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Use of Stochastic Models in RBG Standards

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NIST Random Bit Generation Workshop 2023

What is a stochastic model?

Stochastic model in a nutshell

A stochastic model

- provides a mathematical description of a noise source using random variables,
- allows the verification of an entropy lower bound for the output data,
- is based on and justified by the understanding of the noise source.

Physical vs. non-physical noise sources

Physical noise sources

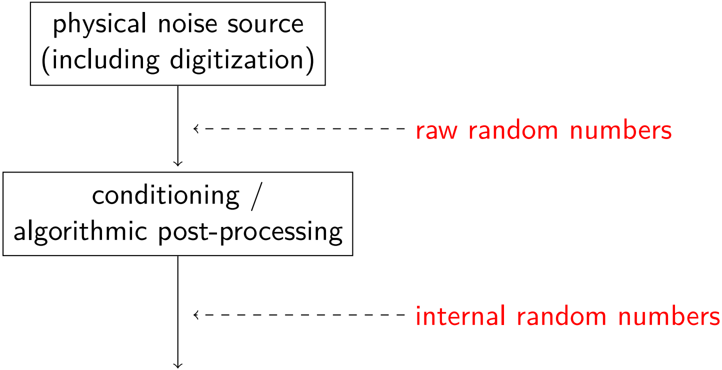
- exploit physical phenomena or physical experiments,
- use dedicated hardware designs.

Non-physical noise sources

- exploit system data or user interaction,
- use general-purpose hardware,
- may run in a variety of operational environments.

→ Stochastic models are only feasible for physical noise sources in general.

Entropy source schematic



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Mathematical definition

- Random numbers are interpreted as realizations of random variables.
- A stochastic model consists of a family of probability distributions that contains the true distribution of the raw random numbers (ideal case).
- This family of distributions usually has 1 to 3 parameters.

- The raw random numbers shall be (time-locally) stationarily distributed.

Stochastic model validation

The stochastic model of a noise source shall be

- substantiated using arguments from physics or electrical engineering,
- validated using empirical data and tailored statistical tests.

Entropy estimation

- The stochastic model shall be used to derive an entropy lower bound per internal random bit (depending on the parameters of the model).
- A set of good parameters for the targeted entropy bound shall be determined.
- The parameters of the noise source shall be estimated under relevant environmental conditions.

Health testing

An online test / health test shall

- detect non-tolerable entropy defects sufficiently soon,
- be tailored to the stochastic model,
- use the raw random numbers, because they contain more information than the internal random numbers.

Stochastic models in RBG standards

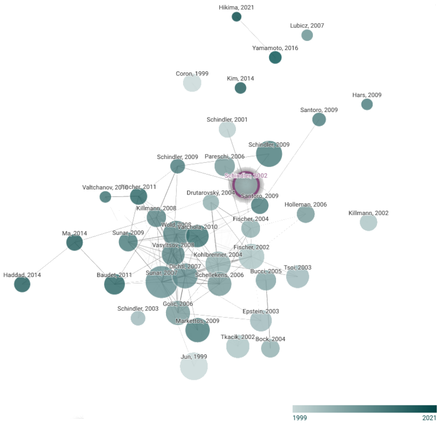
- AIS 20/31: Stochastic model mandatory for certification of physical noise sources in the German Common Criteria scheme since 2001 (functionality classes PTG.2, DRG.4, and PTG.3).
- ISO/IEC 20543: Stochastic model required for evaluation of physical noise sources.
- NIST SP 800-90B: Stochastic model recommended as entropy justification for physical noise sources. NIST intends to make stochastic models mandatory.

Stochastic models in the scientific literature

Stochastic models

- have become the state of the art in the analysis of physical noise sources,
- have influenced the design of physical noise sources.

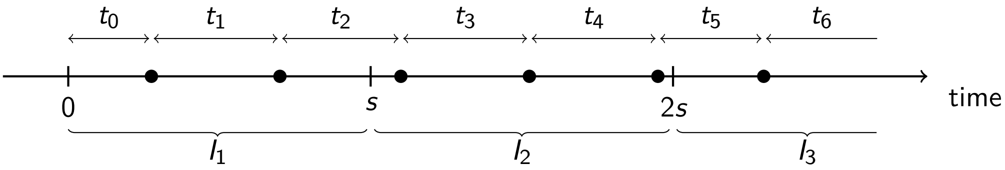
→ Stochastic model should already be considered at the design stage.



Source: `https://www.connectedpapers.com`

Example: Counting random events

Counting random events



- Intermediate times between events ($\bullet$): $t_0, t_1, t_2, \ldots$
- Time intervals ($\vdash\!\!\dashv$): $I_n = \big((n-1)s, ns\big]$ with fixed length s

- Raw random numbers: $r_n = \#\big\{\text{events occuring in } I_n\big\}$
- Internal random numbers: $y_n = r_n \bmod 2$

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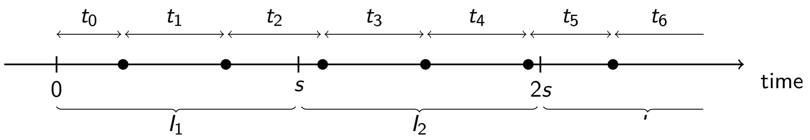
Example: Noisy diodes



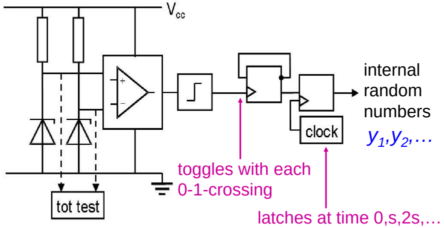


Figure: Two noisy diodes: schematic design (Killmann & Schindler, CHES 2008)

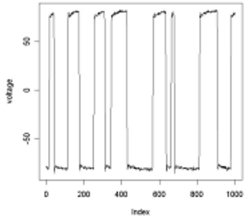


Figure: Random event: Up-crossing of amplified voltage.

Further examples of random events

- Rising edges of a ring oscillator.
- Photons emitted from an LED.
- Decays of a radioactive source.

Stochastic model (generic)

- The intermediate times $t_1, t_2, \ldots$ are interpreted as realizations of iid non-negative random variables $T_1, T_2, \ldots$
- The time intervals $I_n = ((n-1)s, ns]$ have fixed length s.
- The raw random numbers are $R_n = \#\{\text{events occuring in } I_n\}$.
- The internal random numbers are $Y_n = R_n \bmod 2$.

- This model is analyzed in AIS 20/31 draft 2022.
- If $s \gg \mathrm{E}(T_j)$, the iid-assumption can be relaxed ($\rightarrow$ noisy diodes).

- A stochastic model for a real-world physical noise source has to be substantiated and validated.

Stochastic model (normal distribution)

From now on:

- The intermediate times $T_1, T_2, \ldots$ are iid $\mathcal{N}(\mu, \sigma^2)$-distributed.
- The time intervals $I_n = ((n-1)s, ns]$ have fixed length $s \gg \mu$.
- The raw random numbers are $R_n = \#\{\text{events occuring in } I_n\}$.
- The internal random numbers are $Y_n = R_n \bmod 2$.

- The parameters of this model can be taken as

$$\frac{s}{\mu} \qquad \text{(expected number of events in } I_n\text{)}$$

$$\frac{\sigma}{\mu} \qquad \text{(coefficient of variation of } T_j\text{)}$$

Statistical properties of the raw random numbers

- The raw random numbers $R_1, R_2, \dots$ are stationary.
- Their mean is

$$E(R_n) = \frac{s}{\mu}\,.$$

- Their variance can be (well) approximated as

$$\mathrm{Var}(R_n) \approx \left(\frac{\sigma}{\mu}\right)^2 \frac{s}{\mu} + \frac{1}{6} + \frac{1}{2}\left(\frac{\sigma}{\mu}\right)^4 .$$

- Their covariances can be (well) approximated as

$$\mathrm{Cov}(R_n, R_{n+1}) \approx -\frac{1}{12} - \frac{1}{4}\left(\frac{\sigma}{\mu}\right)^4$$

and $\mathrm{Cov}(R_n, R_{n+k}) \approx 0$ for $k \geq 2$.

Entropy of the internal random numbers

- We require an entropy lower bound for the internal random numbers $Y_1, Y_2, \dots$
- We consider the (worst-case) conditional min-entropy

$$\mathrm{H}_\infty(Y_n \mid Y_{n-1}) = -\log_2 \max_{y_n, y_{n-1} \in \{0,1\}} \Pr(Y_n = y_n \mid Y_{n-1} = y_{n-1}) .$$

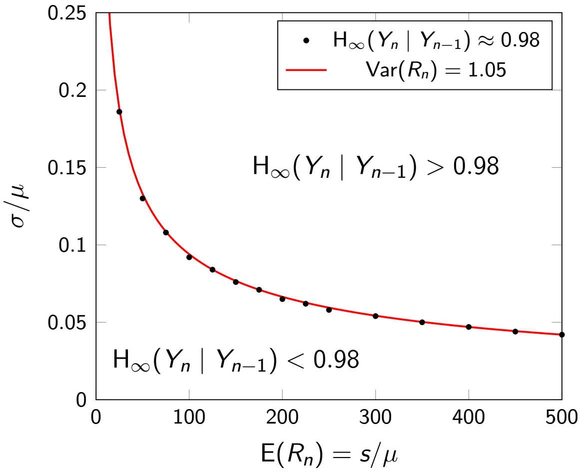
- We want to find parameters for which $\mathrm{H}_\infty(Y_n \mid Y_{n-1}) \geq 0.98$.

Entropy estimation by simulation

The cond. min-entropy $H_\infty(Y_n \mid Y_{n-1})$

- increases with s/μ (more events per time interval),
- increases with σ/μ (more variation per event),
- is determined by $\mathrm{Var}(R_n)$.

$\rightarrow$ Online test / health test should be based on $\mathrm{Var}(R_n)$.



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Wrap-up

Summary

Stochastic models

- help to understand physical noise sources,
- enable to derive entropy lower bounds,
- enable effective and lean health tests,
- are mandatory in German CC certifications according to AIS 31 since 2001,
- are recommended as justification of entropy estimates in SP 800-90B validations,
- should be considered already at the design stage.

Thank you for your attention!

Questions?

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