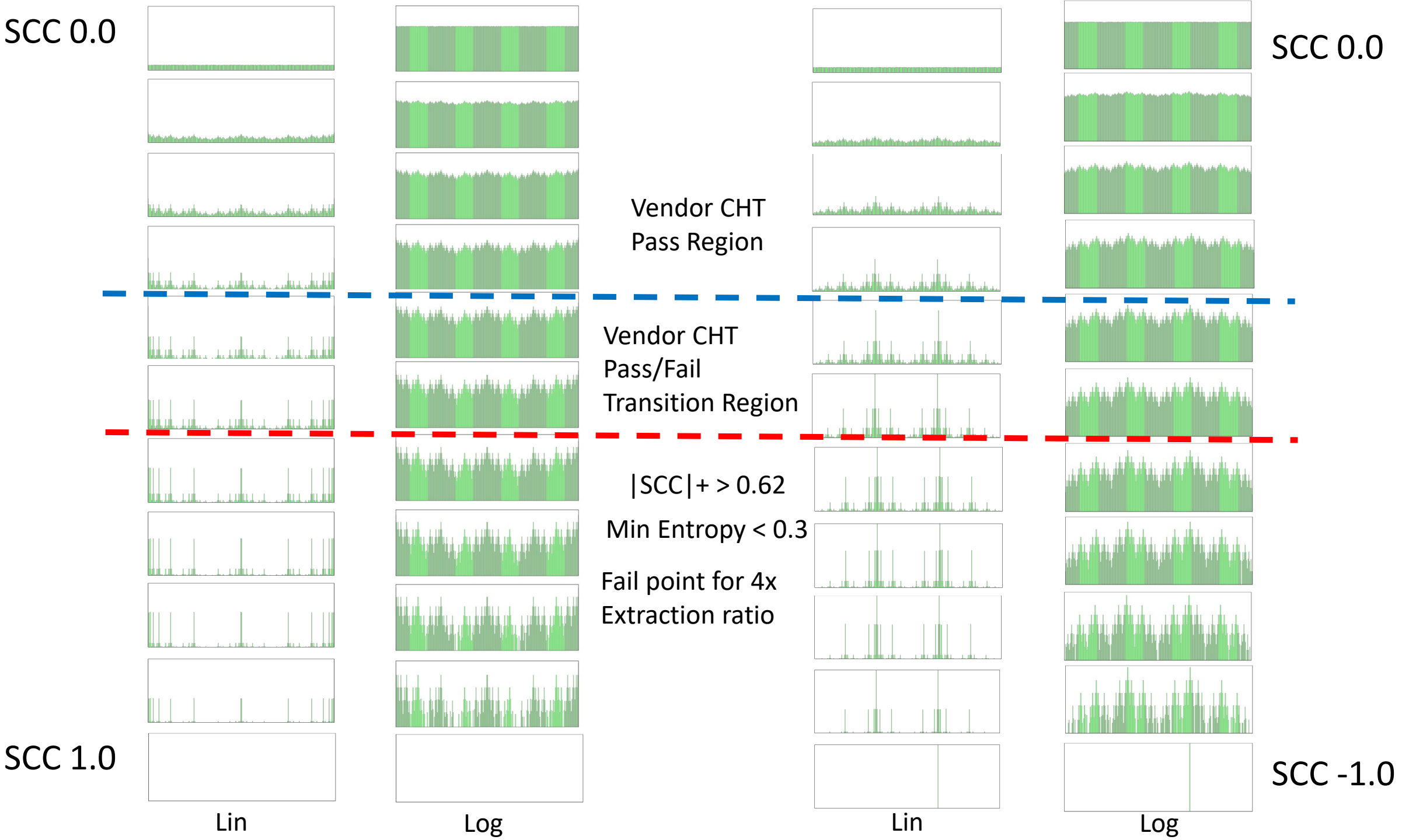
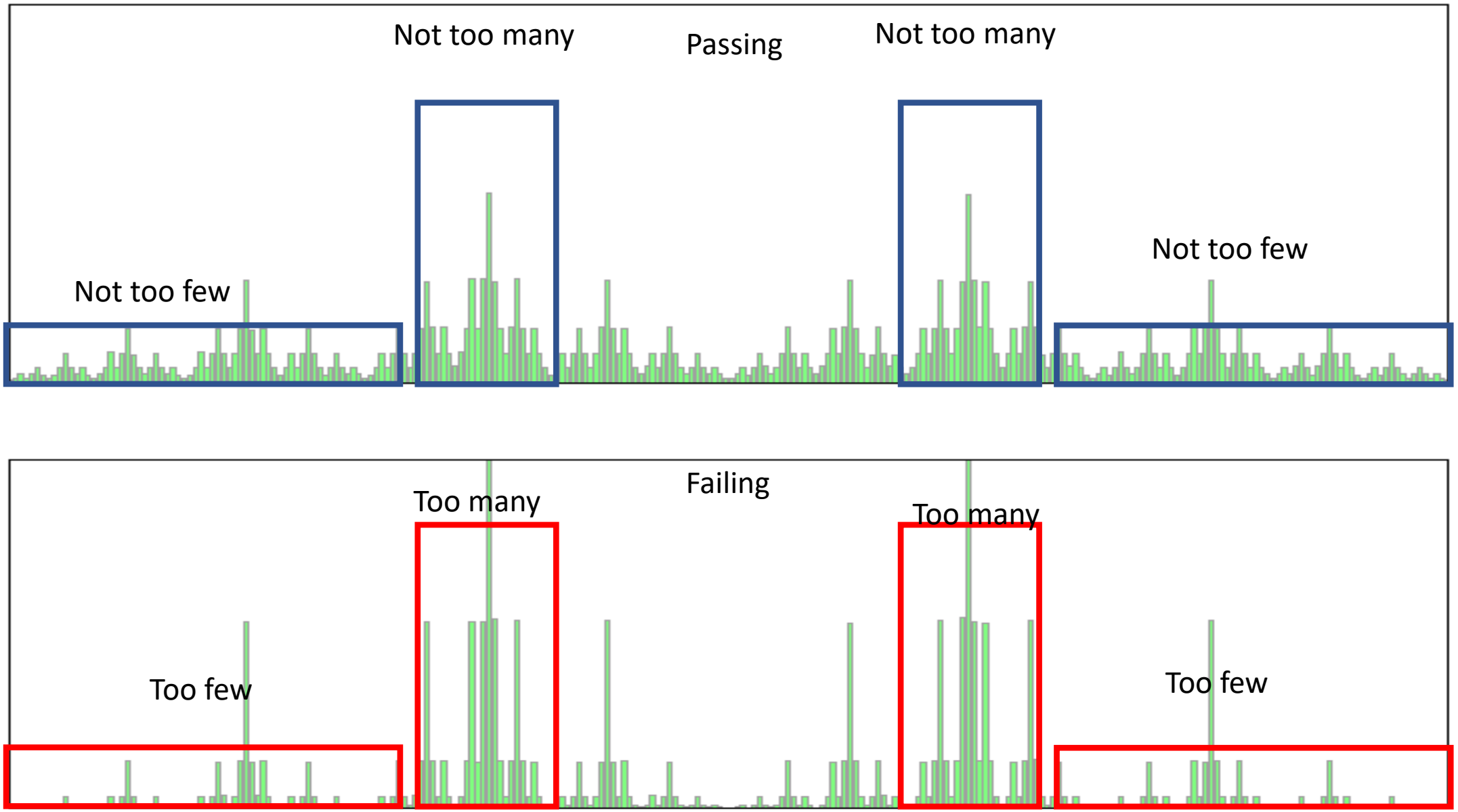
Vendor CHTs

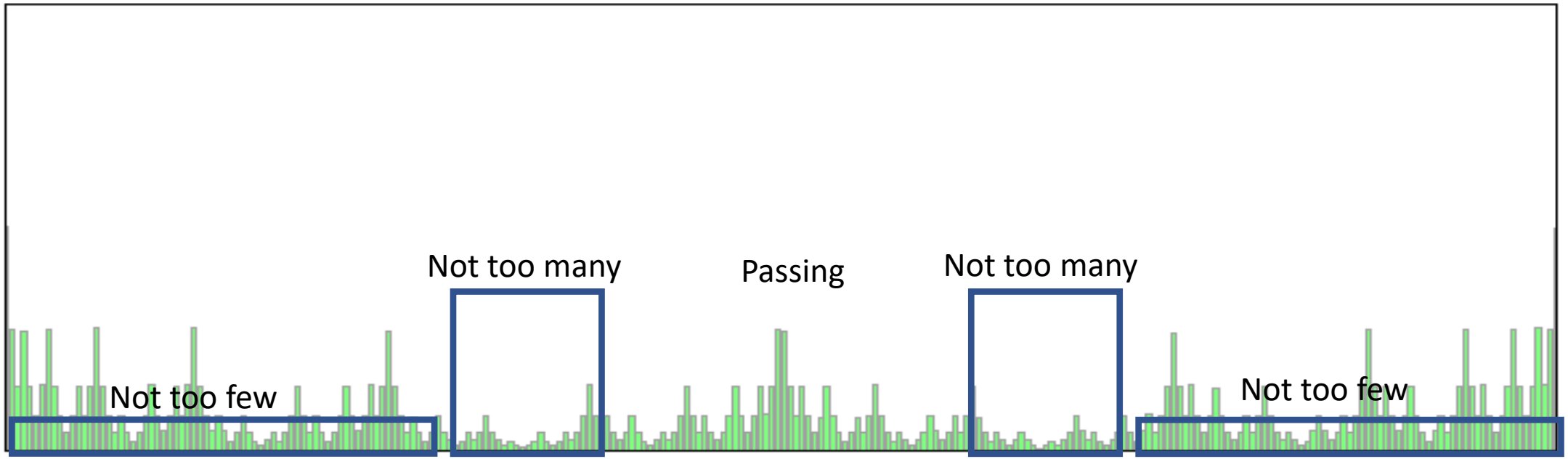
David Johnston

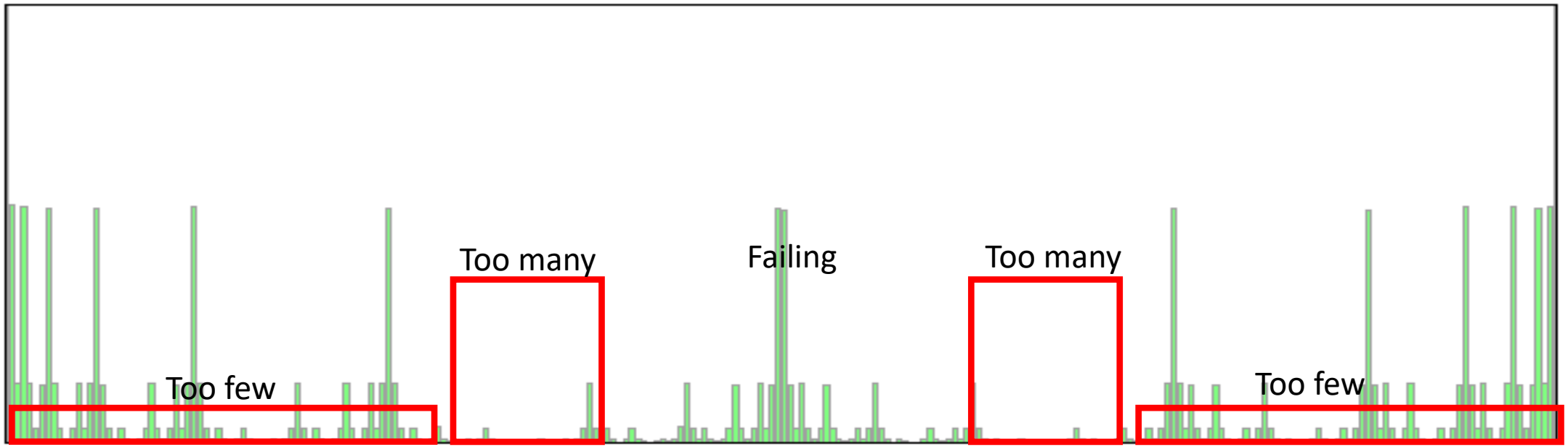
dj.johnston@intel.com

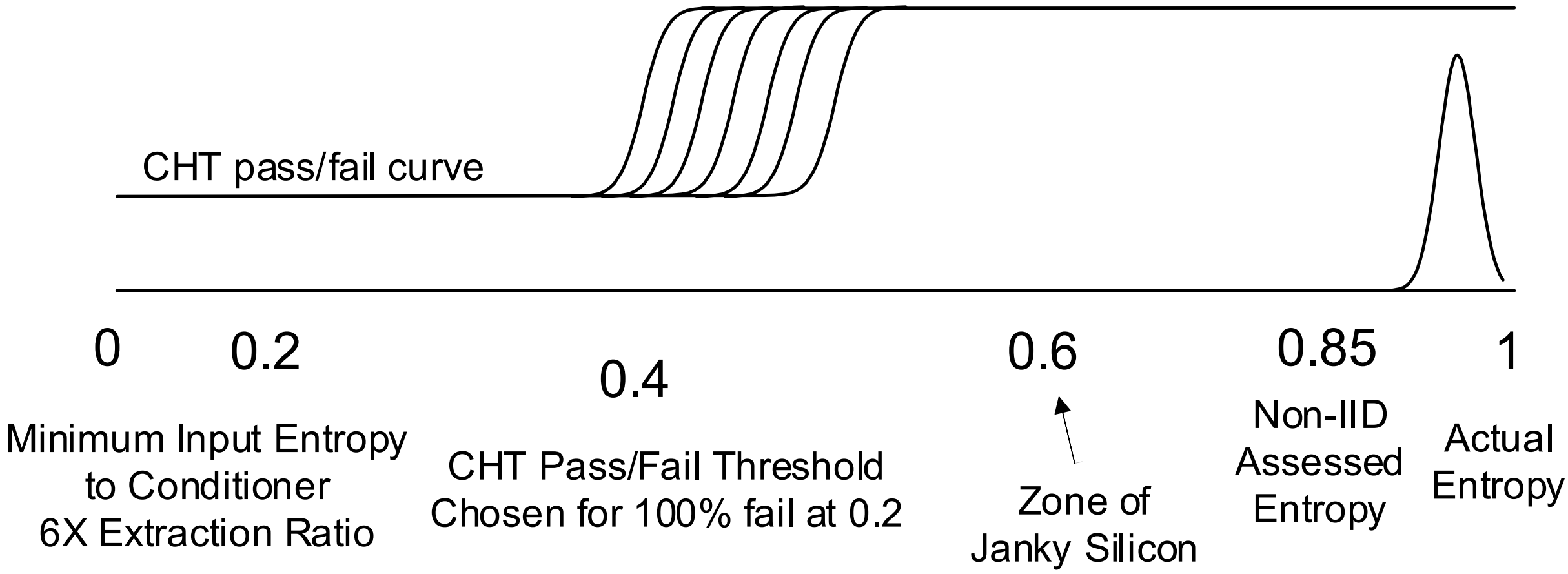
For : NIST RBG Workshop. May 31st – June 1st 2023


SCC 0.0
SCC 1.0
Lin
Log
Vendor CHT
Pass Region
Vendor CHT
Pass/Fail
Transition Region
|SCC|+ > 0.62
Min Entropy < 0.3
Fail point for 4x
Extraction ratio
SCC 0.0
SCC -1.0
Lin
Log


Not too many
Passing
Not too many
Not too few
Not too few
Failing
Too many
Too many
Too few
Too few

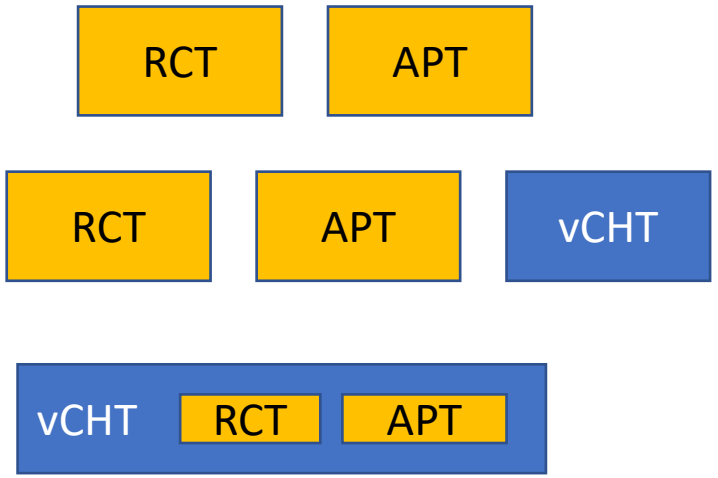

Not too many
Passing
Not too many
Not too few
Not too few


Too many
Failing
Too many
Too few
Too few


CHT pass/fail curve
0
0.2
0.4
0.6
0.85
1
Minimum Input Entropy
to Conditioner
6X Extraction Ratio
CHT Pass/Fail Threshold
Chosen for 100% fail at 0.2
Zone of
Janky Silicon
Non-IID
Assessed
Entropy
Actual
Entropy

4.5

1. The continuous tests **shall** include either:
   a. The **approved** continuous health tests, described in Section 4.4, or
   b. Some developer-defined tests that meet the requirements for a substitution of those approved tests, as described in Section 4.5. If developer-defined health tests are used in place of any of the **approved** health tests, the tester **shall** verify that the implemented tests detect the failure conditions detected by the **approved** continuous health tests, as described in Section 4.4. The need to use the two **approved** continuous health tests can be avoided by providing convincing evidence that the failure being considered will be reliably detected by the developer-defined continuous tests. This evidence may be a proof or the results of statistical simulations.
   c. The continuous tests **may** include additional tests defined by the developer.



Inadequate for negative SCC based failure

Vendor CHT must catch what the RCT and APT Cannot
+ also meet 4.5a & 4.5b

Vendor CHT must catch what the RCT and APT Cannot
+ also meet 4.5a & 4.5b
+ also catch what RCT and APT catch

4.5 Developer-Defined Alternatives to the Continuous Health Tests

a. If a single value appears more than $\lceil 100/H \rceil$ consecutive times in a row in the sequence of noise source samples, the test **shall** detect this with a probability of at least 99 %.

- If the CHTs are required to detect all failures the RCT would detect why is this requirement on a vendor CHT?
- A vendor CHT is there to catch the cases that the RCT and APT cannot catch and replace the RCT and APT if not present. Thus the RCT capture set is mandatory.
- The capturing runs of ceil(100/H) bits is like an RCT with C=ceil(100/H). The RCT's C is always smaller and is going to detect this condition 100% of the time.

H	RCT C	Ceil(100/H)
0.1	68	1000
0.2	35	500
0.3	24	334
0.4	18	250
0.5	15	200
0.6	13	167
0.7	11	143
0.8	10	125
0.9	9	112

Proposal for 4.5 part a

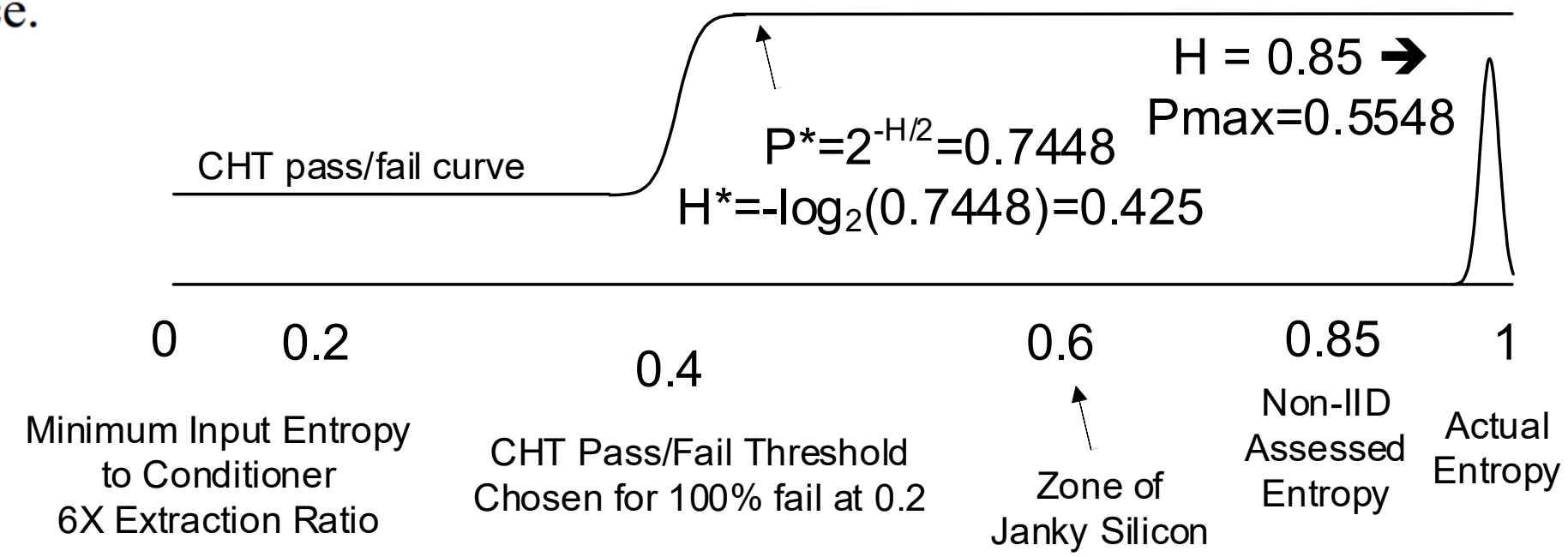
- (1)Instead of requiring a vendor CHT to do it, amend the RCT definition to require C is at least small enough to detect a repeated sequence of ceil(100/H) symbols. This will be a no-op for any binary RCT since it already meets this requirement.
- (2)Remove 4.5 part a.
- (3) or just do (2) since (1) is moot

4.4.1 Repetition Count Test

Given the assessed min-entropy H of a noise source,

4.5 Developer-Defined Alternatives to the Continuous Health Tests

b. Let $P = 2^{-H}$. If the noise source's behavior changes so that the probability of observing a specific sample value increases to at least $P^* = 2^{-H/2}$, then the test **shall** detect this change with a probability of at least 50 % when examining 50 000 consecutive samples from this degraded source.



Proposal for 4.5 part b

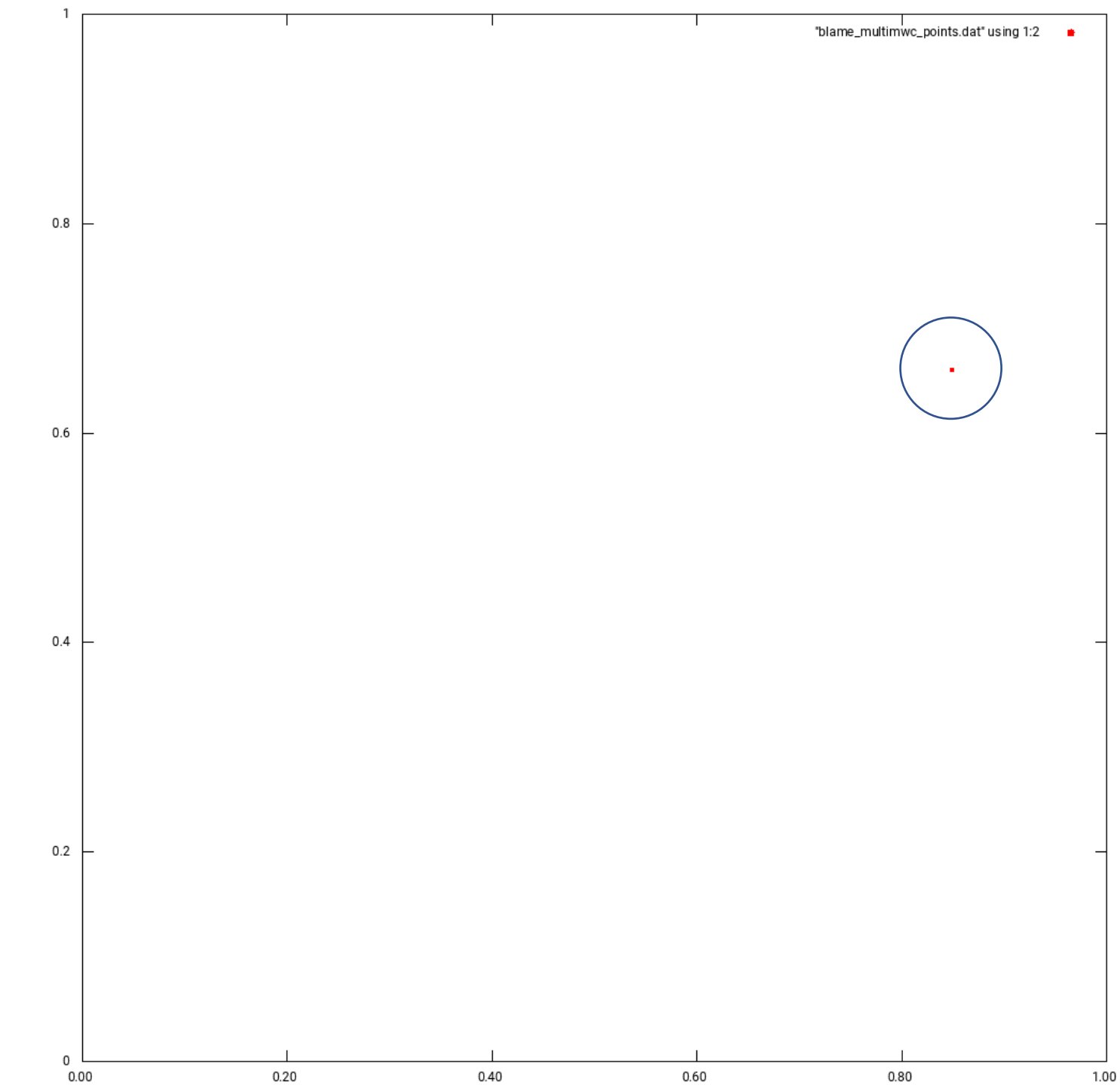
- The current rule make a threshold that is a function of how good the noise source is, rather than a function of the failure point. Instead make it a statement that the test will almost always fail at the lowest tolerable entropy rate and make it valid over the number of bits that need to be tested before they go into the conditioner. I.E. n_in.
- E.G. “If the source entropy rate falls below the minimum required entropy rate into the conditioning chain to achieve full entropy at the conditioning chain output, the vendor defined test shall detect this with probability greater than 1-(10E-6) over n_in bits to the conditioning chain.”
- This means that the output of the conditioner chain will be full entropy and the vendor can use an honest H_submitter value rather than an artificially low number.

Proposal for RCT 4.4.1 & APT 4.4.2

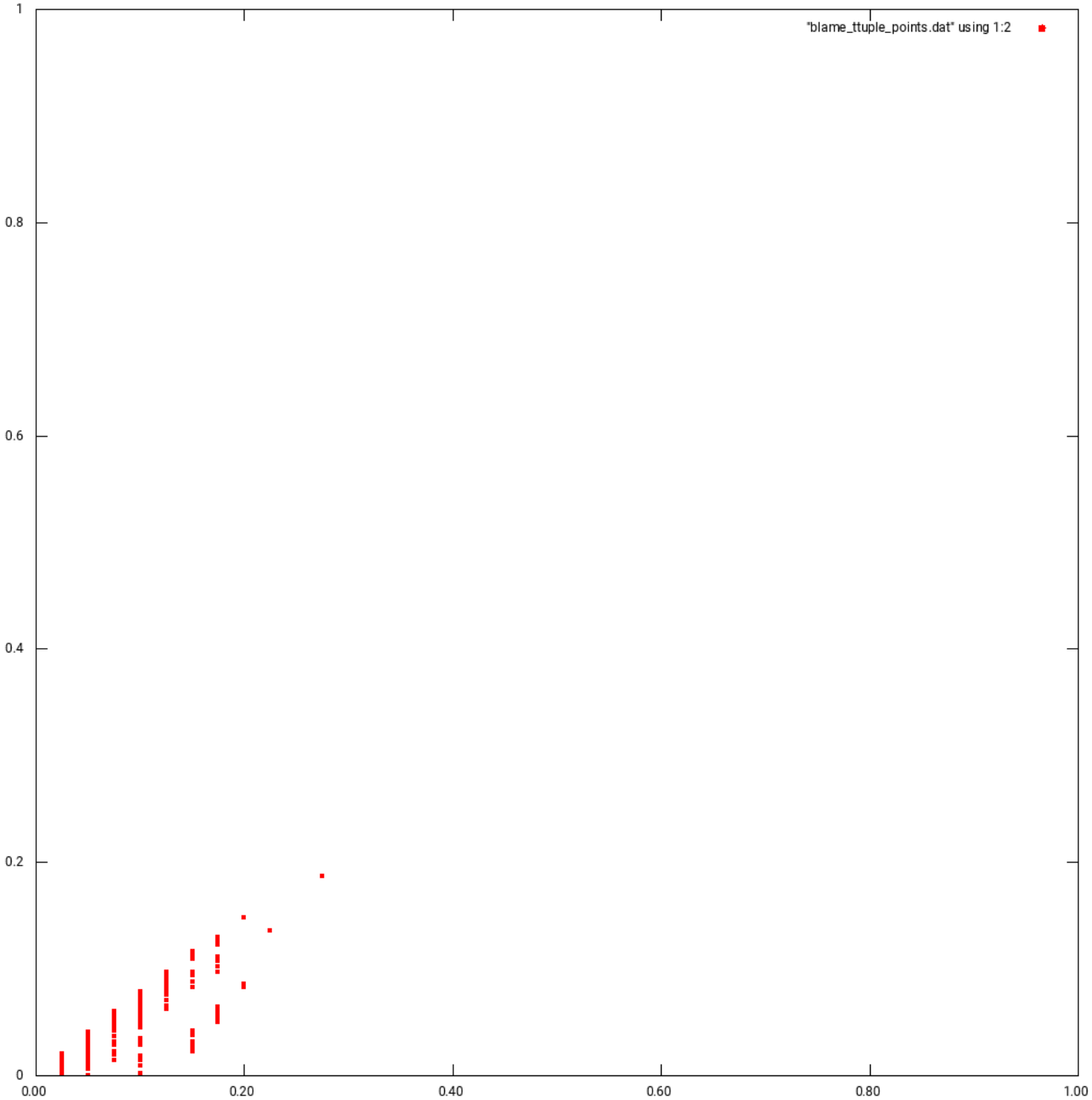
> Given the assessed min-entropy H of a noise source, the probability[9] of that source generating n identical samples consecutively is at most $2^{-H(n-1)}$. The test declares an error if a sample is repeated C or more times. The cutoff value C is determined by the acceptable false-positive probability α and the entropy estimate H using the following formula

- The RCT text calls for H to be set to the assessed entropy.
- The assessed entropy is the wrong place to set a test threshold
- The specification is ambiguous as to whether the H used in the RCT is the same H used in the APT and in vendor defined CHTs
- H should be set per test based on the properties of the test and the optimum cutoff points for a test given the expected distribution of entropy rate from a functioning source and the lowest tolerable entropy rate.
- Change "*the assessed min-entropy H*" to "*a chosen test cutoff entropy* H_{rct}" and similarly use H_{apt} in 4.4.2.
- The false positive error rate should be close to 100% at the minimum tolerable entropy rate.

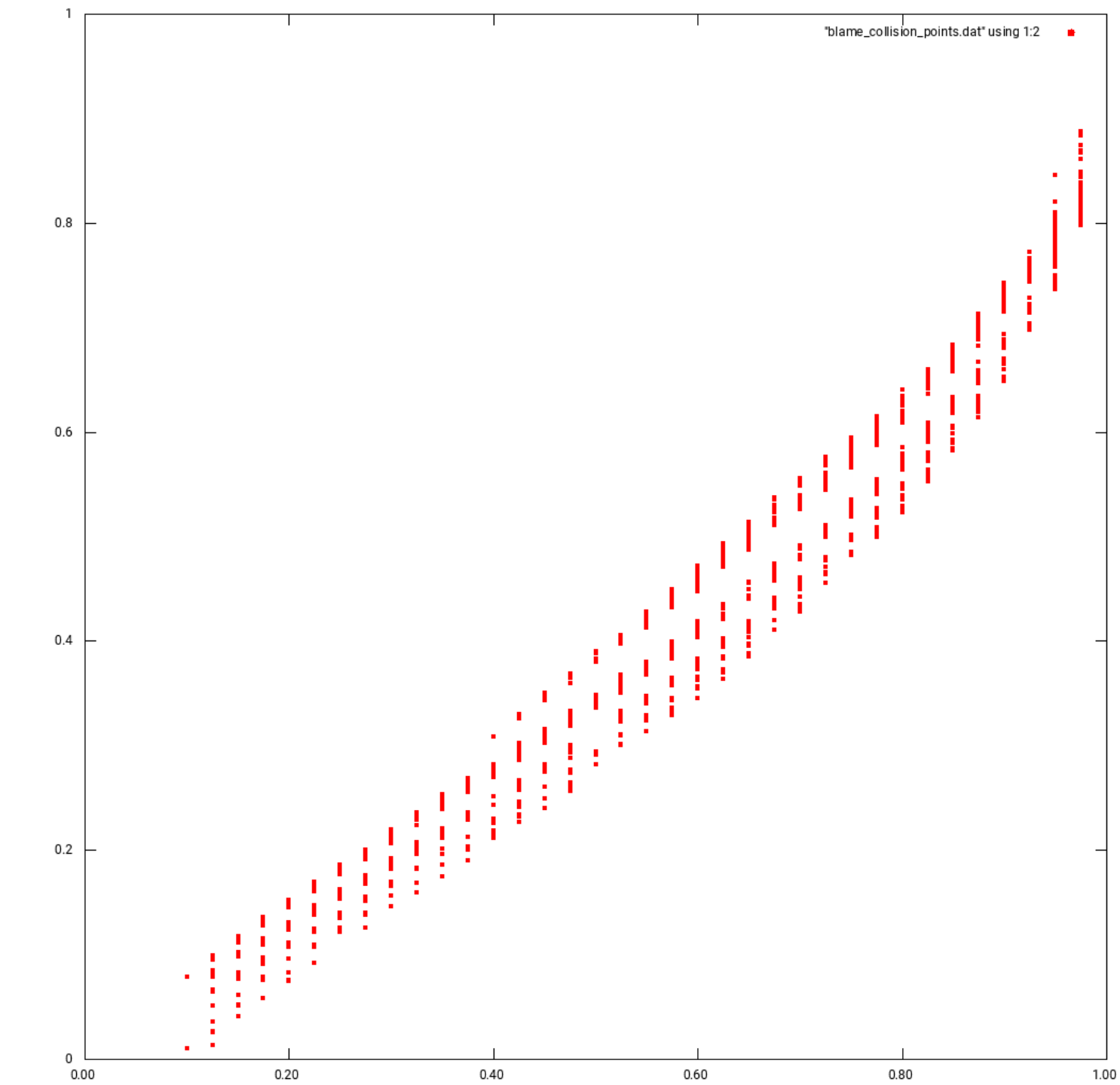
- Since we have to use the results of ea_non_iid in our CHT parameters (E.G. in 4.4.1) we care that these results are realistic. In many cases, they are not.
- In this synthetic test 7800 samples of entropy data were input to the test at entropy levels from 0.025 to 0.975 in 0.025 increments. The actual entropy, the bias, scc and H_original, along with the estimation error and the test with the lowest estimate were recorded for each data point. 200 runs per entropy level. 39 entropy levels.
- `ea_non_iid -i -v -t -l 0,1000000 <filename> 1` was used.
- The next 4 plots give the set of points where a specific test gave the lowest estimate.



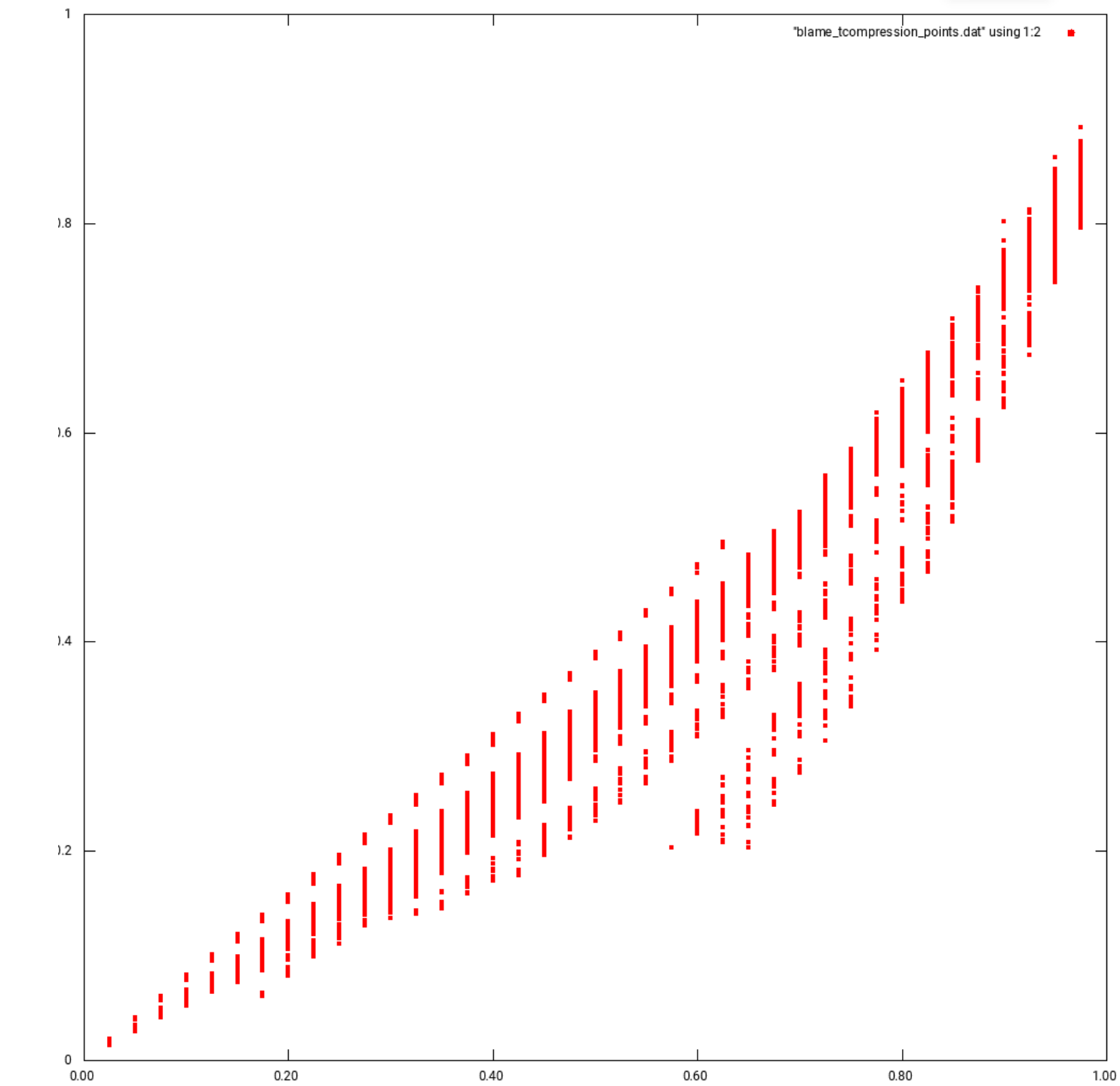
- The lonely MultiMWC Estimate
- 1 case in 7800



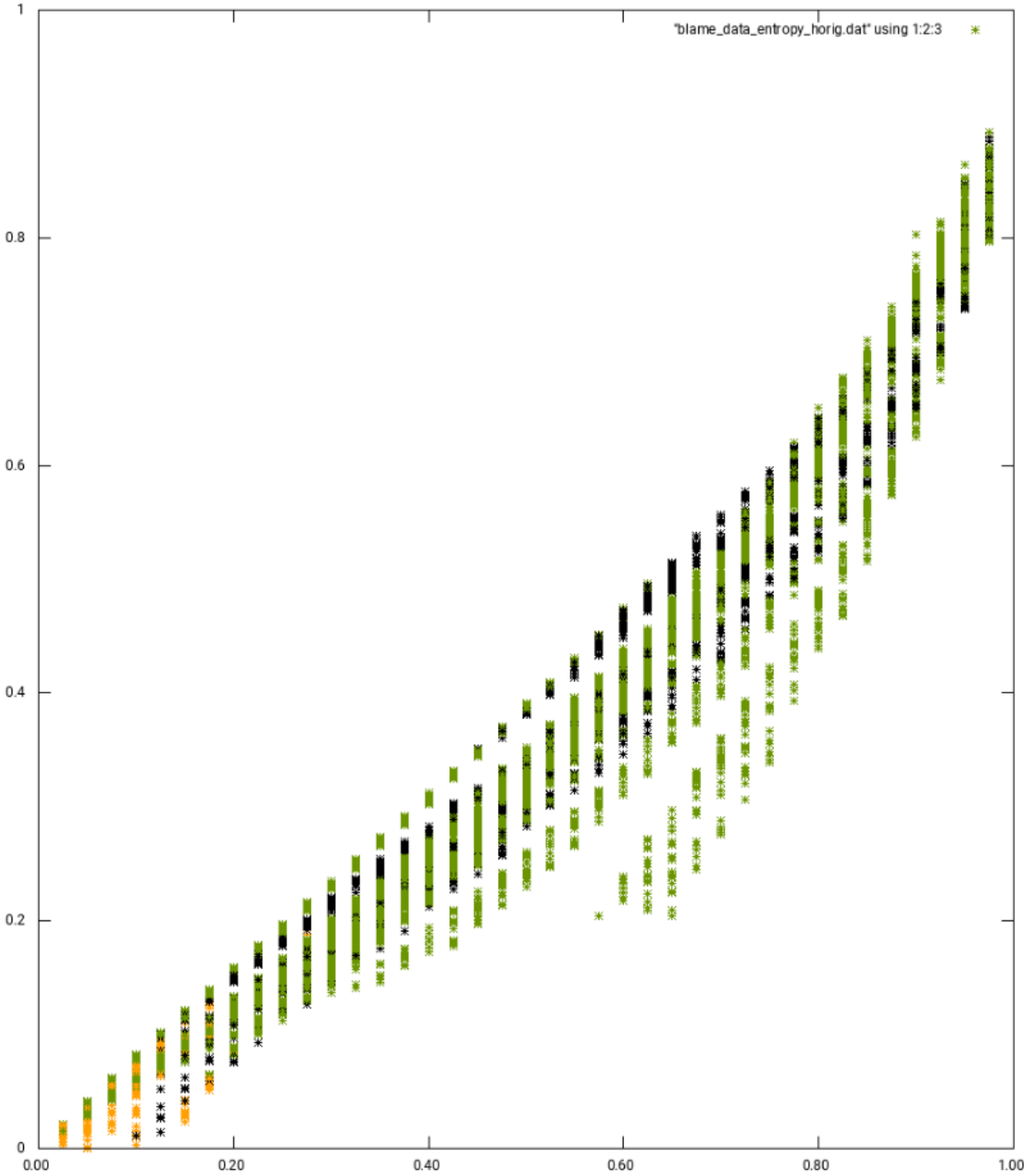
- The tTuple estimate only gives the lowest estimate when entropy is below 0.2ish
- 339 cases in 7800
- If you are claiming entropy of 0.1 you can be judged as having entropy of 0.01 by the tTuple test.



- The collision test contributes the lowest estimate in about 18% of cases, when entropy > 0.1
- The underestimation is largest around entropy from 0.6 to 0.8
- 1434 out of 7800

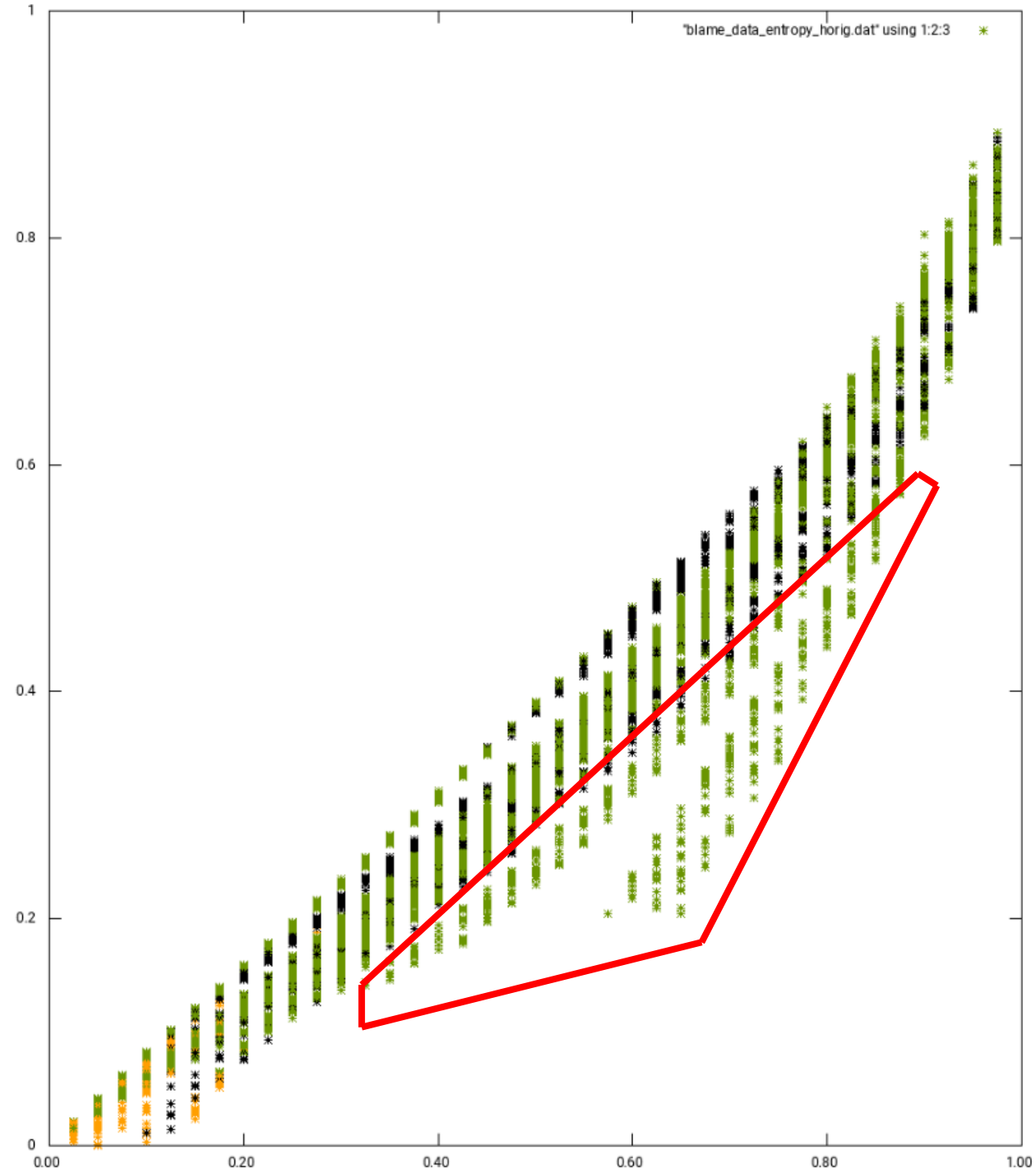


- Winner Winner! Chicken Dinner!
- The tCompression test contributes the lowest estimate for 77.2% of cases across the full entropy range.
- The underestimation is largest at entropy 0.65
- 6026 out of 7800



alg_idx	Count	Test Name
1	0	Most Common Value
2	1434	Collision Test Estimate
3	0	Markov Test Estimate
4	6026	tCompression Test Estimate
5	339	T-Tuple Test Estimate
6	0	LRS Test Estimate
7	1	Multi Most Common in Window (MultiMWC) Prediction Test Estimate
8	0	Lag Prediction Estimate
9	0	Multi Markov Model with Counting (MultiMMC) Prediction Test Estimate
10	0	LZ78Y Prediciton Test Estimate

Collision	Black
Tcompression	Green
T-Tuple	Yellow
MultiMWC	Red



Average Err	-0.15339
largest Err	-0.44592
Smallest Err	-0.00334
median Err	-0.15689

Over all 7800 tests

tCompression total	-911.636
tCompression count	6026
tCompression Av err	-0.15128
smallest tC err	-0.00352
largest tC Err	-0.44592

Over tests where tCompression gave the lowest estimate

not tCompression total	-238.804
not tCompression count	1774
not Tc Av Err	-0.13461
smallest not Tc Err	-0.00334
largest not Tc Err	-0.27637

Over tests where any other test gave the lowest estimate

Maurer's Universal Statistical Test Abstract:

"A new statistical test for random bit generators is presented which, in contrast to presently used statistical tests, is universal in the sense that it can detect any significant deviation of a device's output statistics from the statistics of a truly random bit source when the device can be modeled as an ergodic stationary source with finite memory but arbitrary (unknown) state transition probabilities. The test parameter is closely related to the device's per-bit entropy which is shown to be the correct quality measure for a secret-key source in a cryptographic application. The test hence measures the cryptographic badness of a device's possible defect. The test is easy to implement and very fast and thus well suited for practical applications. A sample program listing is provided."

The assumption of ergodicity and stationarity does not hold for physical sources.

The Maurer's Universal Statistic is questionable for physical sources.

Also. the lower bound adjustment is based on unsound assumptions:

Comments on Cryptographic Entropy Measurement

Anna M. Johnston
Juniper Networks
amj at juniper dot net

October 30, 2019

"The central limit theorem does not hold and confidence intervals are not well defined for H_∞."

Proposal : Remove SP800-90B 6.3.4 (Compression Estimate)

Some Statistics!

Simplifying The SCC Equation for Binary Data

Start with

$$scc = \frac{n\left(\sum_{i=0}^{n-1} x_i x_{(i+1 \mod n)}\right) - \left(\sum_{i=0}^{n-1} x_i\right)^2}{n\left(\sum_{i=0}^{n-1} x_i^2\right) - \left(\sum_{i=0}^{n-1} x_i\right)^2}$$

$$\left(\sum_{i=0}^{n-1} x_i\right)$$

Count occurrences of 1

$$\left(\sum_{i=0}^{n-1} x_i^2\right)$$

When xi == 1 or 0, xi = $(xi)^2$
So is identical to above count.

n is the number of bits. We can fix this for the test HW.

$$\left(\sum_{i=0}^{n-1} x_i x_{(i+1 \mod n)}\right)$$

*0*0=0*
*0*1=0*
*1*0=0*
*1*1=1*

So just count occurrences of 11

Reduce degrees of freedom by 1 to eliminate the mod n.

End With

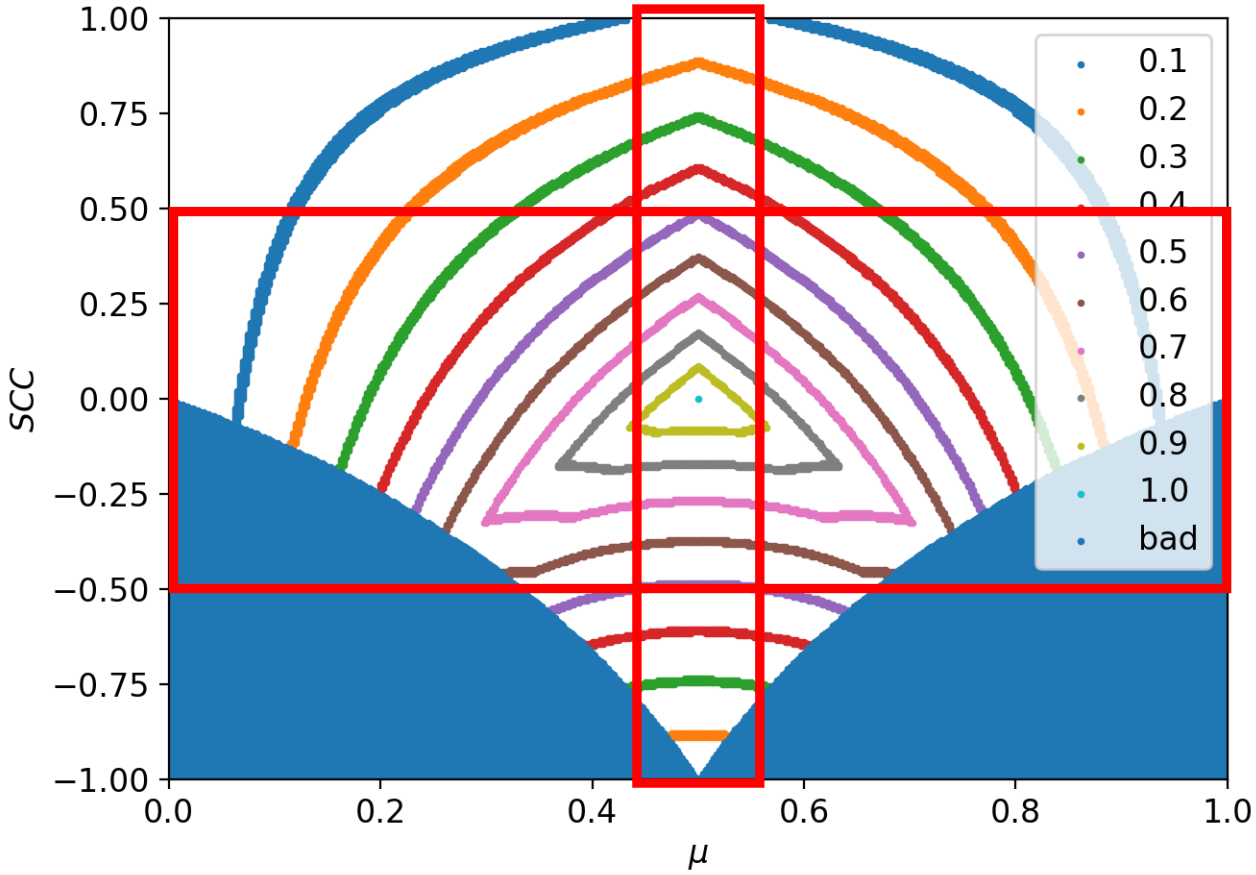
$$scc = \frac{(n-1)\ count11\ - count1^2}{(n-1)\ count1\ - count1^2}$$

$$scc = \frac{(n-1)\,count11\ - count1^2}{(n-1)\,count1\ - count1^2}$$

Rearrange for 11s and 1s

Test puts bounds on 11s and 1s counts

What entropy is the test testing for?

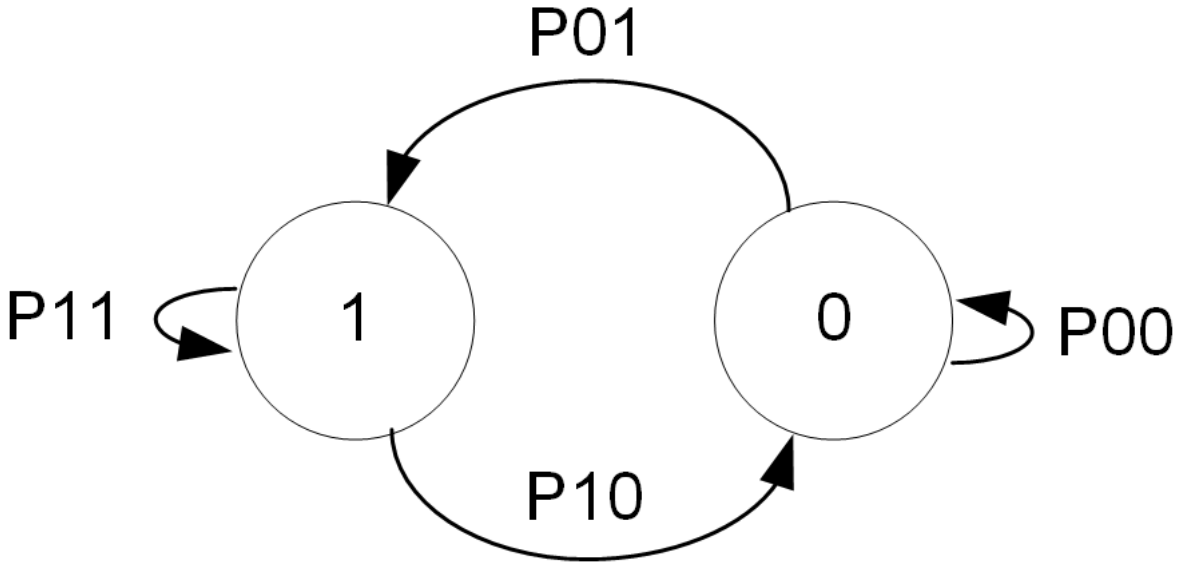


- The mean and SCC are separate things that can be tested for (efficiently in hardware)

- But they are related in ways that make swoopy lines when plotted against actual entropy levels as a function of mean and SCC making it hard to test for entropy levels

- Let’s use a really simple Markov models to link the two….

- A two state Markov model has two degrees of freedom.
- P01+P00 = 1
- P11+P10 = 1
- As a generator, the P01, P10 pair can be set to generate data with any combination of bias and SCC using the equations below

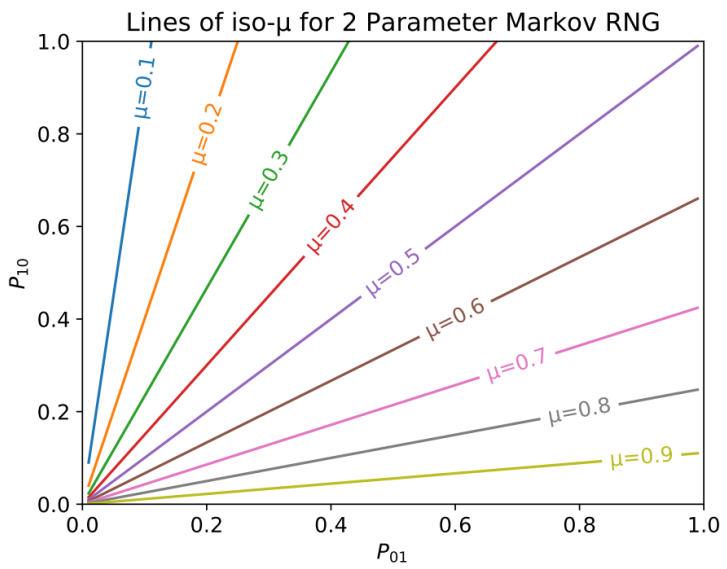


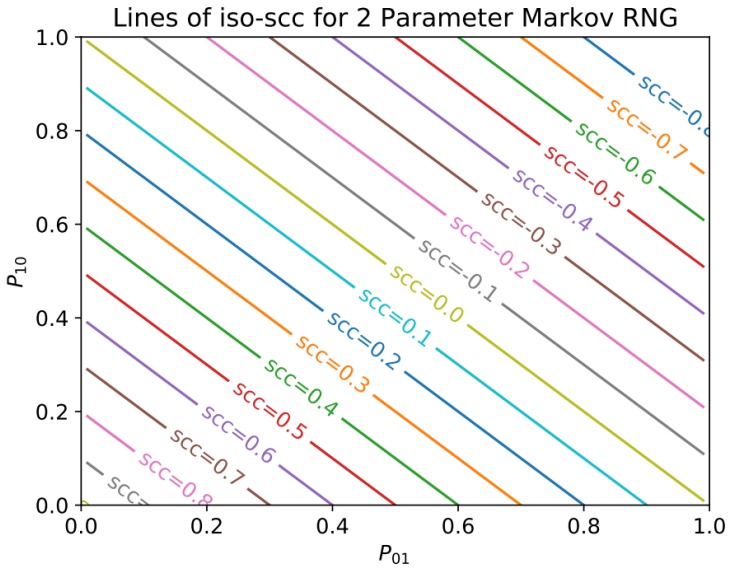
$$\mu = \frac{P_{01}}{P_{10} + P_{01}}$$

$$SCC = 1 - P_{10} - P_{01}$$

$$P_{10} = (1 - \mu)(1 - SCC)$$

$$P_{01} = \mu(1 - SCC)$$





Onto Entropy!

Unexpected MCVs turned out to be real

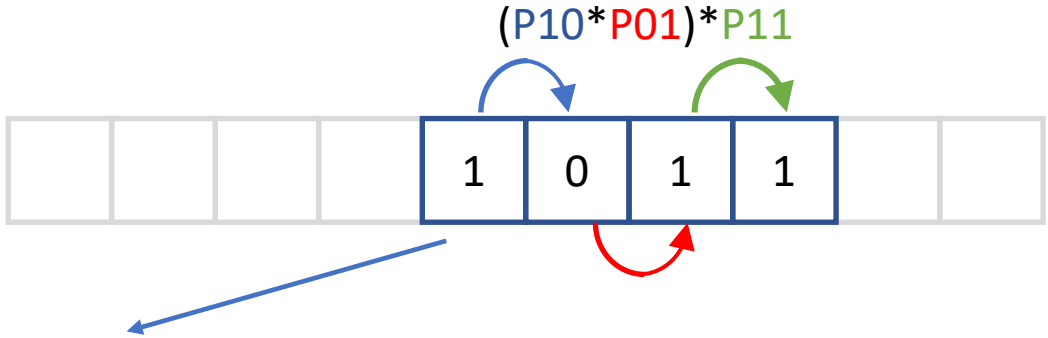
P_{10} \ P_{01}	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.95	0	0	0	0	0	0	0	2	4	A	A	A	5	5	A	A	5	A	A
0.9	0	0	0	0	0	0	0	0	4	A	A	A	5	5	5	5	A	5	A
0.85	0	0	0	0	0	0	0	0	4	A	5	5	5	5	5	5	A	5	5
0.8	0	0	0	0	0	0	0	0	4	A	A	A	5	5	5	5	5	5	A
0.75	0	0	0	0	0	0	0	0	4	5	A	A	5	A	A	5	5	5	5
0.7	0	0	0	0	0	0	0	0	2	2	5	A	A	A	A	A	A	A	5
0.65	0	0	0	0	0	0	0	0	0	5	5	A	5	A	A	A	A	A	5
0.6	0	0	0	0	0	0	0	0	0	2	A	A	5	A	A	A	A	5	A
0.55	0	0	0	0	0	0	0	0	0	A	A	5	A	A	A	A	A	A	A
0.5	0	0	0	0	0	0	0	0	0	E	B	A	5	D	5	5	B	5	A
0.45	0	0	0	0	0	0	0	0	F	F	F	F	F	B	D	B	B	B	B
0.4	0	0	0	0	0	0	0	F	F	F	F	F	F	F	F	F	F	D	B
0.35	0	0	0	0	0	0	0	F	F	F	F	F	F	F	F	F	F	F	F
0.3	0	0	0	0	0	F	F	F	F	F	F	F	F	F	F	F	F	F	F
0.25	0	0	0	0	0	F	F	F	F	F	F	F	F	F	F	F	F	F	F
0.2	0	0	0	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
0.15	0	0	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
0.1	0	0	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
0.05	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F

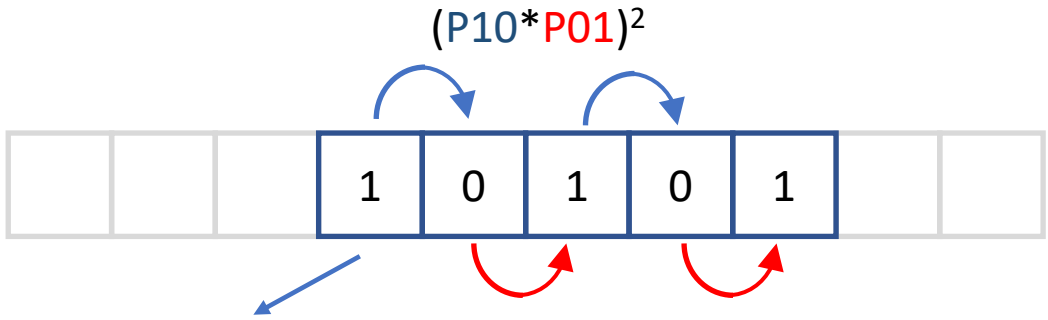
Each point is the most common 4 bit symbol in 1MiB of data generated from the Markov model

MCV AT $P_{10} = 0.55, P_{01} = 0.45$

bitwidth	MCV
4	1011
5	10101
6	101011
7	1010101
8	10101011
9	101010101

E.G. 101 is the most probable two step path. But P11 > P10





P01, P10, n

ALL THE POSSIBLE EVEN BITWIDTH MCV SYMBOLS

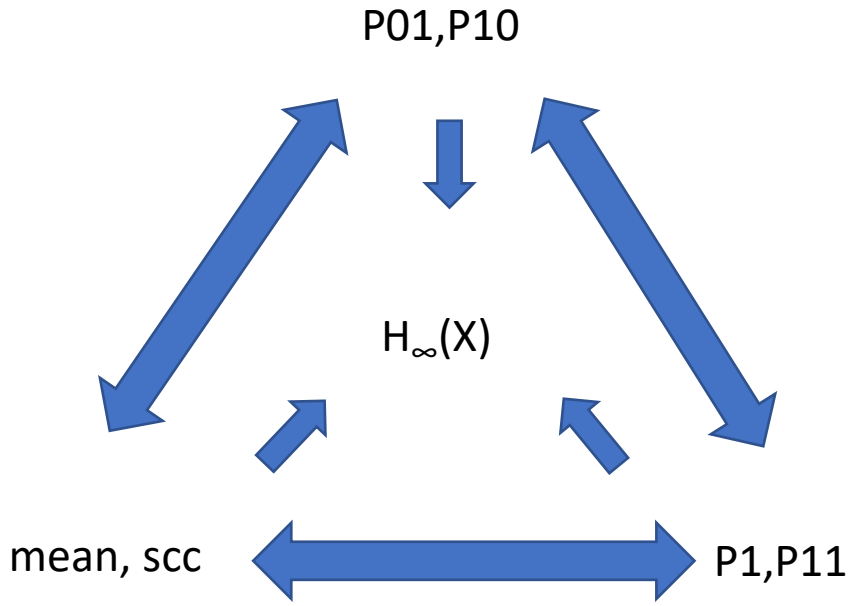
Symbol	Probability
$\{1\}^n$	$(P_{01}+P_{11})(P_{11}^{n-1})$
$\{0\}^n$ repeated n times	$(P_{00}+P_{10})(P_{00}^{n-1})$
$\{01\}^{\frac{n-2}{2}}\{00\}$	$(P_{00}+P_{10})(P_{01}P_{10})^{\frac{n-1}{2}}P_{00}$
$\{01\}^{\frac{n}{2}}$	$(P_{00}+P_{10})(P_{01}P_{10})^{\frac{n}{2}}$
$\{10\}^{\frac{n-2}{2}}\{11\}$	$(P_{01}+P_{11})(P_{10}P_{01})^{\frac{n-1}{2}}P_{11}$
$\{10\}^{\frac{n}{2}}$	$(P_{01}+P_{11})(P_{10}P_{01})^{\frac{n}{2}}$

ALL THE POSSIBLE ODD BITWIDTH MCV SYMBOLS

Symbol	Probability
$\{1\}^n$	$(P_{01}+P_{11})(P_{11}^{n-1})$
$\{0\}^n$	$(P_{00}+P_{10})(P_{00}^{n-1})$
$\{01\}^{\frac{n-2}{2}}\{0\}$	$(P_{00}+P_{10})(P_{01}P_{10})^{\frac{n-1}{2}}$
$\{10\}^{\frac{n-2}{2}}\{1\}$	$(P_{01}+P_{11})(P_{10}P_{01})^{\frac{n-1}{2}}$

MCV=max(those cases)

H_inf(X) = -log2(P(MCV))



The Markov model can be used to generate data with known entropy

By fitting to the model, the min entropy can be measured

`djent` will give you Markov parameters from data
`markov2p.py` will compute entropy from Markov parameters

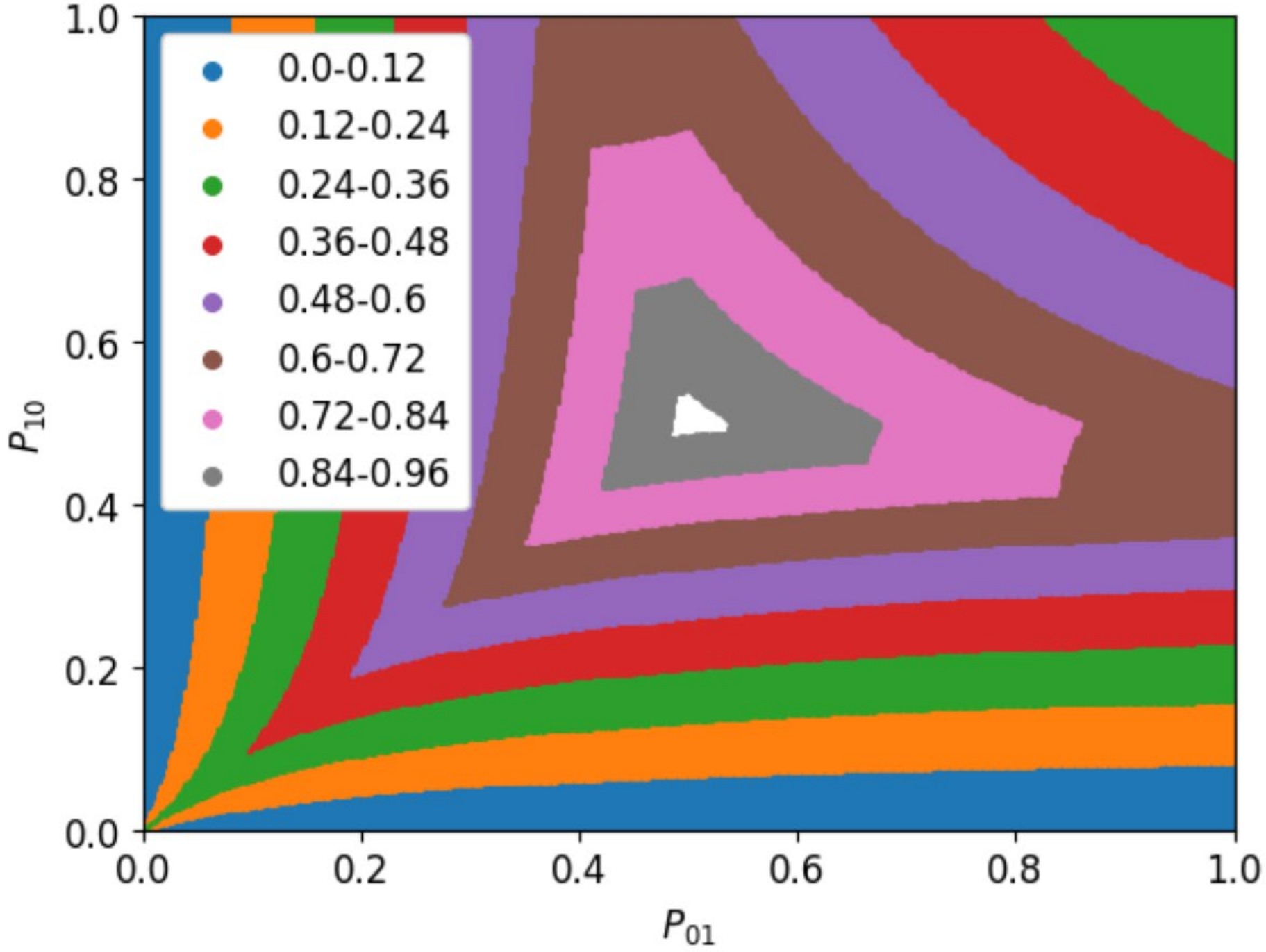
```
$ djenrandom -m markov_2_param --entropy=0.6 -k 1024 -b -s > file.bin

$ djent -b file.bin
 opening file.bin as binary
 Symbol Size(bits) = 1
   Min Entropy (by max occurrence of symbol 0) = 0.601632
   Analysing 8388600 1-bit symbols
   Shannon IID Entropy = 0.925765 bits per symbol
   Optimal compression would compress by 7.423505 percent
   Chi square: symbol count=8388601, distribution=848372.77, randomly exceeds 0.00 percent of the time
   Mean = 0.340992
   Monte Carlo value for Pi is 3.798721 (error 20.92 percent).
   Serial Correlation = 0.002939
   Longest Run Symbol = 0. Run Length = 38
   Probabilty of longest run being <= 38 = 0.999985
   Position of Longest Run = 892812 (0xd9f8c). Byte position 111601 (0x1b3f1)
   A 2 state Markov generator with transition probabilities P01=0.339990, P10=0.657071 would generate data
   with the same mean and serial correlation

$ python3
Python 3.7.0 (v3.7.0:1bf9cc5093, Jun 26 2018, 23:26:24)
[Clang 6.0 (clang-600.0.57)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import markov2p
>>> er,_,mcv = markov2p.p_to_entropy(p01=0.339990, p10=0.657071, bitwidth=8)
>>> er
0.5997142477854777
>>> mcv
0
```

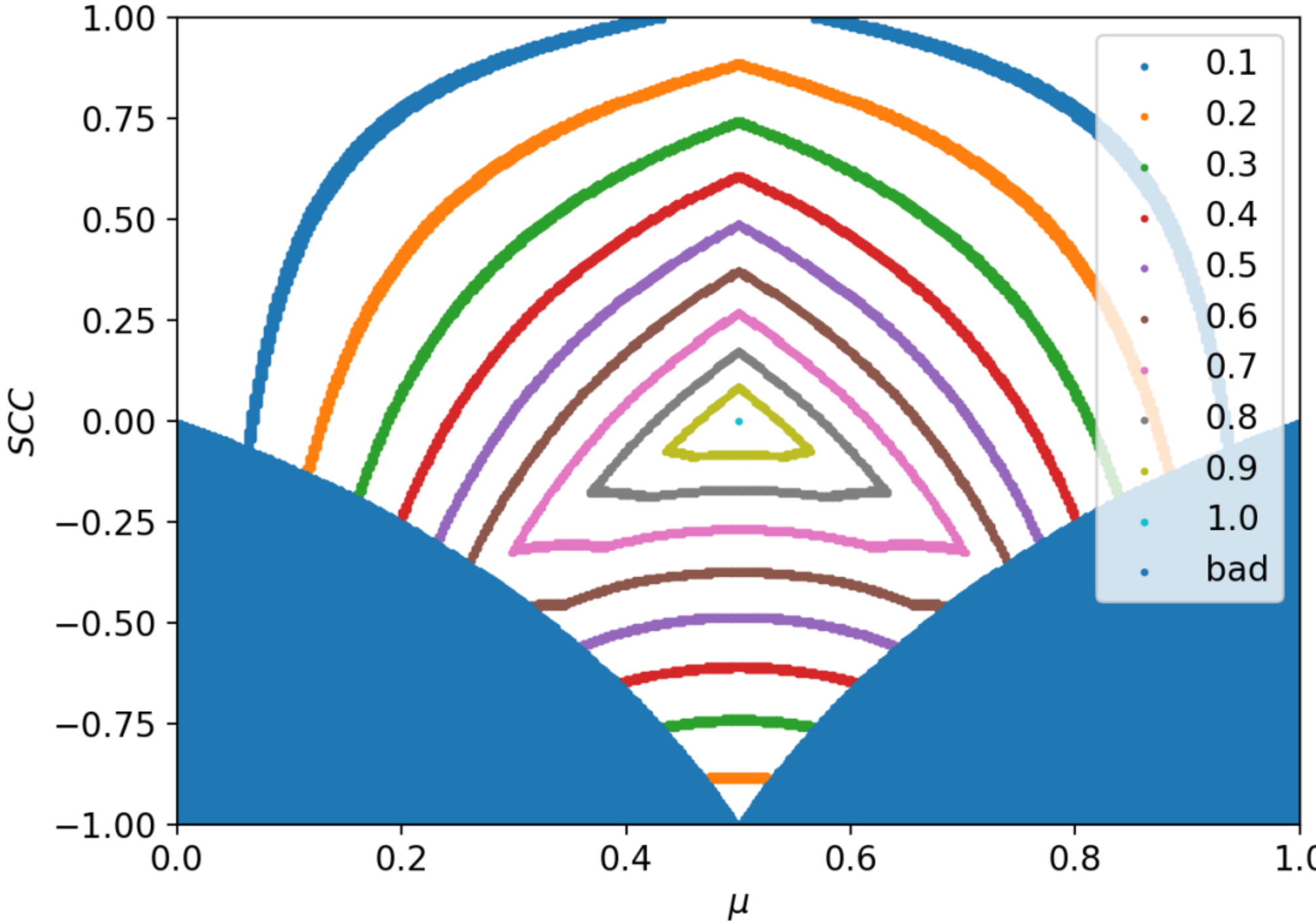
- Entropy vs p10,p01

- Swoopy lines. Not great for making online entropy tests

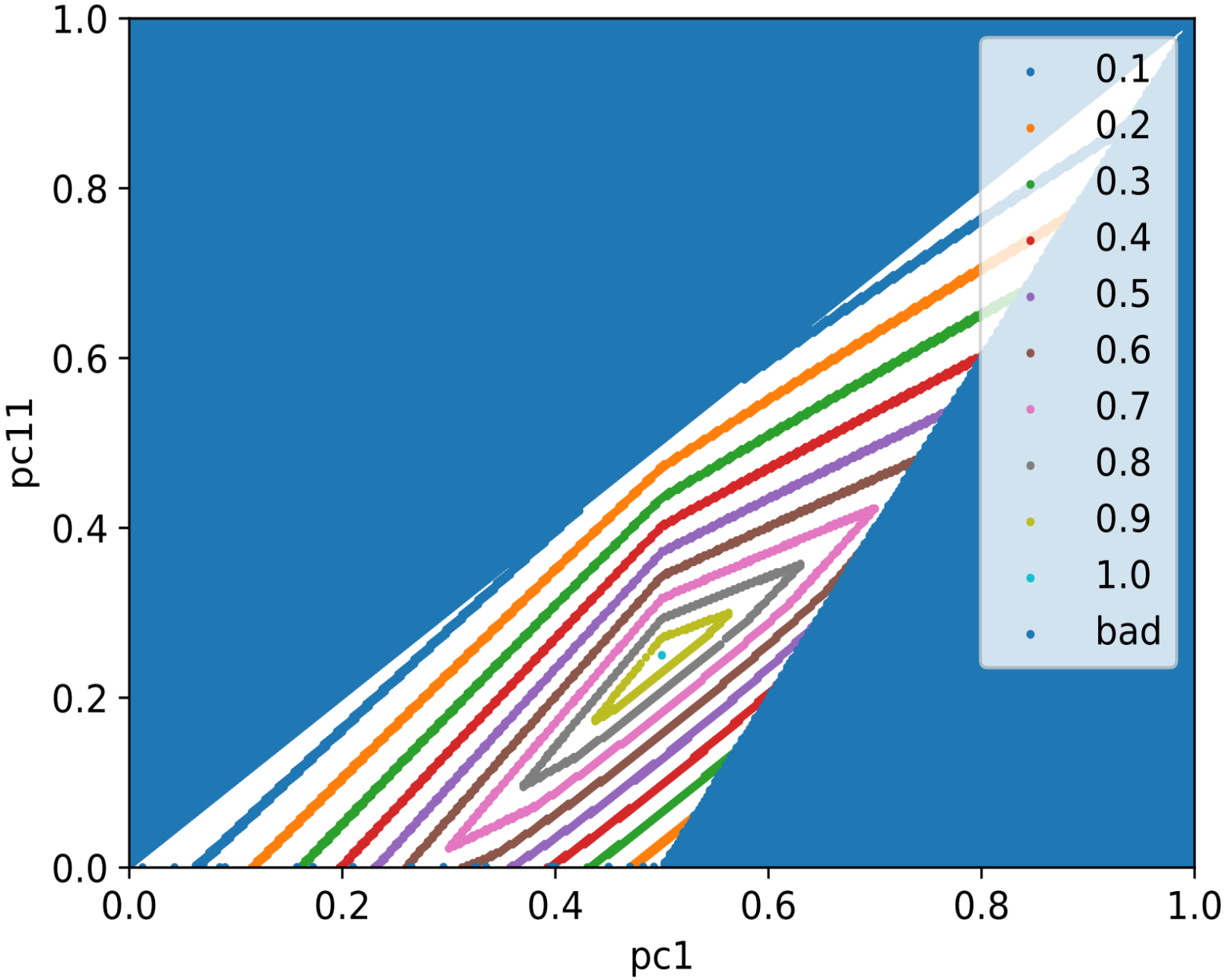


- Min Entropy vs mean,scc

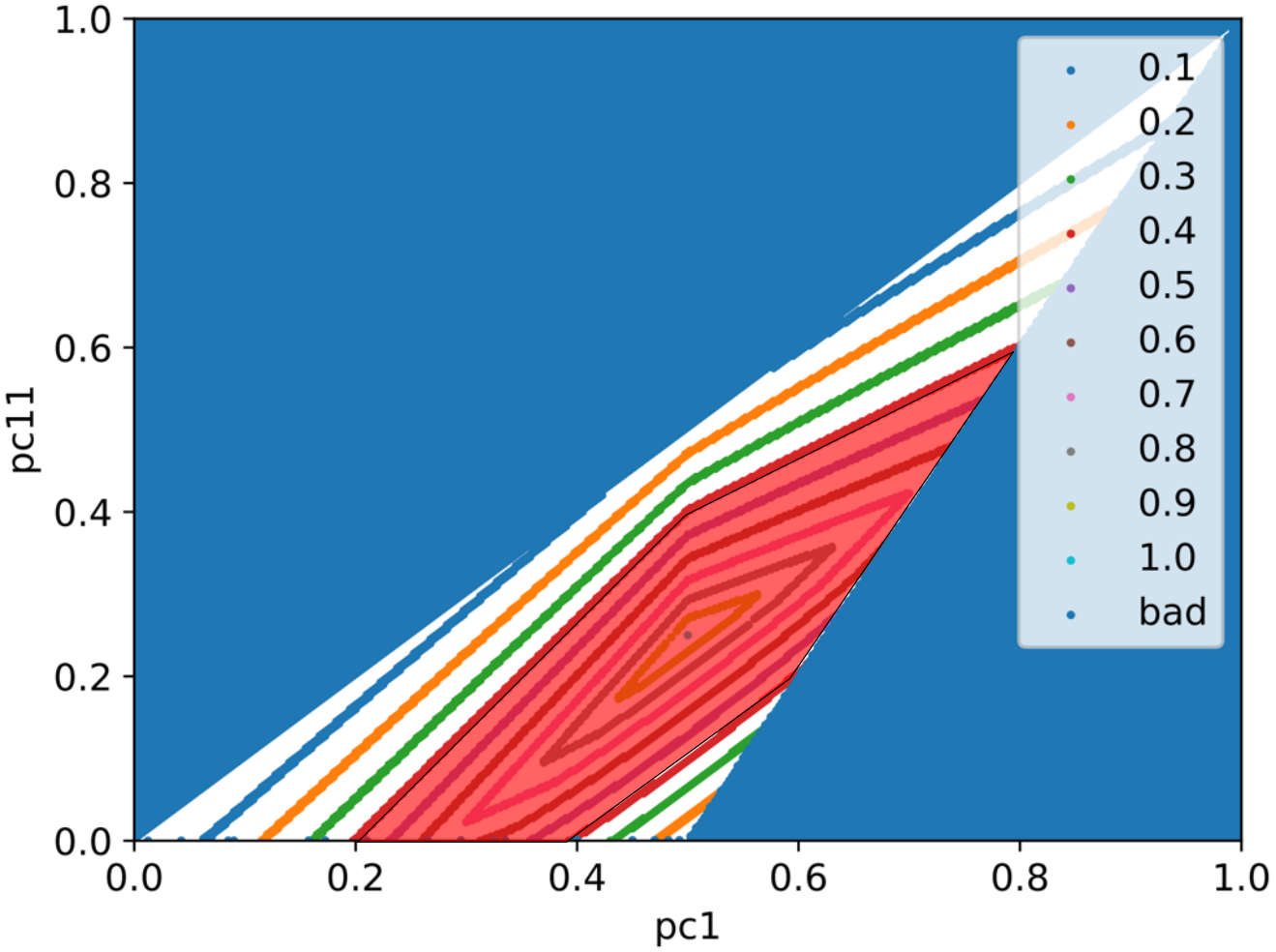
- Again, swoopy lines, OK for testing with bounds on P1,P11 as long as mean is close to 0.5 but loses selectivity if mean varies more.



- Plotting entropy against p1,p11 normalized to (0,1) interval shows mostly straight lines. The lower edge of each iso-entropy line is convex, meaning when approximated to a straight line, it is conservative, excluding good points, rather than including bad points

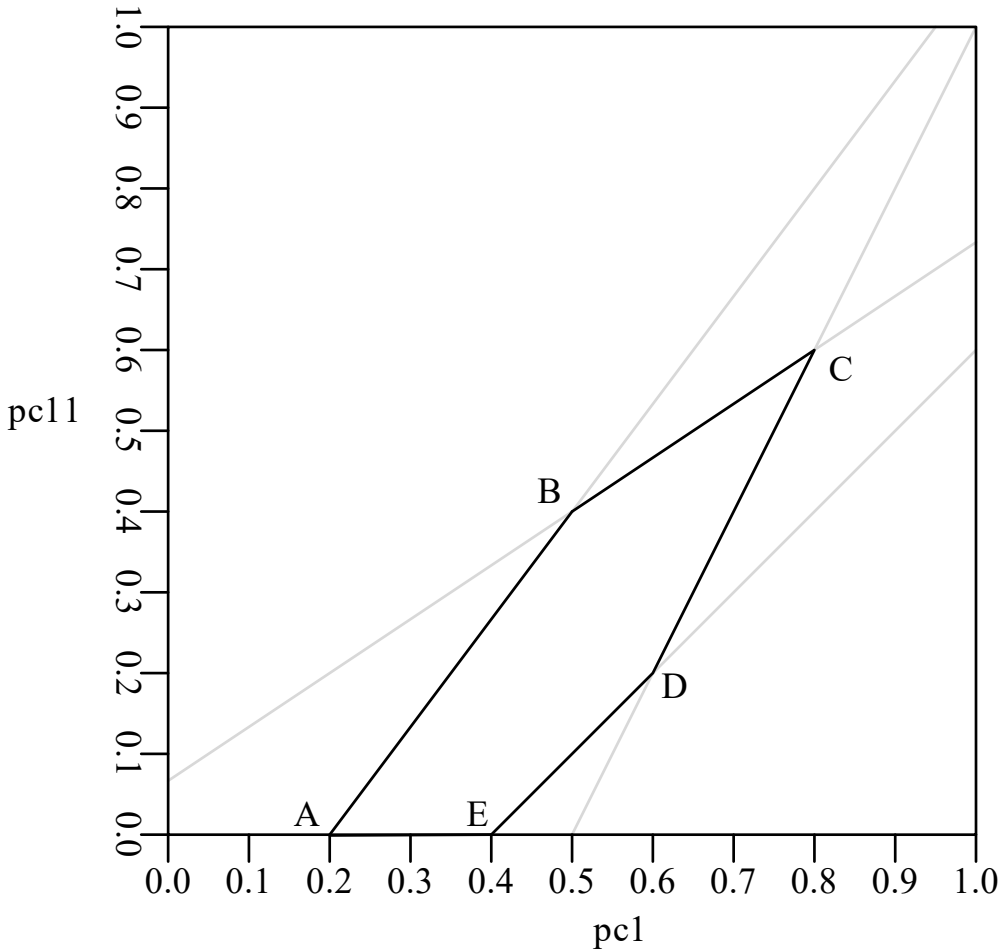


Test For Points Within an Entropy Contour



- Red region covers set of points with entropy rate 0.4 or higher.

Straight Edges Allow Linear Comparison Tests



AB $pc11 < \frac{4}{3}pc1 - \frac{4}{15}$

BC $pc11 < \frac{2}{3}pc1 - \frac{1}{15}$

DC $pc11 > 2pc1 - 1$

ED $pc11 > pc1 - 0.4$

Make The Coefficients Integer and Scale by n-1

AB $15pc11 < 15pc1 - 4$

BC $15pc11 < 10pc1 + 1$

DC $pc11 > 2pc1 - 1$

ED $5pc11 > 5pc1 - 2$

AB $15count11 < 15count1 - 9212$

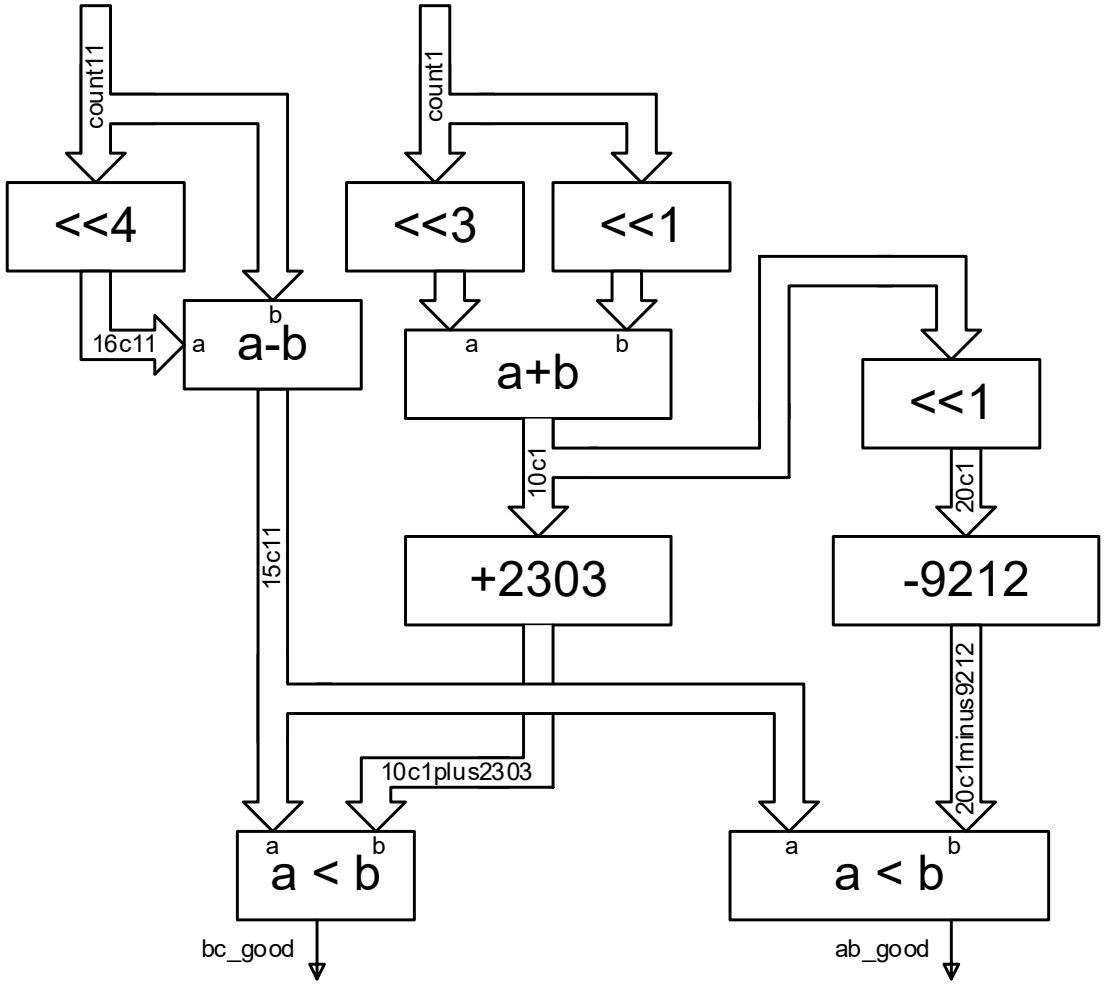
BC $15count11 < 10count1 + 2303$

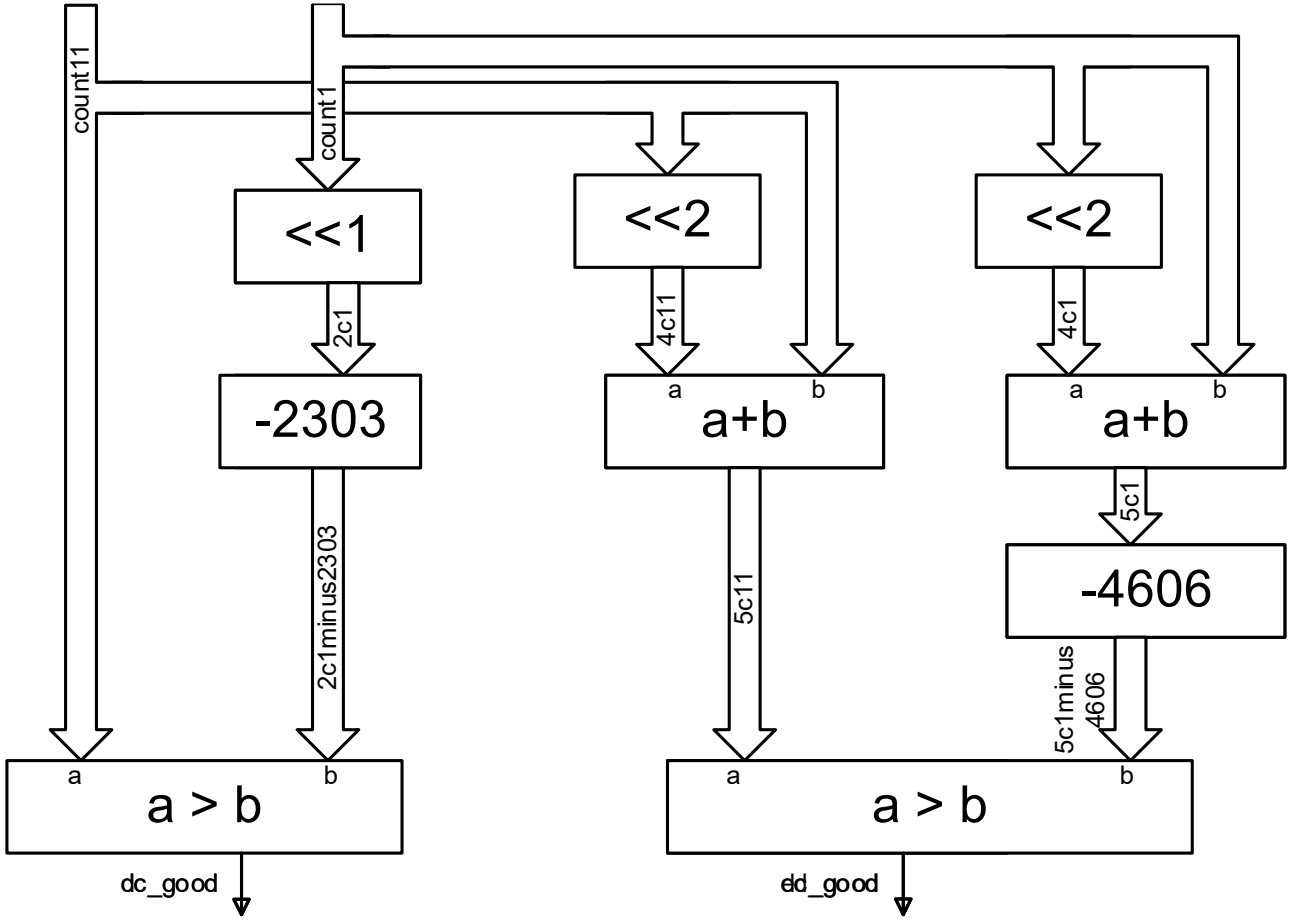
DC $count11 > 2count1 - 2303$

ED $5count11 > 5count1 - 4606$

- Now we have a test for entropy > 0.4, as a function of count1 and count11
- This is the Polygon Test

Efficient in Hardware – 17 bit signed numbers





Polygon Test Summary

- If mean and SCC describe the statistics of your noise source well
  - Hint: If you use feedback to maintain a random variable in a goodly random place, mean and SCC probably do describe the statistics of your noise source well
- Then it is possible to implement real time, online tests based on pattern counts, that have a tight pass/fail threshold at some entropy level.
- This is an improvement over tests aimed at distinguishing a failed from a functional source, which inevitably lead to high false positive and/or high false negative errors because they depend on splitting a uniform universe of symbols into a bad set and a good set.
- For obvious reasons, I dubbed this instance of such a test as the “polygon test” and it will feature in upcoming Intel RNGs as the vendor defined CHT.

Summary

- Vendor CHTs are hard because
  - The non-iid tests are unreliable and can give massive underestimation. The resulting number is references in the parameter calculations for tests that work against actual entropy, not the lower bound estimate we get from the non-iid tests
  - 4.5a and 4.5b requirements on vendor CHTs do not help. A test needs to be designed to catch failure modes. Re-creating what the RCT does and adding entropy bounds that are not real is a distraction from what is important in the test
- With 4.5a and 4.5b out the way
  - Given a free hand to design a test, tests that are not looking to meet 4.5a and 4.5b (although they might) can be designed with very sharp entropy cut-offs that are simple to implement and can run at full speed, while the RCT and APT contribute nothing to the detection capability of the CHT system.