.mit.edu>
Sent: Friday, February 16, 2018 10:43 PM
To: Alperin-Sheriff, Jacob (Fed)
Cc: pqc-comments; pqc-forum@list.nist.gov; sd lattice
Subject: Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

On Thu, Feb 15, 2018 at 2:22 PM, Alperin-Sheriff, Jacob (Fed) <jacob.alperin-sheriff@nist.gov> wrote:
> Coming back to this point "Yes, [Hardness of SIS and LWE with Small
> Parameters] gets closer to the mark for plain LWE with binary (or
> ternary) errors. But to prevent any confusion: it would not support
> the plain-LWE analogue of LAC either, because the number of samples is
> limited to smaller than what LAC reveals to the attacker."
>
> If I'm understanding everything properly, the paper doesn't say
> anything at all about the plain LWE analogue of LAC and more
> generally, the public key in any ring-LWE (or plain LWE) scheme using
> the Lindner-Peikert formulation of primal Regev, because that paper
> requires m >= n + omega(log n) in order to use the Micciancio-Mol SIS
> equivalence, and we have m=n for LAC. Is this correct?

Well, those m parameters don't appear to mean the same thing.

I think the main issue is this: for plain LWE with binary (or ternary) errors, [Hardness ... with Small Parameters] proves hardness when the number of LWE samples with *uniform secret over Zq* is

m < n(1 + 1/log n). (See discussion following Thm 4.6.)

However, Lindner-Peikert encryption uses *normal form* LWE, where the entries of the secret are chosen from the error distribution (not uniformly).

Transforming from uniform-secret to normal-form LWE costs at least n samples, which for the above m leaves < n/log n normal-form samples.
But the Lindner-Peikert system reveals at least n normal-form samples.
(So does the original Regev cryptosystem.)

Sincerely yours in cryptography,
Chris

From:	sd lattice
To:	Alperin-Sheriff, Jacob (Fed); luxianhui@iie.ac.cn
Cc:	pqc-comments; pqc-forum@list.nist.gov; Christopher J Peikert
Subject:	Re: OFFICIAL COMMENT: LAC
Date:	Friday, February 23, 2018 5:44:32 AM

Dear Alperin-Sheriff, Jacob

I agree with you and Peikert that, existing result cannot guarantee the worst-case hardness of the Ring-LWE problem with ternary errors used in LAC. I think it is an interesting question to study the worst-case hardness of Ring-LWE problem or plain LWE problem with ternary errors and 2n samples. I will try to solve this question.

I checked your analysis of the CCA security of LAC256. Although the attacker can find enough ciphertexts with Hamming weights of (r, e_1) of 1024+310, I do not understand how to recover s based on the analysis in https://eprint.iacr.org/2016/085.pdf.

The main difficulty is that, this analysis needs to change the error-reconciliation vector. However, in the decryption algorithm of CCA LAC256, there is a re-encryption process to check whether the ciphertext is valid. This means that, if the attacker changes the error-reconciliation vector, the cipherrtext will be rejected.

Another difficulty is that, in this analysis, the attacker needs to set e_1 to be special vectors with 1 in one coefficient and 0 elsewhere. However, (r, e_1) are generated as the output of the hash function, and the attacker cannot control the value of e_1.

It seems that, when the attacker finds one failure decryption it can only get the information that there are more than 55 bits errors.

Best Wishes.

LAC Team

发件人: Christopher J Peikert
发送时间: 2018年2月17日 11:43
收件人: Alperin-Sheriff, Jacob (Fed)
抄送: pqc-comments; pqc-forum@list.nist.gov; sd lattice
主题: Re: [pqc-forum] Re: OFFICIAL COMMENT: LAC

On Thu, Feb 15, 2018 at 2:22 PM, Alperin-Sheriff, Jacob (Fed)
<jacob.alperin-sheriff@nist.gov> wrote:
> Coming back to this point "Yes, [Hardness of SIS and LWE with Small
> Parameters] gets closer to the mark for plain LWE with binary (or ternary)
> errors. But to prevent any confusion: it would not support the plain-LWE
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> If I'm understanding everything properly, the paper doesn't say anything at
> all about the plain LWE analogue of LAC and more generally, the public key
> in any ring-LWE (or plain LWE) scheme using the Lindner-Peikert formulation
> of primal Regev, because that paper requires m >= n + omega(log n) in order
> to use the Micciancio-Mol SIS equivalence, and we have m=n for LAC. Is this
> correct?

Well, those m parameters don't appear to mean the same thing.

I think the main issue is this: for plain LWE with binary (or ternary) errors, [Hardness ... with Small Parameters] proves hardness when the number of LWE samples with *uniform secret over Zq* is

m < n(1 + 1/log n). (See discussion following Thm 4.6.)

From:	luxianhui@iie.ac.cn
Sent:	Friday, February 23, 2018 6:01 AM
To:	mt@post-quantum.com; pqc-comments
Cc:	pqc-forum; Alperin-Sheriff, Jacob (Fed)
Subject:	Re:OFFICIAL COMMENT: LAC

Hi Mike and Martin,

I will try to implement a constant-time decoding algorithm according to the suggestion of Martin. Just as Martin pointed out, there are extensive research results about McEliece system in the last decade. Maybe I can use the result in [1].

I think it is a good idea to use Goppa codes in LAC and I will try it.

[1] Daniel J. Bernstein, Tung Chou, Peter Schwabe: **McBits: Fast Constant-Time Code-Based Cryptography.** CHES 2013: 250-272.

Best Wishes.

LAC Team

From: Mike Hamburg <mike@shiftleft.org>
Sent: Thursday, April 12, 2018 2:52 PM
To: Alperin-Sheriff, Jacob (Fed)
Cc: pqc-comments; pqc-forum@list.nist.gov
Subject: Re: [pqc-forum] OFFICIAL COMMENT: LAC

By the way, this analysis comes from several productive discussions at PQC, with the likes of Hart, Sauvik, Aemon, and others.

— Mike

> On Apr 12, 2018, at 2:50 PM, Mike Hamburg <mike@shiftleft.org> wrote:
>
> Hello Jacob and other PQCers,
>
> Further analysis suggests that this failure mode of LAC is worse than previously expected.
>
> I assume that an attack constitutes a “certificational weakness” if it requires < 2^255 classical work (vs Class V), 2^64 total decryption queries, and 2^64 different target public keys, and breaks at least one public key with probability > 1/2. Note that this excludes attacks that eg send a single message and pray that it causes a failure; such attacks would constitute certificational weaknesses in many systems but don’t translate well to the real world.
>
> The following analysis is for LAC256, but the attacks it proposes work are likely to take < 2^128 work.
>
> LAC’s failure probability is influenced by both the Hamming weight of the key and of the ciphertext. An adversary can spend about 2^62 work to find a ciphertext with Hamming weight 9 standard deviations above the mean, which would be 0.77 per position (instead of 0.5). If my estimates are correct, such a ciphertext fails with probability about 2^-62 for a random key. This would mean an attack using about 2^124 work and 2^62 queries to find the first failure. But this might not work against a single key, because a significant fraction of the failure probability comes from the possibility that the key may have high Hamming weight.
>
> However, LAC doesn’t hash the public key when expanding an encryption seed. Therefore, the attack actually only takes 2^62 work to find an evil seed, and then that seed can be turned into a ciphertext for each of 2^62 public keys. (We are assuming here that each bacterium inside the gut of each human on Earth has a public key.) Send one evil query to each key, and probably one of them will fail to decrypt.
>
> Afterward, the adversary can be confident that the victim’s key has a high Hamming weight (median ~2.8 standard deviations above 0.5), and that it has a large correlation with the (secret,noise) vectors produced by the evil ciphertext. Finding enough further ciphertexts that have a high correlation with the initial one might be feasible, though I haven’t calculated it. Just going on Hamming weight would give 2^108 work per additional failure (with < 2^60 additional decryption queries per failure), but I expect that considering the correlation would greatly reduce the required work.
>
> The adversary doesn’t know which 56 of the 1024 positions failed, nor whether the failures were positive or negative. But he can align the failing ciphertexts with each other by computing their mutual correlations. Then averaging the failing ciphertexts gives an approximation of the (reverse of the) key. Some non-rigorous experiments with single-failure systems suggests that a few dozen failing ciphertexts may be enough to recover the key entirely, or at least bring it within range of a practical hybrid attack.
>
> Overall, these back-of-the-envelope calculations suggest that CCA failure attacks on LAC could succeed with high probability for less than 2^128 work. If many targets are available, the effort might even approach a feasible level.
>

> These calculations do not account for the possibility that the key and/or ciphertext may also have a high autocorrelation. This might increase the correlation between failures, and thus further increase the probability that > 56 failures occur.
>
> I suggest that LAC should retune in the following ways:
> * Smaller noise variance, though this increases the risk of a hybrid attack.
> * Fixed Hamming weight noise instead of IID weighted ternary noise. This increases complexity and risk of side-channel attacks, but LAC's large error correction already induces similar risk of side-channel attacks. Also, whether this works depends on how much autocorrelation affect probability of decryption failure, because fixed Hamming weight doesn't mean fixed autocorrelation.
> * Hash the public key into the FO mode's seed, to prevent multi-target attacks.
>
> Hashing the public key into the seed probably makes the attack infeasible, but it would probably still be present as a certificational weakness, so at least one of the other two countermeasures is probably also required.
>
> Finally, note that this concern applies also to ThreeBears and to HILA5, and neither of these has a completely rigorous failure analysis yet. However, all the threats described in this comment have already been accounted for in the ThreeBears failure analysis, including:
>
> * Correlation due to larger-than-expected secret key.
> * Correlation due to larger-than-expected ciphertext.
> * (After publication) Correlated failures due to autocorrelation of key/ciphertext.
> * Multi-target attacks (because ThreeBears hashes part of the pubkey into the seed).
>
> These attacks are not accounted for in the HILA5 failure analysis, but HILA5's failure analysis is very conservative (Saarinen estimates 2^-135, but I think the real answer is more like 2^-165), and 5-error-correction is much less than 55-error-correction. So I believe both ThreeBears and HILA5 to be safe from this attack, but further investigation is warranted.
>
> Cheers,
> — Mike Hamburg
>
>> On Feb 16, 2018, at 3:02 PM, Mike Hamburg <mike@shiftleft.org> wrote:
>>
>> Hi Jacob,
>>
>> I think Fluhrer's attack https://eprint.iacr.org/2016/085.pdf doesn't apply here, because it assumes unconstrained chosen messages and not random ones. Furthermore, while "HILA5 Pindakaas" discussed attacking a 5-ECC with unconstrained chosen messages, attacking a 55-ECC with random high-weight (but otherwise not maliciously chosen) messages would be much more complicated.
>>
>> On the negative side, LAC256 private keys have somewhat low entropy of only 1.5kibit (1.5 bits per position). So having 2^14 known failures and 2^64 non-failures might open up information-theoretic approaches. For example with 2^14 failures you may be able to reduce the entropy quite a bit by constraining that it has a large with the known failure cases. I think more analysis is warranted here.
>>
>>
>>
>> You also asked whether I had measured the BCH code. I haven't, I was just guessing by reading it, especially since it uses very different numbers of iterations depending on the number of errors.
>>
>> If the BCH code's behavior is visible to a software attacker, then LAC might be attackable with practical complexity. If there are enough errors for Chien search to be used, an adversary would know with reasonable confidence exactly how many errors are present and where they are, or at the very least where the last one is. Furthermore, the ciphertexts

From: Jan-Pieter D'Anvers <janpieter.danvers@esat.kuleuven.be>
Sent: Thursday, April 12, 2018 6:23 PM
To: pqc-forum@list.nist.gov
Subject: Re: [pqc-forum] OFFICIAL COMMENT: LAC

Hi Mike, Hi all,

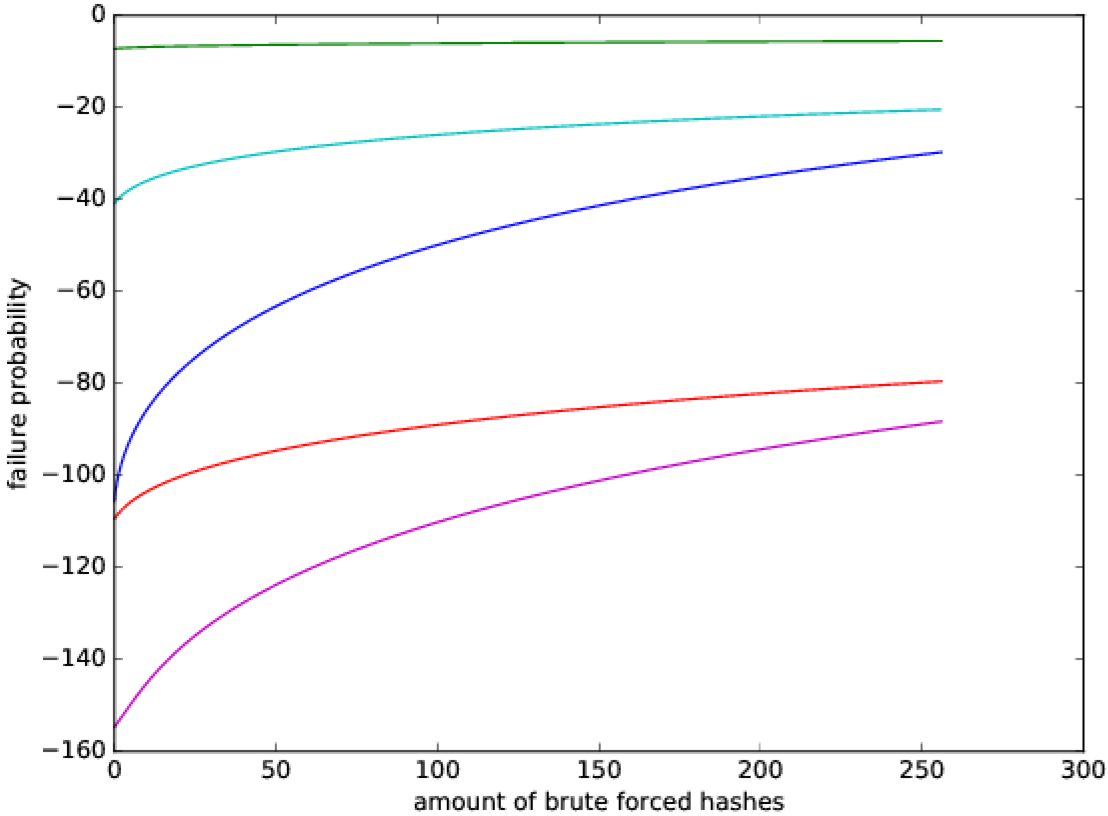
Short version

- the error correction of LAC makes it more prone to attacks where the attacker precomputed weak values of (e', s') (Bob's secret and noise) with high hamming weight
- however, because of the error correction and the relatively low error rate of the individual coefficients, it is seems hard to determine which coefficients are failing and whether they are positive or negative failure coefficients. This makes it difficult to get any information about the secret (e, s) from the failures (or: low failure rate is not necessarily a bad thing)
- Using enough failures (2^10? maybe lower), we could construct classifiers that we can use to roughly estimate the secret (e, s)

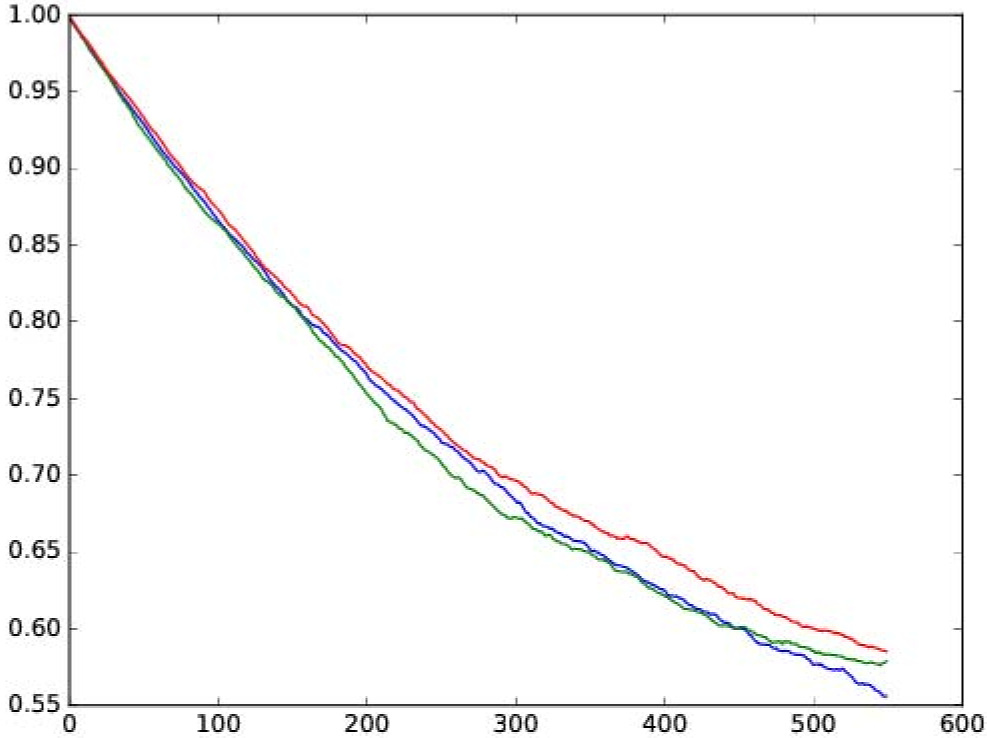
Long version

I have been working on an analysis of the attack angle where the adversary influences the hamming weight of his secrets (e', s') to obtain information about (e, s). In the graph beneath you can see how searching through the hashes for weak high hamming weight (e', s') influences the failure probability. The different schemes depicted are LAC256 (dark blue), LAC256 without error correction (green), a LAC256-alike scheme without error correction but adjusted to have similar error so that you can see the influence of the error correction (red), Frodo (light blue), FrodoKEM (purple). Here you can clearly see that the error correction of LAC heavily decreases the failure probability, but also that it makes is way more susceptible for searching for high (e', s').

Note that this search can be sped up with Grover, and that as you noted, the search has to be done only once if no multitarget protection is in place. Fixed hamming weight secrets (e', s') would also defeat this approach, but they might of course have other security implications.



If the failure rate would be around 2^-128, a failure would have very high correlation with the secrets, so that you could run an autocorrelation between the samples and recover the positions of the failure coefficients and determine whether they are positive or negative. Then it is relatively easy to get a good estimate of the key using any statistical technique. In the picture a simple non optimized Maximum Likelihood estimation is used to plot the entropy reduction vs the amount of failure vectors that are used. Note that one failure sample in case of LAC would amount to 56 failure vectors that give information.



Once a decent estimate of the key has been made, we can compute a new RLWE problem by subtracting b'=As_est+e_est from b, which would result in the RLWE problem A(s-s_est)+(e-e_est), with smaller noise and error terms, and therefore more susceptible to attacks on the RLWE problem.

However, this all assumes that it is easy to distinguish positive, negative and no failure coefficients. This is not necessarily the case! Since the failure rate of one single coefficient is rather low (+/-2^-8), the correlation of the secret (e, s) with the input (e', s') is not high. From preliminary experiments, it seems not trivial to distinguish the autocorrelations leading to a failure coefficients from these leading to a no failure coefficient, and therefore it seems not trivial to get information about the secret (e, s) from a failure sample!

If this information could be obtained from side channels, then the adversary can use the statistical methods from above to estimate the key and thus recover (e, s). If not, the adversary needs to find another way to gain information about (e, s). We now that in case of a failure (e, s) has a slightly higher correlation with the absolute value of 56 coefficients of (e', s') than usual. Using this information we can construct a (very noisy) classifier that can (with very low probability of success) distinguish between a sample (e_est, s_est) that is close to the secret (e, s) and one that is not close to (e, s) by summing the absolute value of the 56 biggest coefficients. As said before, this classifier will not be very informative, but by combining (averaging) several different classifier from different failure samples, we can improve the quality of our classifier. Early calculations indicate that we need to have around 2^10 failures to construct a such a decent classifier to get a very rough estimate of (e, s), but I'm not sure yet. I'm currently working on a more exact estimate of the needed amount of failures.

Once we have a decent classifier, we can do a search for a rough estimate of (e, s). which in turn will enable us to 1) or directly make a better estimate of (e, s) 2) or use (e, s) to classify our coefficients into positive, negative and no failure coefficients, so that the statistical techniques from above can be used to get a better estimate of (e, s).

As I see it now, two factors determine the success probability of the attack:
1) the probability of finding failures though searching for weak (e, s) keys as depicted in the first figure
2) the amount of failures we need to make a rough estimate of the secret (e, s) which can be used to directly make a good estimate of (e, s) or to distinguish positive/negative/no failure coefficients.

For most schemes (1) will be the bottleneck and (2) will not pose any problem because of the low failure rate and thus high correlation between the failure (e', s') and the secret (e, s). For LAC (2) might also make the attack more difficult since multiple failures might be needed.

I will keep you posted with updates.

Regards,

Jan-Pieter

By the way, this analysis comes from several productive discussions at PQC, with the likes of Hart, Sauvik, Aemon, and others.

— Mike

On Apr 12, 2018, at 2:50 PM, Mike Hamburg <mike@shiftleft.org> wrote:

Hello Jacob and other PQCers,

Further analysis suggests that this failure mode of LAC is worse than previously expected.

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The following analysis is for LAC256, but the attacks it proposes work are likely to take < 2^128 work.

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However, LAC doesn't hash the public key when expanding an encryption seed. Therefore, the attack actually only takes 2^62 work to find an evil seed, and then that seed can be turned into a ciphertext for each of 2^62 public keys. (We are assuming here that each bacterium inside the gut of each human on Earth has a public key.) Send one evil query to each key, and probably one of them will fail to decrypt.

Afterward, the adversary can be confident that the victim's key has a high Hamming weight (median ~2.8 standard deviations above 0.5), and that it has a large correlation with the (secret,noise) vectors produced by the evil ciphertext. Finding enough further ciphertexts that have a high correlation with the initial one

From: LU XIANHUI <luxianhui@outlook.com>
Sent: Wednesday, October 24, 2018 3:31 AM
To: pqc-comments; pqc-forum@list.nist.gov
Subject: Formal Analysis of LAC

Dear All,

We give a formal analysis of subfield attack and high hamming weight attack on LAC, detailed result can be found at https://eprint.iacr.org/2018/1009 .

Briefly, we reduce the session key length to 256bits for all the three security levels (just like Kyber). As a result we have enough space to promote the error correction ability, especially for the case of LAC256. Finally, we show that decryption error rate is small enough to resist the High hamming weight attack.

We remark that, when the session key length or the message space is reduced to 256, we can use D2 and BCH together in LAC256. With the assist of D2, the BCH error correction code only need to correct 20bist errors at most, which is much smaller than the current version of 55bits. This makes the LAC256 more efficient. For the sake of simplicity, we can use the same parameter set for all of the three security levels.

LAC Team
Best Regards.