Efficacy and Mitigation of the Cryptanalysis on AIM

Seongkwang Kim[1]
Jincheol Ha[2]
Mincheol Son[2]
Byeonghak Lee[1]

[1] Samsung SDS, Seoul, Korea
[2] KAIST, Daejeon, Korea

The 5th NIST PQC Standardization Conference

SAMSUNG SDS KAIST

Overview

- Cryptanalysis on AIM
  - AIMer is a NIST PQC round 1 candidate based on MPC-in-the-Head paradigm and symmetric primitive AIM
  - AIM has been analyzed recently up to 15-bit security degradation
  - We re-analyze the complexity of exhaustive search on AIM, and re-calculate the amount of the security degradation

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- AIM2 and AIMer v2.0
  - To mitigate the analyses, we propose a new symmetric primitive AIM2 which inherits the design rationale of AIM
  - We extensively analyze the security of AIM2
  - Despite of the patch, AIMer v2.0 enjoys faster performance

Symmetric Primitive AIM

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- $\mathrm{AIM}: \{0,1\}^n \times \mathbb{F}_{2^n} \rightarrow \mathbb{F}_{2^n}$ is the one-way function in AIMer v1.0
- It was designed to be efficiently proved by BN++

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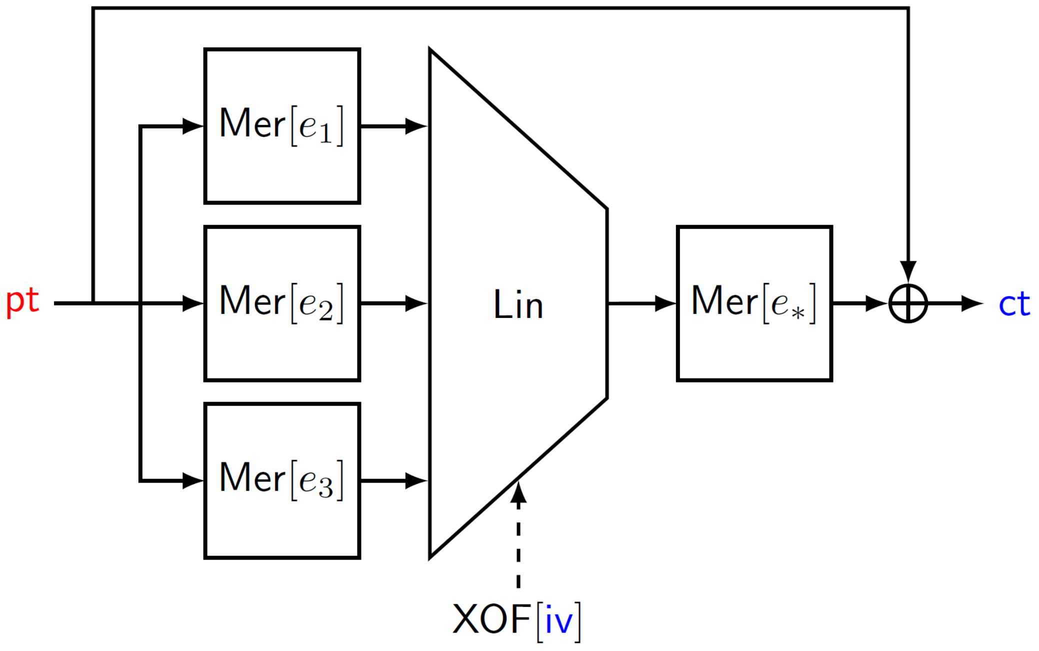
- $\mathrm{AIM}: \{0,1\}^n \times \mathbb{F}_{2^n} \rightarrow \mathbb{F}_{2^n}$ is the one-way function in AIMer v1.0
- It was designed to be efficiently proved by BN++
- Given a single pair $(\mathrm{iv}, \mathrm{ct})$ such that $\mathrm{iv} \leftarrow_{\$} \{0,1\}^n$ and $\mathrm{AIM}(\mathrm{iv}, \mathrm{pt}) = \mathrm{ct}$, it should be hard to find $\mathrm{pt}^* \in \mathbb{F}_{2^n}$ such that

$$\mathrm{AIM}(\mathrm{iv}, \mathrm{pt}^*) = \mathrm{ct}$$

- In AIMer, $pk = (\mathrm{iv}, \mathrm{ct})$ and $sk = (pk, \mathrm{pt})$

Symmetric Primitive AIM



- Mersenne S-box
  - $\mathrm{Mer}[e](x) = x^{2^e - 1}$
  - Invertible, high-degree, quadratic relation
  - Requires a single multiplication
  - Produces $3n$ quadratic equations

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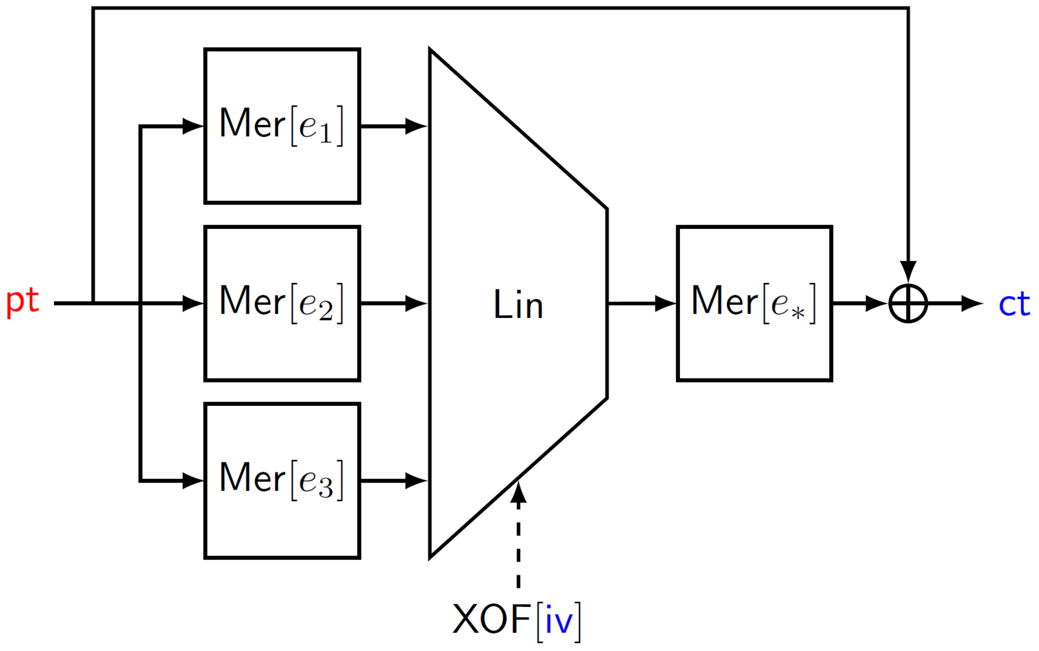

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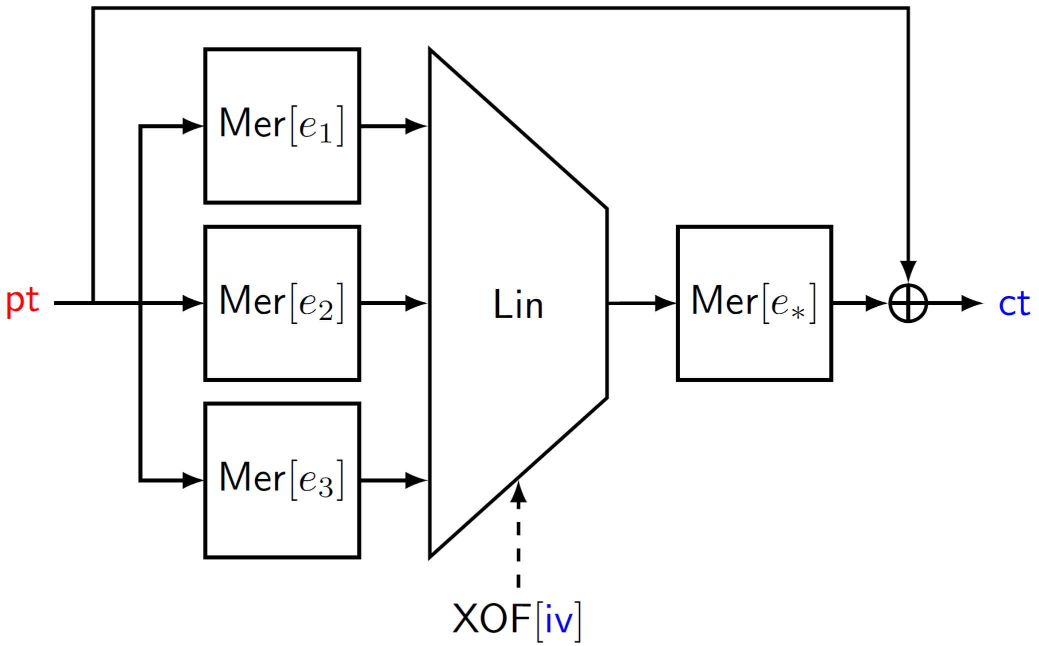

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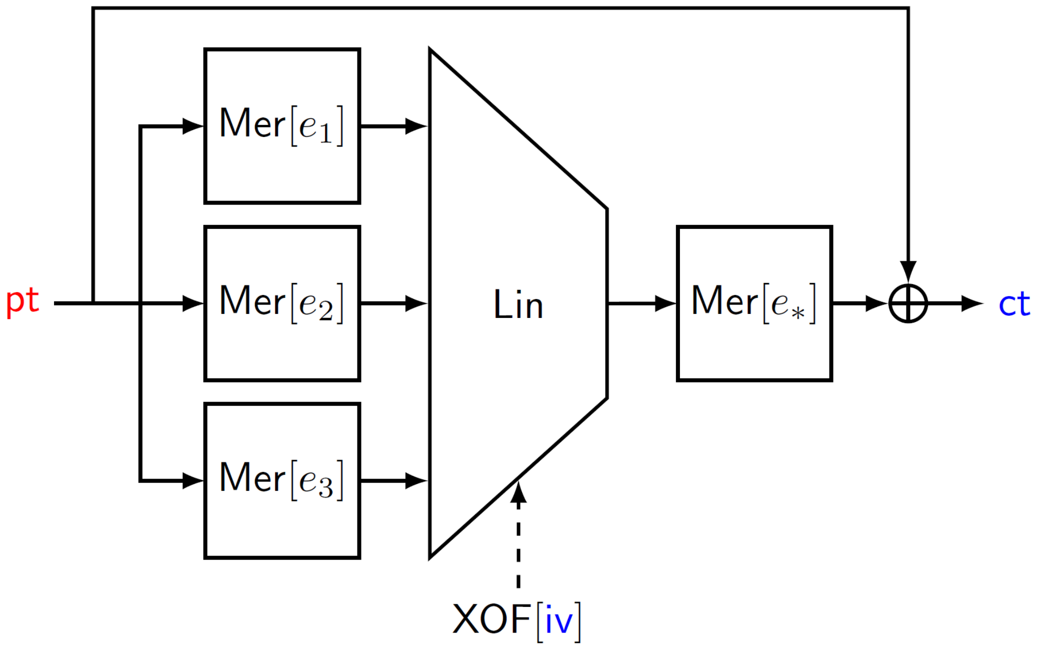

Scheme	λ	n	ℓ	e_1	e_2	e_3	e_*
AIM-I	128	128	2	3	27	-	5
AIM-III	192	192	2	5	29	-	7
AIM-V	256	256	3	3	53	7	5

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Analyses on AIM

Exhaustive Search on AIM

- In the conference version, the complexity of exhaustive search on AIM was overestimated
- The reason is the addition chain structure of AIM
- For example, AIM-I requires only 6 multiplications for evaluating 2 S-boxes

$$x \to x^{2^2-1} \to x^{2^3-1} \to x^{2^6-1} \to x^{2^{12}-1} \to x^{2^{24}-1} \to x^{2^{27}-1}$$

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	Previous Cost	**Current Cost**	AES Cost
AIM-I	149.0	146.3	143
AIM-III	214.4	211.8	207
AIM-V	280.0	276.7	272

Table. Complexity of exhaustive search attack on AIM and AES in log

Recent Analyses on AIM

- Recent analysis on AIM
  - [LMOM23] Fast exhaustive search, claiming up to 15-bit security degradation
  - [Liu23] Less costly algebraic attack, but not broken
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- Mainly, there are two vulnerabilities in the structure of AIM
  - Low degree representation in n variables ⇒ Fast exhaustive search attack
  - Common input to the parallel Mersenne S-boxes ⇒ Structural vulnerability

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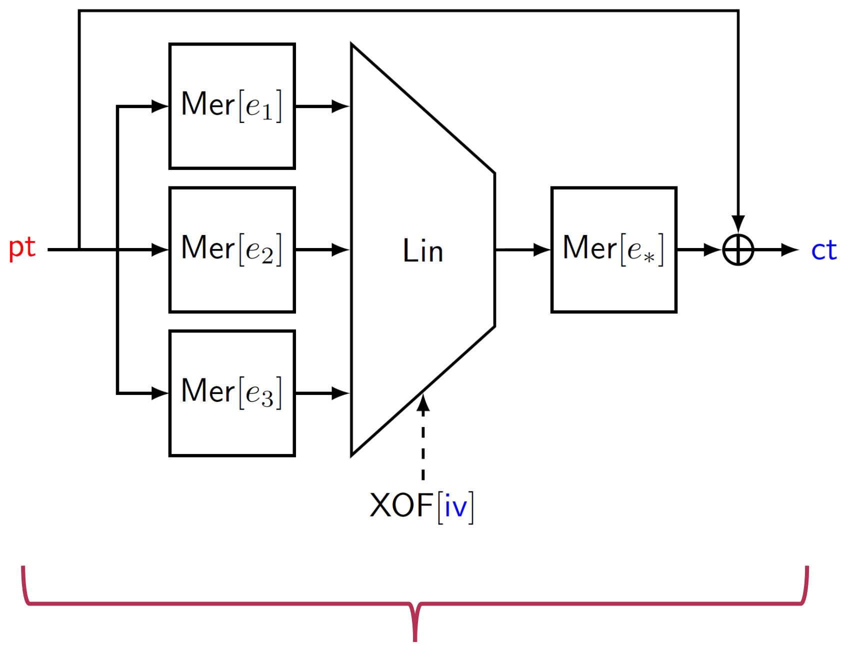
Fast Exhaustive Search Attack (Liu et al.)



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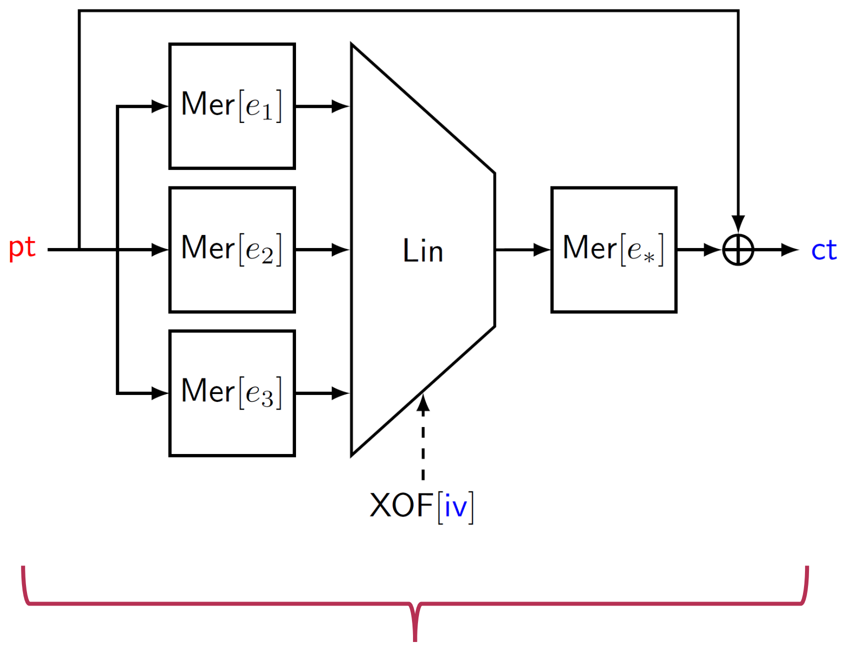

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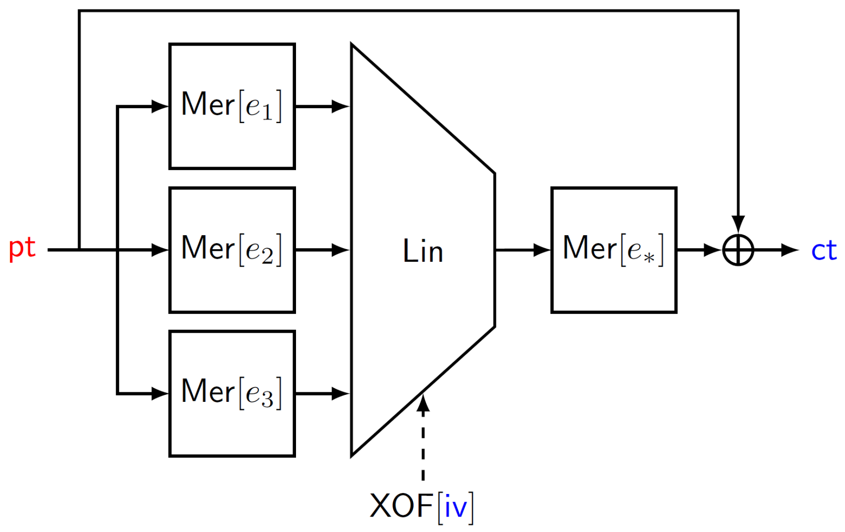

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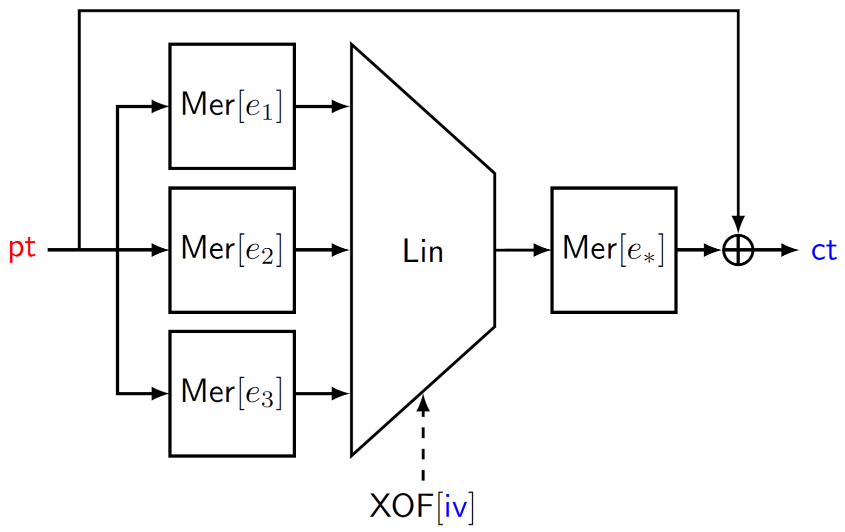

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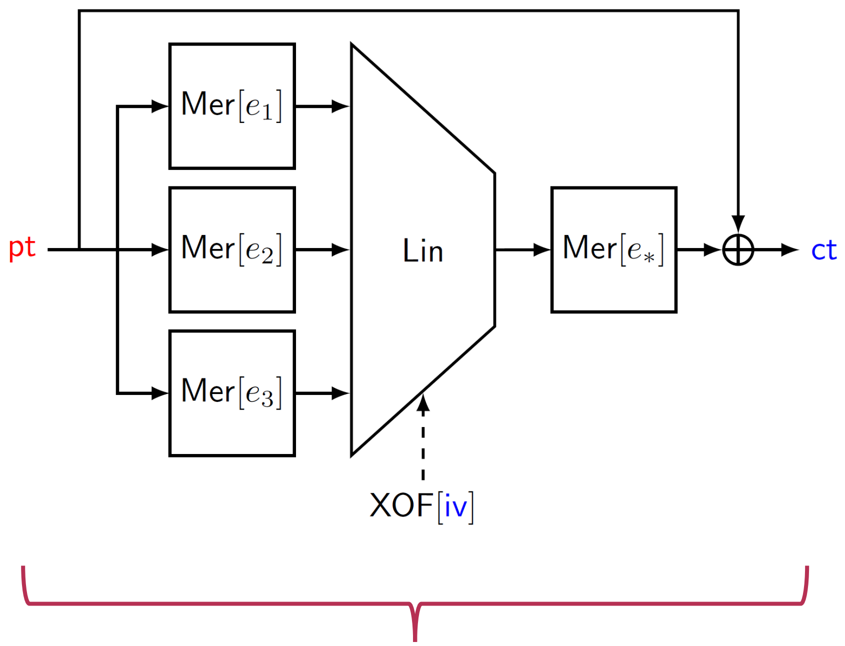
- If degree d is small enough, this fast exhaustive search is faster than naive brute-force search
- The result of Liu et al. (updated security degradation)

	n	Deg	Log(Time) [bits]	Log(Mem) [bits]
AIM-I	128	10	136.2 (−10.1)	61.7
AIM-III	192	14	200.7 (−11.1)	84.3
AIM-V	256	15	265.0 (−11.7)	95.1

Easier System to Solve (Liu)



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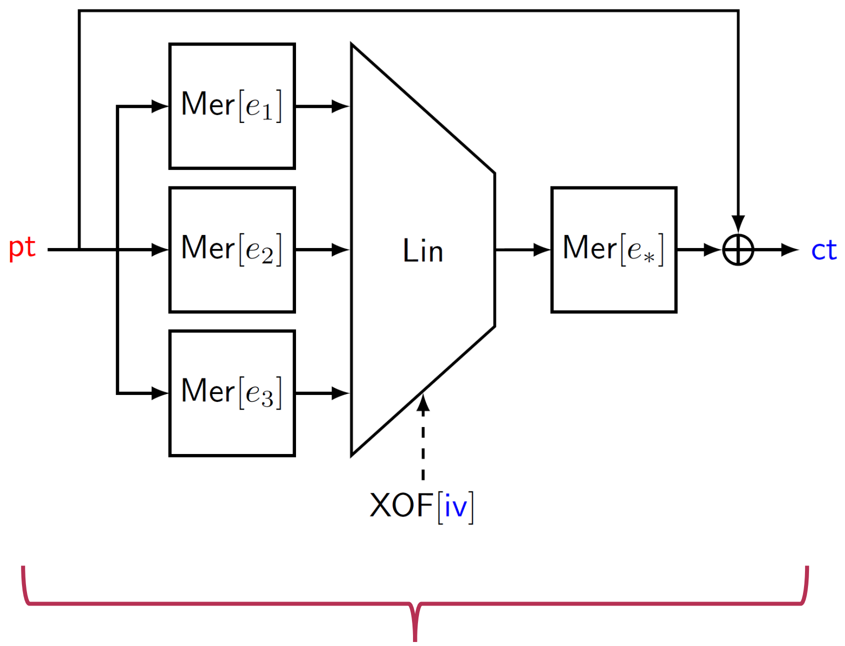

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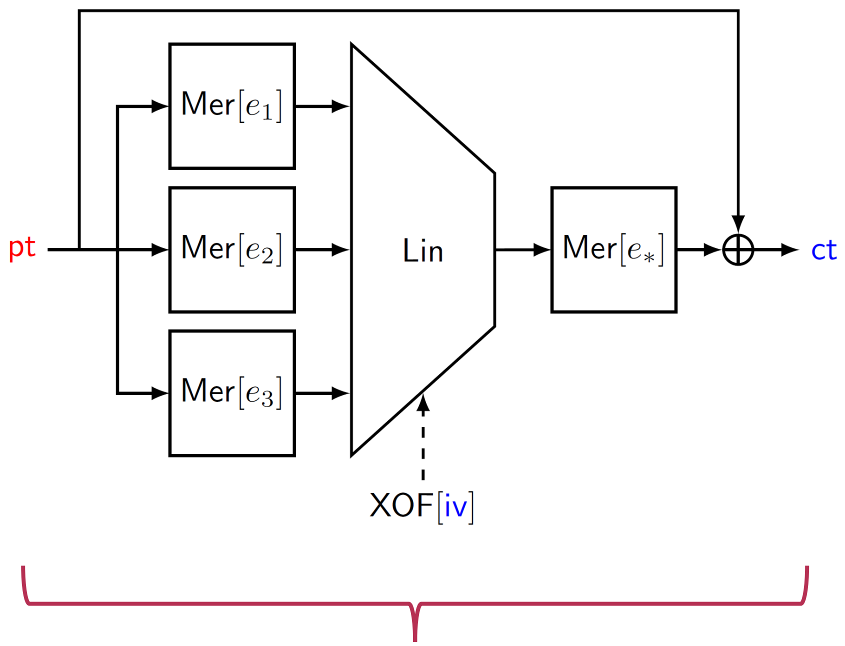

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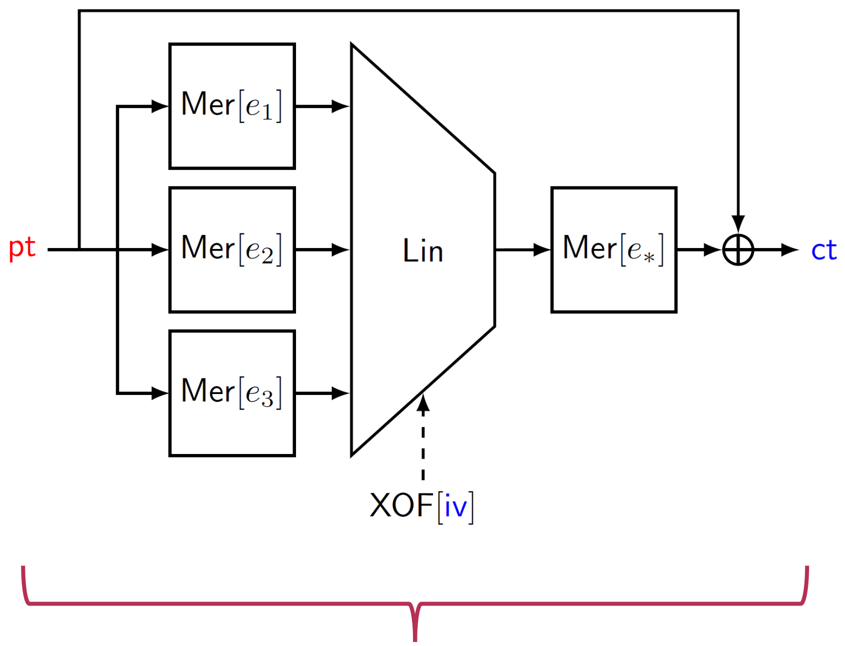

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- The result of Liu (our estimation)

	n	Log(Time*) [bits]	Log(Time**) [bits]
AIM-I	128	124.8 (−18.8)	158.3 (+14.4)
AIM-III	192	157.5 (−54.3)	226.5 (+14.7)
AIM-V	256	188.9 (−87.8)	290.2 (+13.5)

*Assumption: Every equations generated by XL are linearly independent (unrealistic)
**Assumption: XL finishes at the degree of regularity

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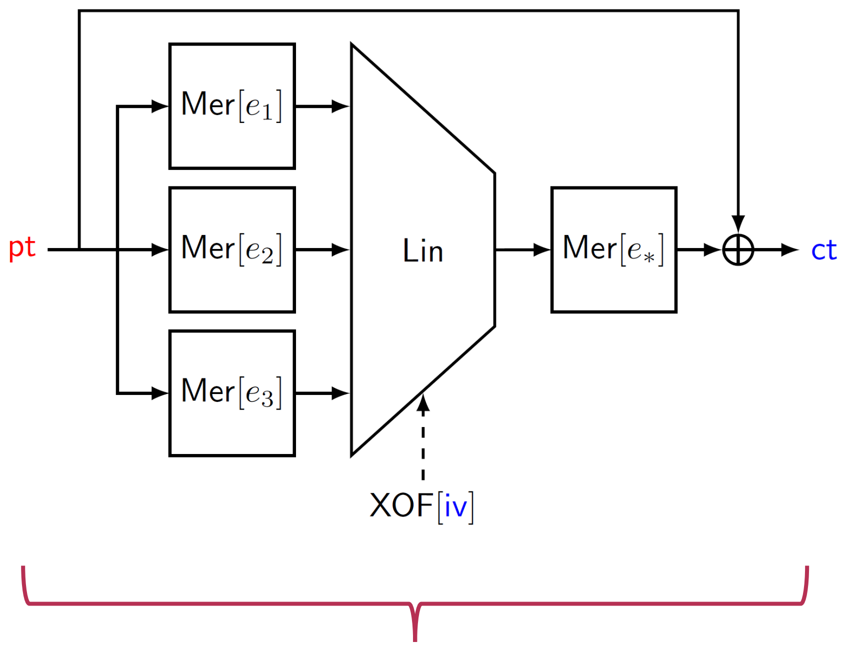

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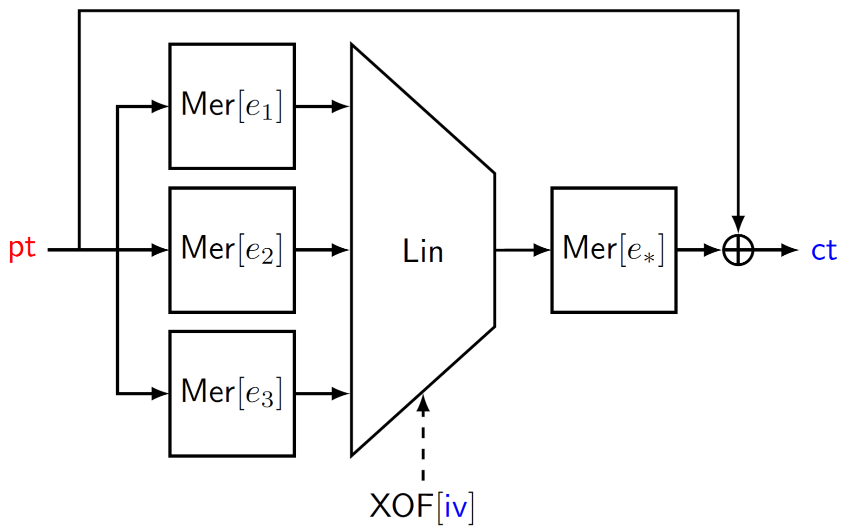

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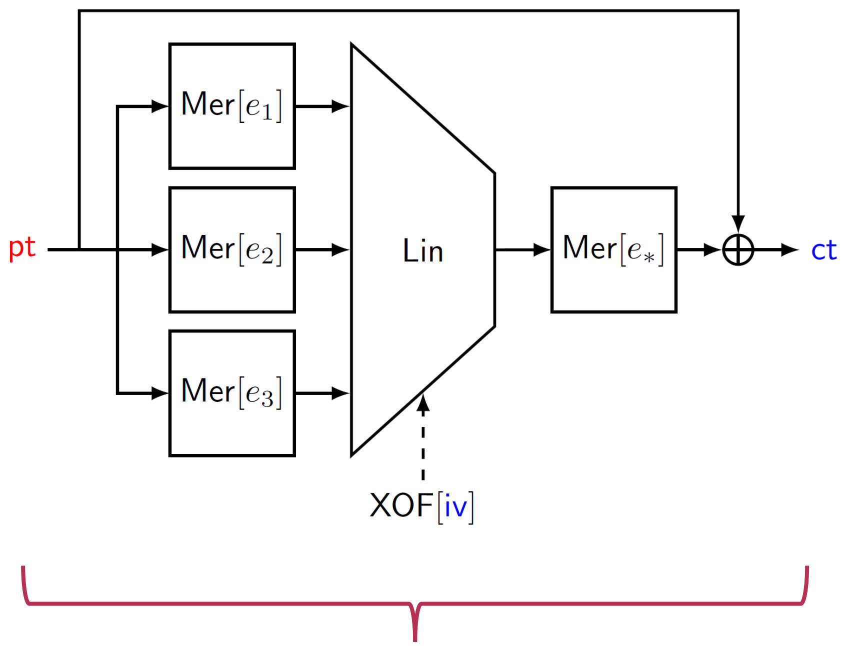

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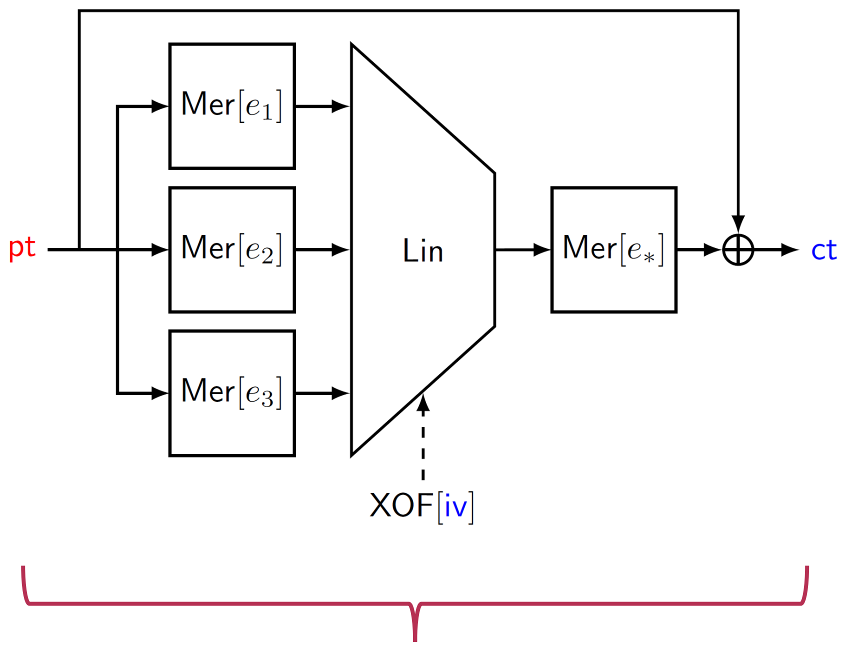

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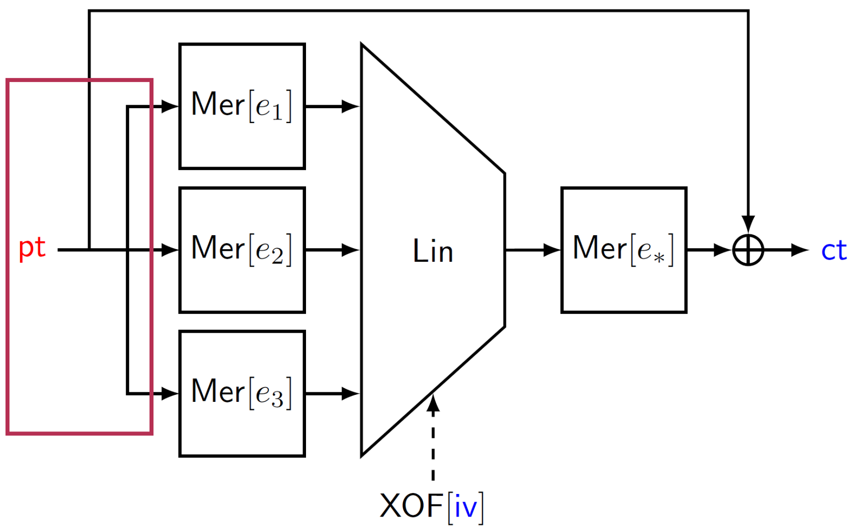
- The result of Saarinen (new estimation)

	n	#mult	Log(Time) [bits]
AIM-I	128	3	145.0 (−1.3)
AIM-III	192	3	210.2 (−1.6)
AIM-V	256	4	275.5 (−1.2)

Structural Vulnerability (Zhang et al.)



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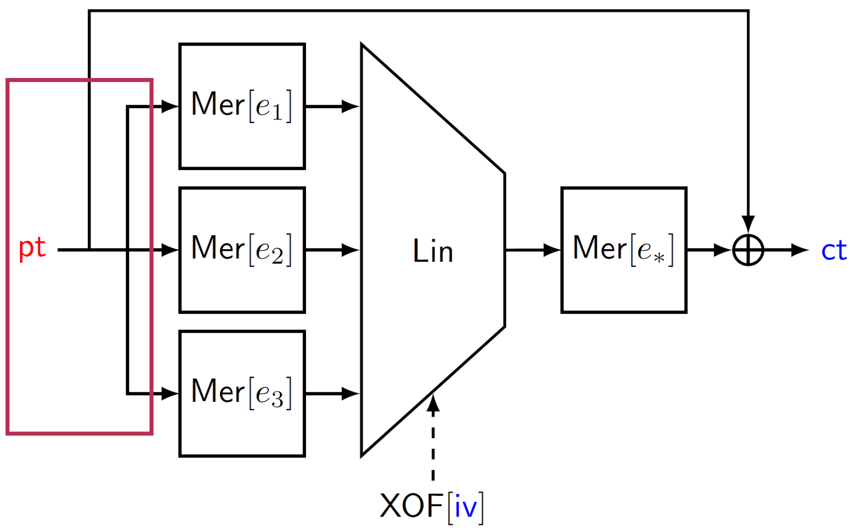

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- Find some $d|(2^n - 1)$ such that

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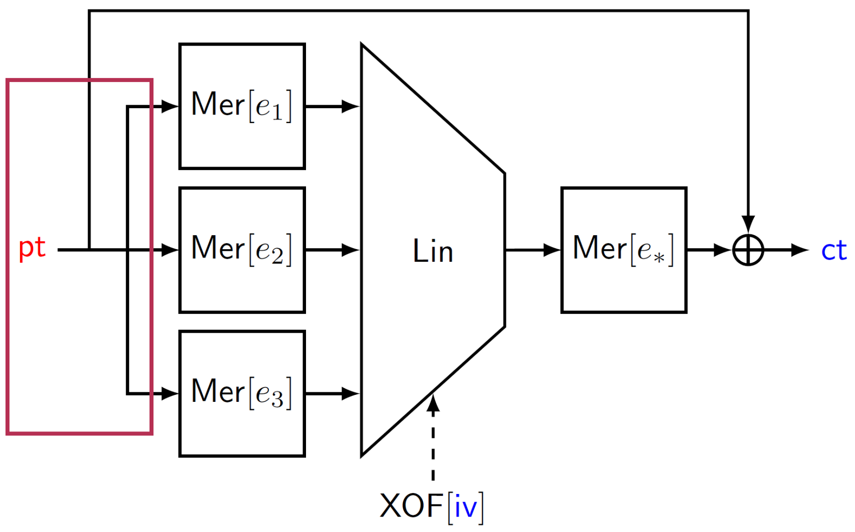

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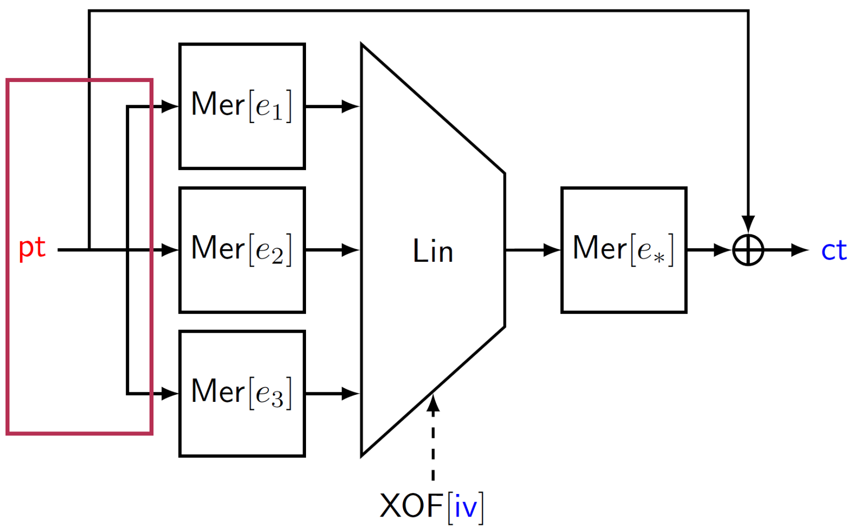

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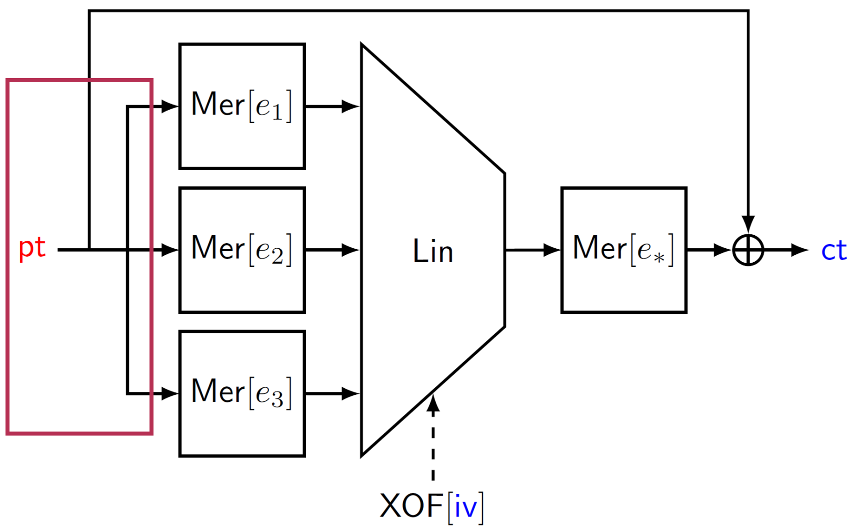

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Summary of Analyses on AIM

- The main vulnerabilities of AIM are:
  - Low algebraic degree
  - No domain separation
- By our complexity estimations, the amount of security degradation is clarified or reduced
- Some turn out to be not as powerful as claimed

	FES (Liu et al.)	Easier System (Liu)	Efficient Search (Saarinen)	Linearization (Zhang et al.)	**Exhaustive Search**	AES Cost
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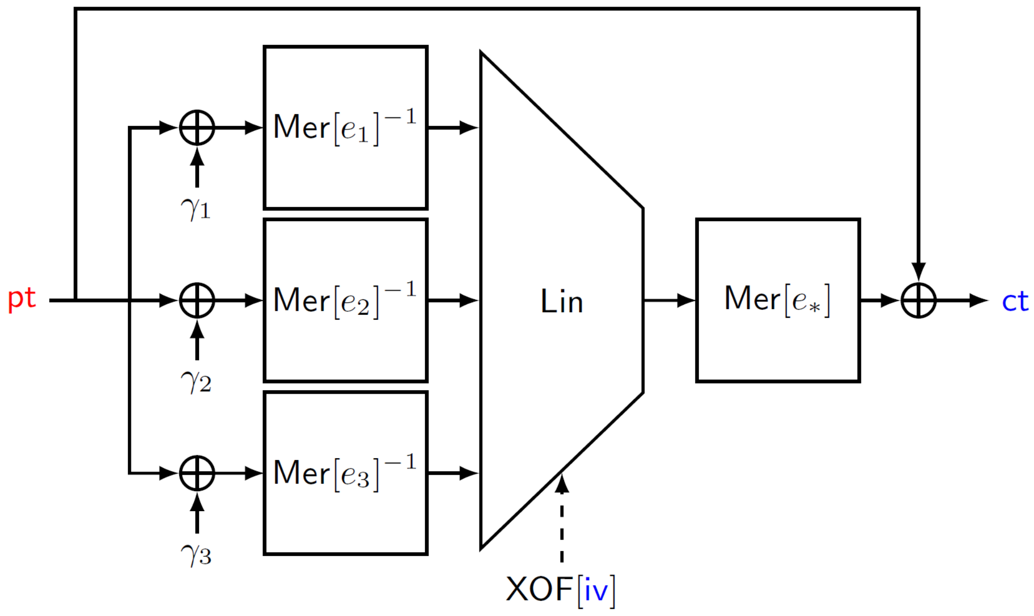
AIM2 and Analysis

AIM2: Secure Patch for Algebraic Attacks



- Inverse Mersenne S-box
  - $\mathrm{Mer}[e]^{-1}(x) = x^a$
  - $a = (2^e - 1)^{-1} \bmod (2^n - 1)$
  - More resistant to algebraic attacks

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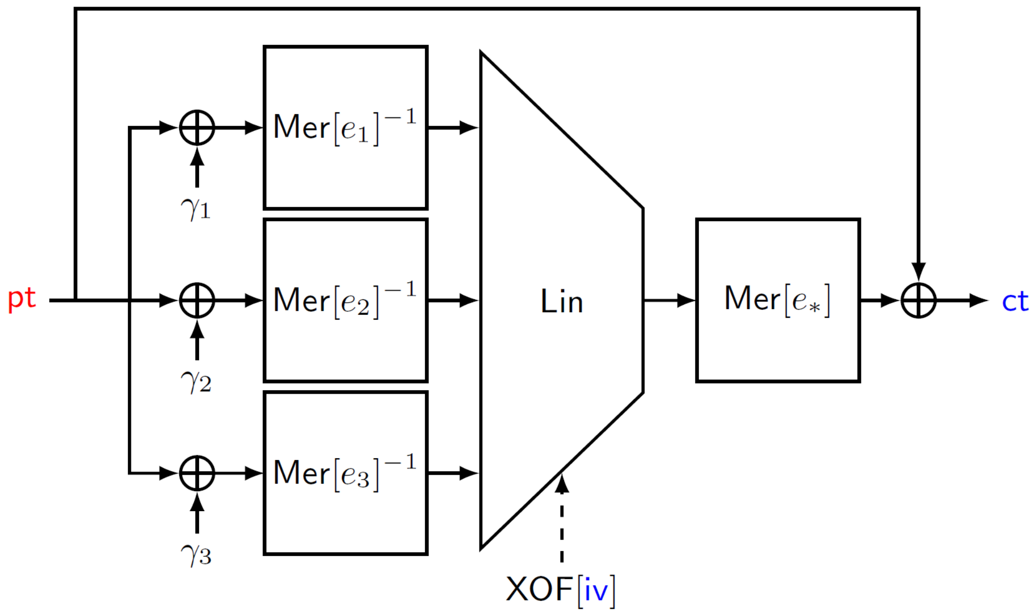

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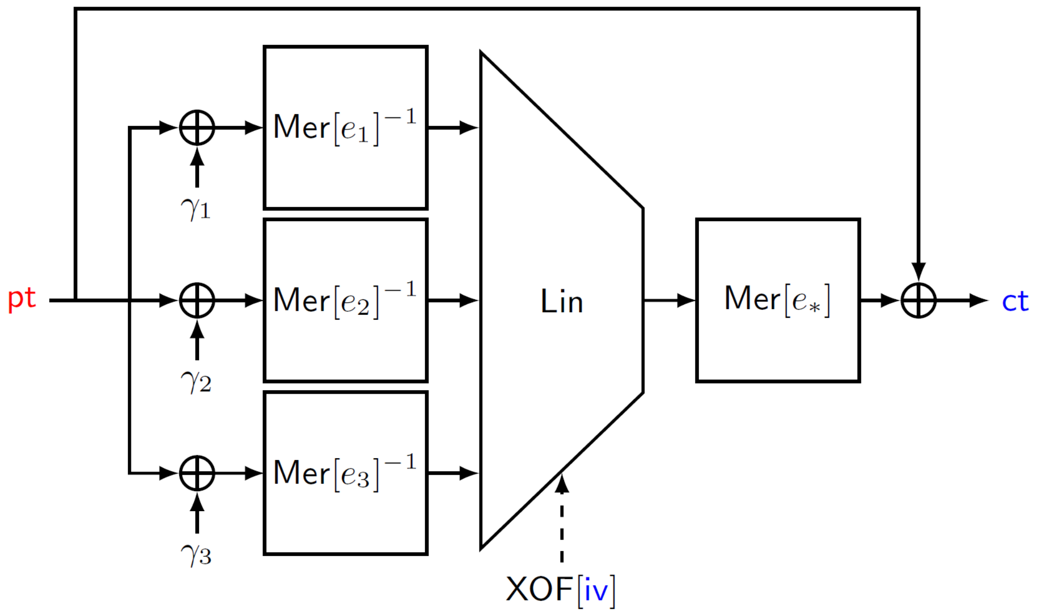

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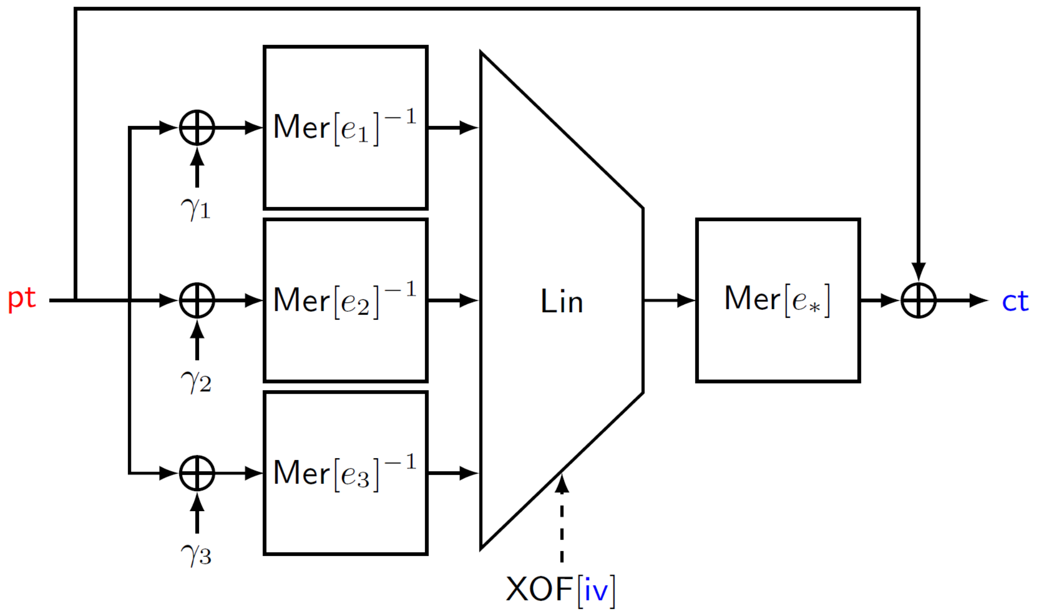
  - $a = (2^e - 1)^{-1} \bmod (2^n - 1)$
  - More resistant to algebraic attacks
- Larger exponents
  - To mitigate fast exhaustive search
- Fixed constant addition
  - To differentiate inputs of S-boxes
  - Increase the degree of composite power function

$$(x^a)^b \text{ vs } (x^a + c)^b$$

AIM2: Secure Patch for Algebraic Attacks



Scheme	λ	n	ℓ	e_1	e_2	e_3	e_*
AIM2-I	128	128	2	49	91	-	3
AIM2-III	192	192	2	17	47	-	5
AIM2-V	256	256	3	11	141	7	3

- Inverse Mersenne S-box
  - $\mathrm{Mer}[e]^{-1}(x) = x^a$
  - $a = (2^e - 1)^{-1} \bmod (2^n - 1)$
  - More resistant to algebraic attacks
- Larger exponents
  - To mitigate fast exhaustive search
- Fixed constant addition
  - To differentiate inputs of S-boxes
  - Increase the degree of composite power function

$$(x^a)^b \text{ vs } (x^a + c)^b$$

Algebraic Analysis on AIM2

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- The resulting system and complexity

	#var	#eq	Log(Time) [bits]
AIM2-I	256	384	207.9 (+60.9)
	384	1536	185.3 (+38.3)
AIM2-III	384	576	301.9 (+89.6)
	576	2304	262.4 (+50.1)
AIM2-V	768	1536	503.7 (+226.0)
	1024	4608	411.4 (+133.7)

Algebraic Analysis on AIM2

- Brute-force search of intermediate variables in a S-box
  - Variable: $x \in \mathbb{F}_{2^n}$, $t = \mathrm{Mer}[e]^{-1}(x)$, and $y = x^a$
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	(e_1, Deg)	(e_2, Deg)	(e_3, Deg)	(e_*, Deg)	Complexity
AIM2-I	$(49,16)$	$(91,15)$	-	$(3,15)$	≥ 176.2 $(+29.2)$
AIM2-III	$(17,17)$	$(47,17)$	-	$(5,26)$	≥ 214.4 $(+2.1)$
AIM2-V	$(11,31)$	$(141,23)$	$(7,25)$	$(3,29)$	≥ 310.4 $(+32.7)$

Other Analysis on AIM2

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  - Saarinen's method is the fastest (by <1 bit)
  - Sliding 2 LFSRs standing for pt and pt^{-1}
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- Quantum attacks
  - Complexities change but not critically
  - Always slower than Grover's algorithm

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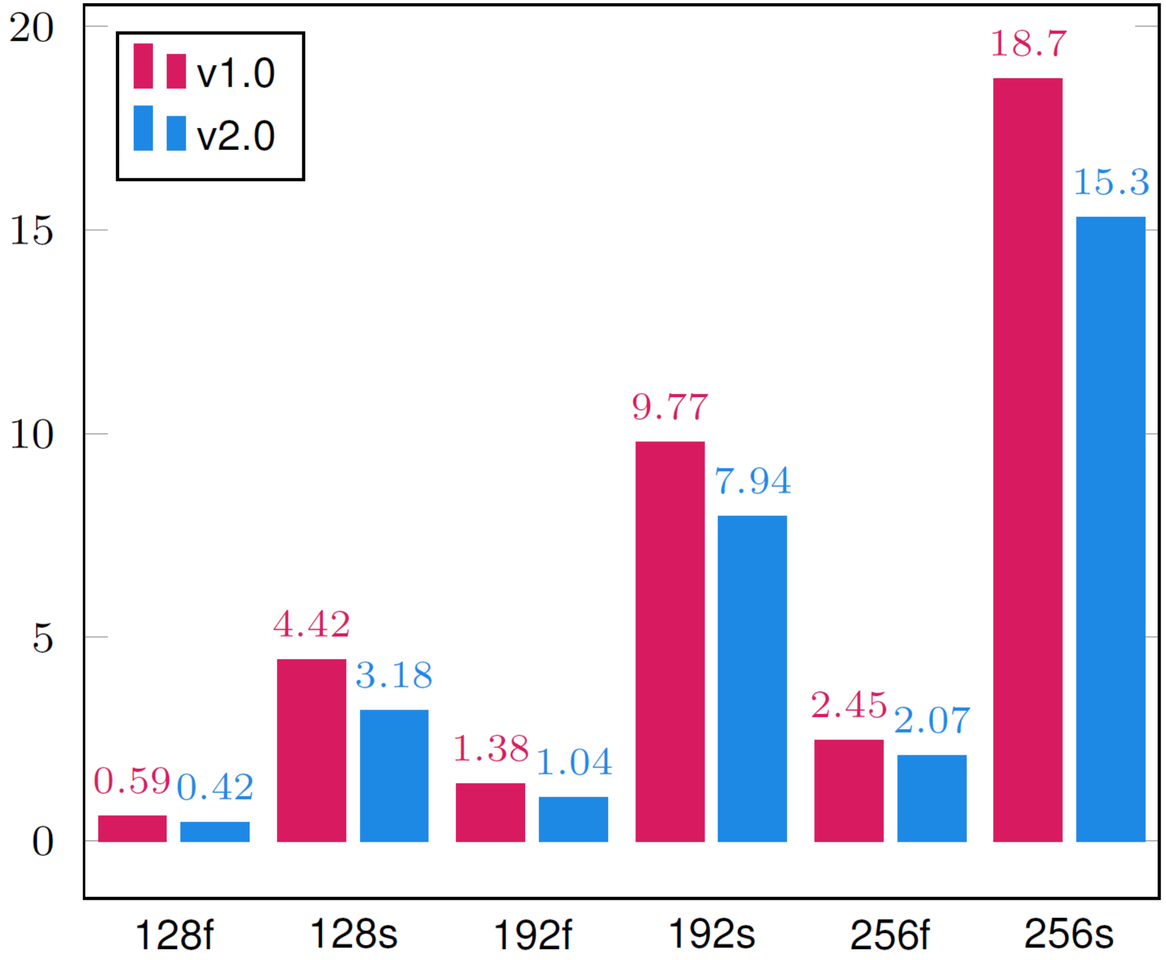
- Editorial Change
  - Improved EUF-CMA security proof (birthday bound → full bound)
  - Implementation-friendly specification

Performance Comparison



Signing time comparison in ms (AVX2)

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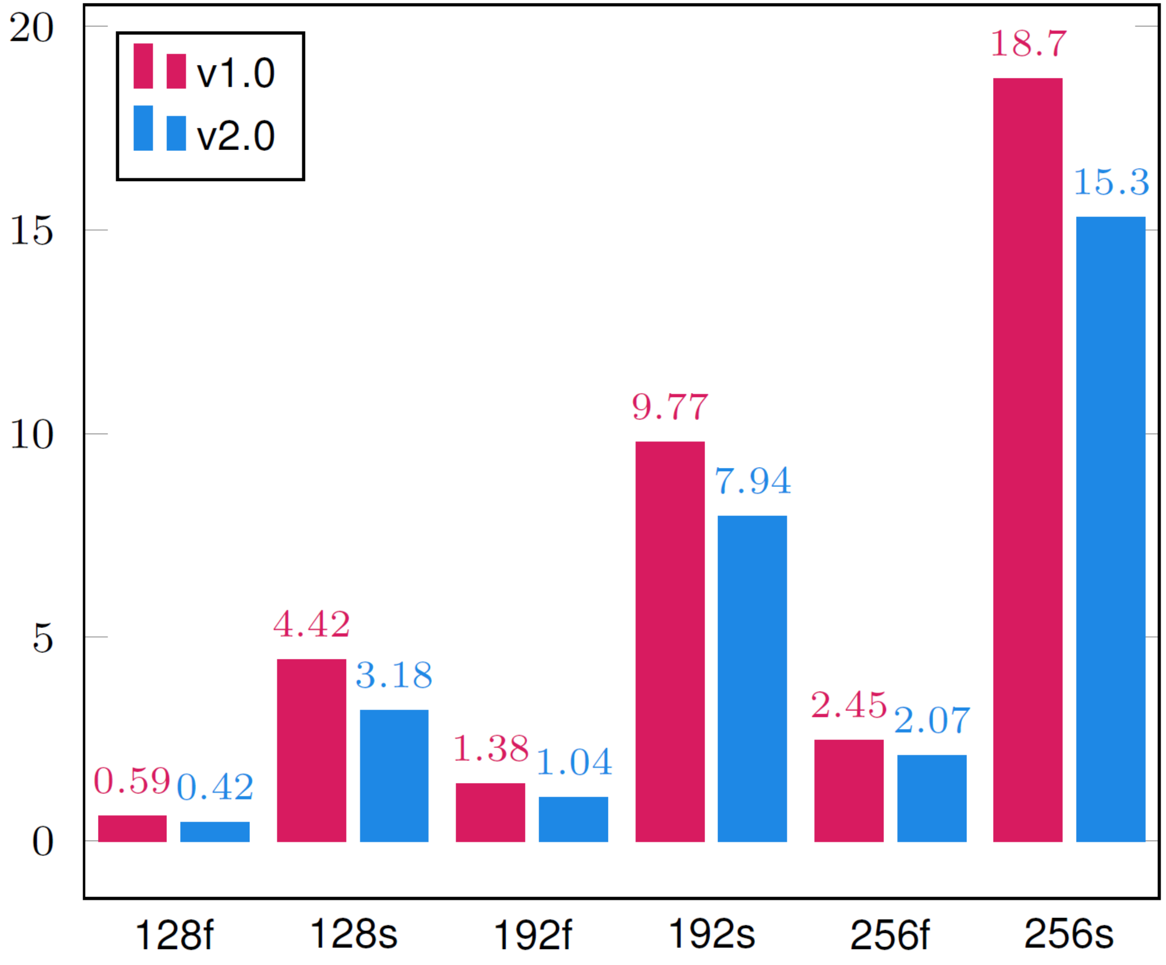

Signing time comparison in ms (AVX2)

Scheme	pk (B)	sig (B)	Sign (ms)	Verify (ms)
Dilithium2	1312	2420	0.10	0.03
Falcon-512	897	690	0.27	0.04
SPHINCS+-128s	32	7856	315.74	0.35
SPHINCS+-128f	32	17088	16.32	0.97
AIMer v1.0	32	5904	0.59	0.53
AIMer v1.0	32	4176	4.42	4.31
AIMer v2.0	32	5888	0.42	0.41
AIMer v2.0	32	4160	3.18	3.13

Measured on Intel Xeon E5-1650 v3 @ 3.50 GHz with 128 GB RAM, TurboBoost and Hyper-threading disabled, gcc 7.5.0 with -O3 option

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  - We re-analyze the efficacy of recent analyses on AIM
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  - We are waiting for **third-party analysis**!
- Work in progress
  - We are implementing AIMer on ARM Cortex-M4 in an optimized form
    - Preliminary result: memory usage $\leq$ 110 KB for all parameter sets
  - We are improving the puncturable PRF in AIMer, and adopting AES-based PRG
    - Preliminary result: 4.8 KB (128f), 3.6 KB (128s)

Thank you!
Check out our website!