One Tree to Rule Them All

Optimizing GGM Trees and OWFs for Post-Quantum Signatures

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Emmanuela Orsini[6] Sebastian Ramacher[7] Christian Rechberger[5] Lawrence Roy[1] Peter Scholl[1]

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[6] Bocconi University [7] AIT Austrian Institute of Technology

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Thank you to Lennart Braun for many slides.

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After Picnic, BBQ, and Banquet . . .

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Ingredients:

- $1\times$ Zero-knowledge proof for "I know $k \in \{0,1\}^{\lambda}$ such that $\mathrm{AES}_k(x) = y$"

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Our submission to the NIST Call for Post Quantum Signatures.

Chefs: Carsten Baum, Lennart Braun, Cyprien Delpech de Saint Guilhem, Michael Klooß, Christian Majenz, Shibam Mukherjee, Emmanuela Orsini, Sebastian Ramacher, Christian Rechberger, Lawrence Roy, Peter Scholl.

Identification Schemes Based on Zero-Knowledge Proofs

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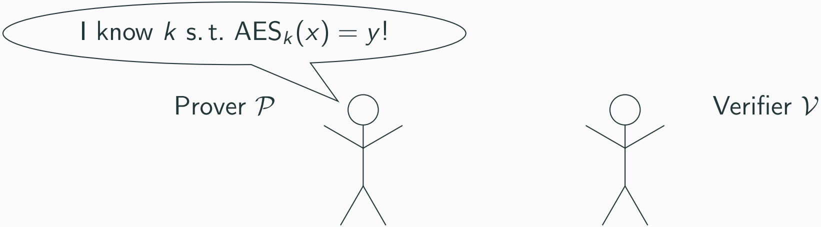

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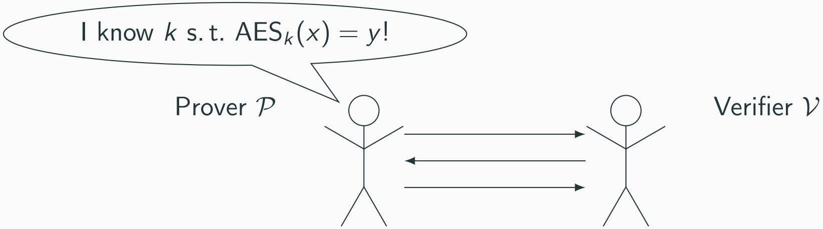

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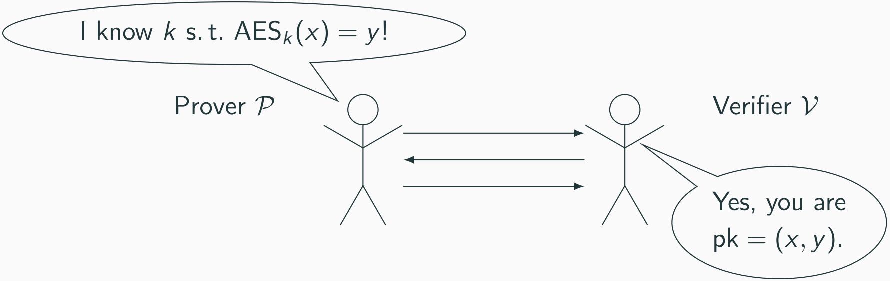

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Prover $\mathcal{P}$

Verifier $\mathcal{V}$

Yes, you are $\mathsf{pk} = (x, y)$.

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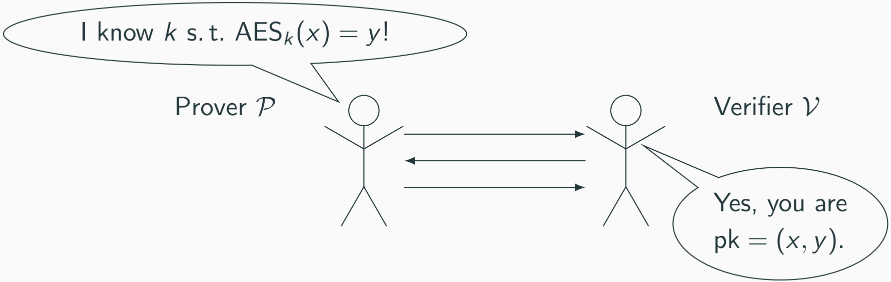
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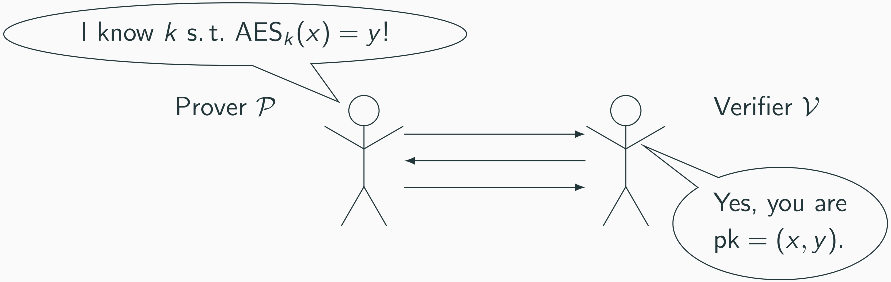
Security Properties

- Soundness: $\mathcal{V}$ cannot be convinced of a false statement
- Zero-Knowledge: $\mathcal{V}$ does not learn anything new from the interaction

Signature Schemes Based on Zero-Knowledge Proofs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Security Properties

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If the verifier has no secrets (i.e., is public-coin), can convert into a signature using Fiat-Shamir.

AES as a ZK-friendly Cipher?

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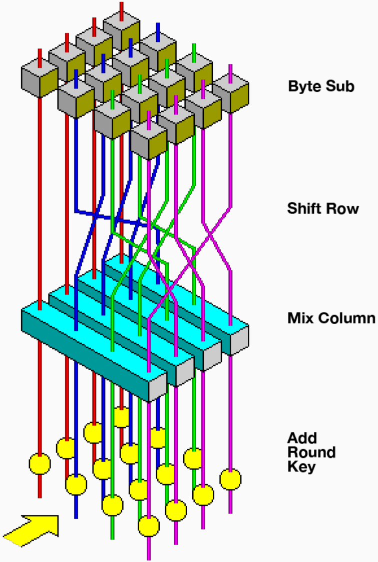
$$x \mapsto y = \begin{cases} 0 & \text{if } x = 0 \\ x^{-1} \in \mathbb{F}_{2^8} & \text{otherwise} \end{cases} \qquad (\star)$$



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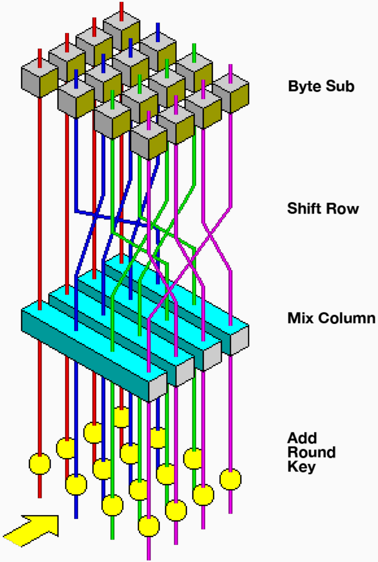
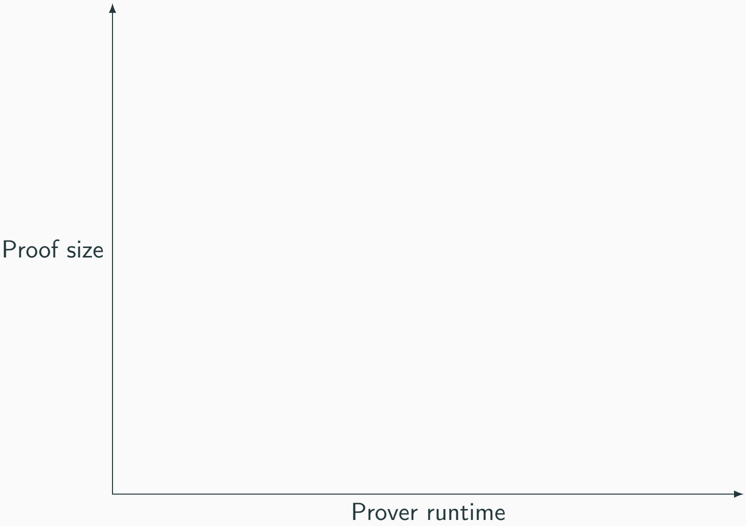
$\Longrightarrow$ Sample keys such that no zeros appear in the S-boxes and just check inversions ($x \cdot y = 1$ over $\mathbb{F}_{2^8}$)

$\rightsquigarrow$ AES-128: 200 quadratic constraints / 1600 bit witness



VOLE-based Zero-Knowledge

Space of Zero-Knowledge Proofs



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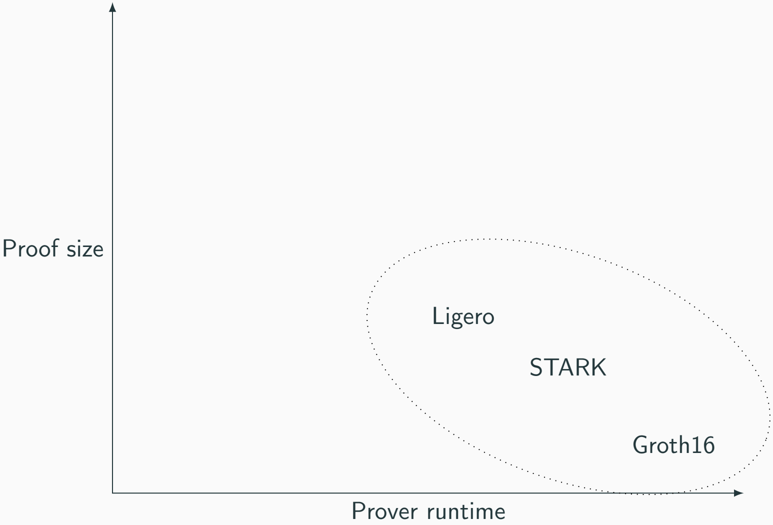

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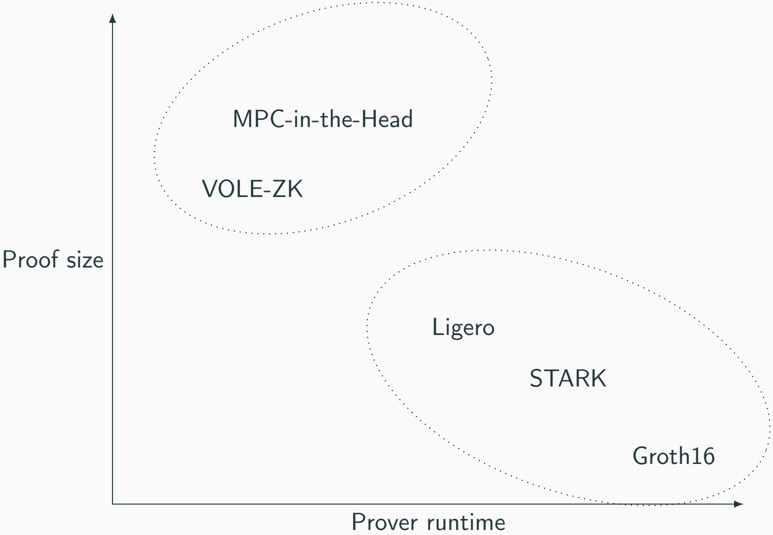

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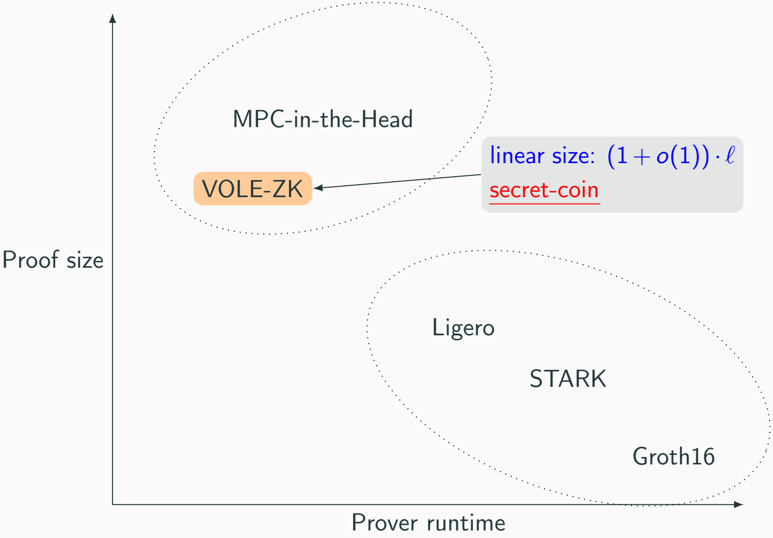

linear size: $(1 + o(1)) \cdot \ell$

secret-coin

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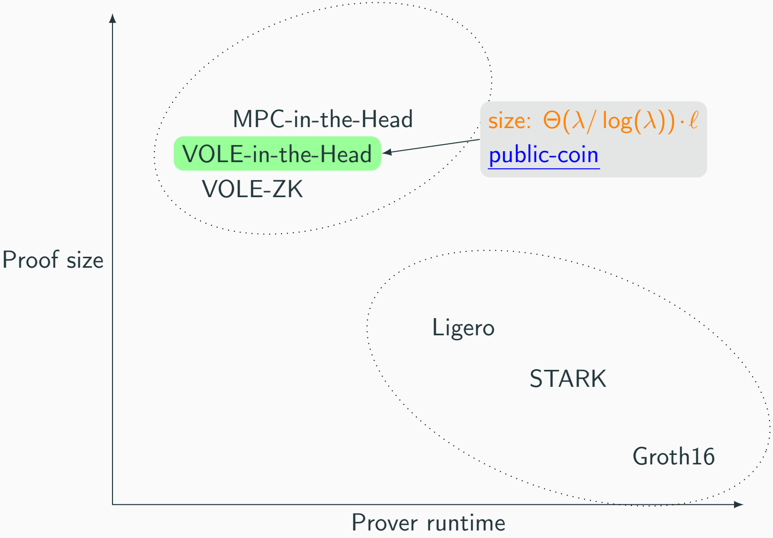

What are VOLEs?

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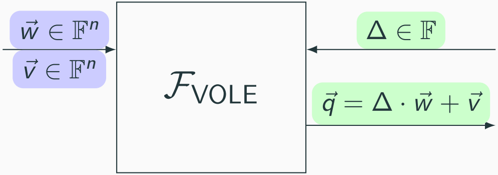
Vector Oblivious Linear Evaluation (VOLE)

Prover $\mathcal{P}$

(VOLE Sender)

Verifier $\mathcal{V}$

(VOLE Receiver)



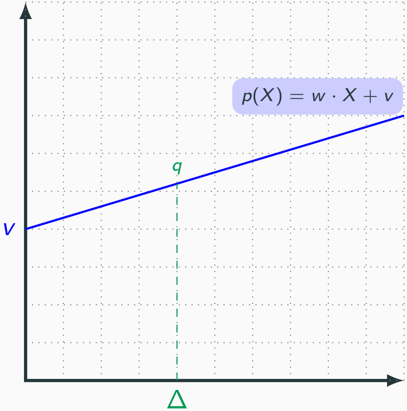
$\vec{w} \in \mathbb{F}^n$

$\vec{v} \in \mathbb{F}^n$

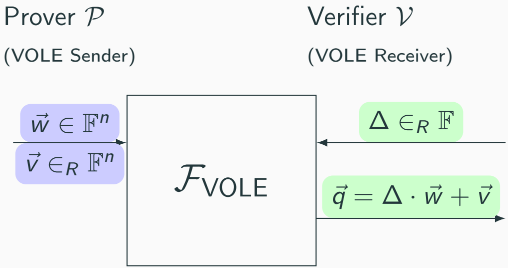
$\mathcal{F}_{\text{VOLE}}$

$\Delta \in \mathbb{F}$

$\vec{q} = \Delta \cdot \vec{w} + \vec{v}$



Vector Oblivious Linear Evaluation (VOLE) as Homomorphic Commitments



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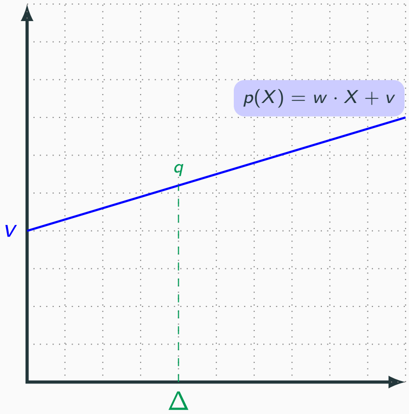

Linearly Homomorphic Commitments

use $q_i = w_i \cdot \Delta + v_i$ as information-theoretic MAC on w_i

- hiding since v_i is random
- breaking binding $\implies$ guessing Δ $\implies$ prob. $1/|\mathbb{F}|$

(cf. **EC:CatFio13 [EC:CatFio13]**, **EC:BDOZ11 [EC:BDOZ11]**)

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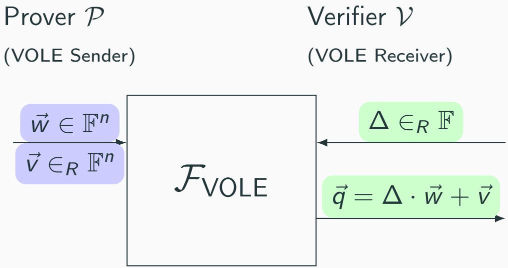

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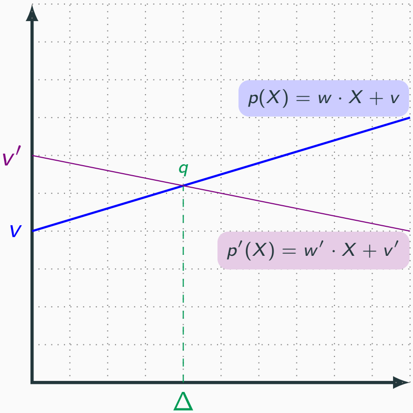

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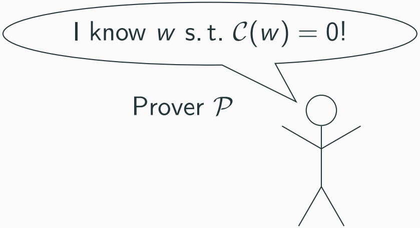
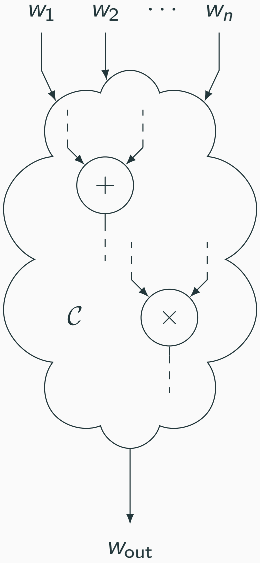


I know w s. t. $\mathcal{C}(w) = 0$!

Prover $\mathcal{P}$



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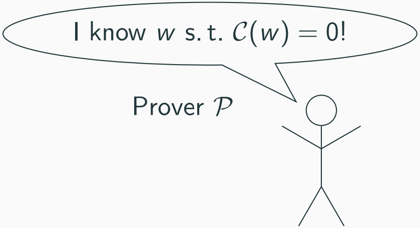

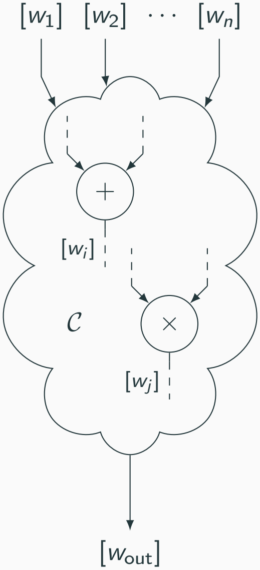

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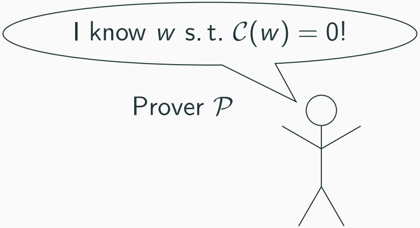

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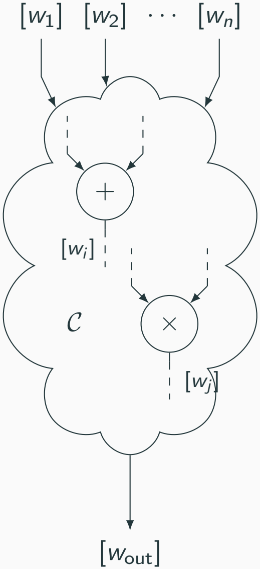
2. multiplication check
   - given $([a],[b],[c])$, verify $a \cdot b \overset{?}{=} c$



Verifying Multiplications – LPZK [ITC:DIO21], QuickSilver [CCS:YSWW21]

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use a random linear combination to verify many multiplications

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$\implies$ p has degree 2 $\implies$ p has at most 2 roots $\implies$ soundness error $^2/_{|\mathbb{F}|}$

VOLE-in-the-Head

VOLE-in-the-Head – The Idea

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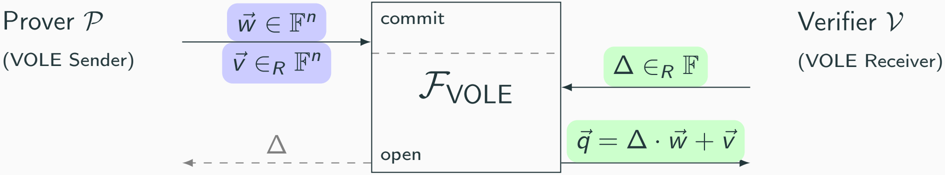
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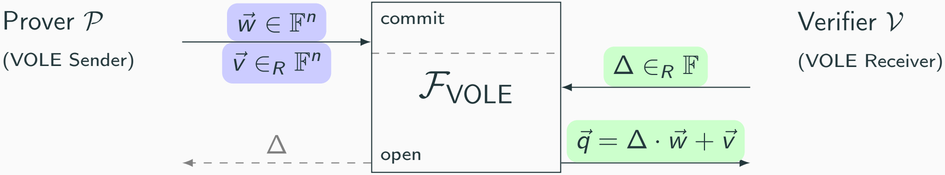

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Implement $\mathcal{F}_{\text{VOLE}}$ with SoftSpoken VOLE [**C:Roy22**].

Small-Field SoftSpoken VOLE

Input: An $\binom{N}{N-1}$-OT, for $N = 2^k \leq \text{poly}(\lambda)$:

$\mathcal{P}$ has seeds sd_x for all $x \in \mathbb{F}_{2^k}$. $\mathcal{V}$ has $\Delta \in \mathbb{F}_{2^k}$ and all seeds except sd_Δ.

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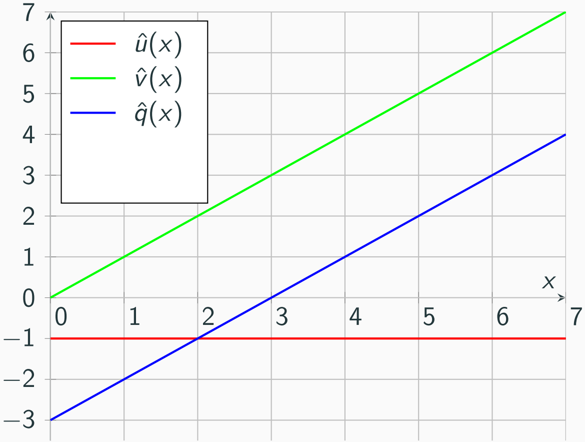
$$\vec{u} = -\sum_{x \in \mathbb{F}_{2^k}} G(\mathrm{sd}_x) = \sum_{x \in \mathbb{F}_{2^k}} \hat{u}(x) G(\mathrm{sd}_x)$$

$$\vec{v} = \sum_{x \in \mathbb{F}_{2^k}} x G(\mathrm{sd}_x) = \sum_{x \in \mathbb{F}_{2^k}} \hat{v}(x) G(\mathrm{sd}_x)$$

$$\vec{q} = \sum_{x \in \mathbb{F}_{2^k}} (x - \Delta) G(\mathrm{sd}_x)$$

$$= \sum_{x \in \mathbb{F}_{2^k}} \hat{q}(x) G(\mathrm{sd}_x)$$



Small-Field SoftSpoken VOLE

Input: An $\binom{N}{N-1}$-OT, for $N = 2^k \leq \mathrm{poly}(\lambda)$:
$\mathcal{P}$ has seeds sd_x for all $x \in \mathbb{F}_{2^k}$. $\mathcal{V}$ has $\Delta \in \mathbb{F}_{2^k}$ and all seeds except sd_Δ.

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$$\vec{v} = \sum_{x\in\mathbb{F}_{2^k}} x G(\mathrm{sd}_x) = \sum_{x\in\mathbb{F}_{2^k}} \hat{v}(x) G(\mathrm{sd}_x)$$

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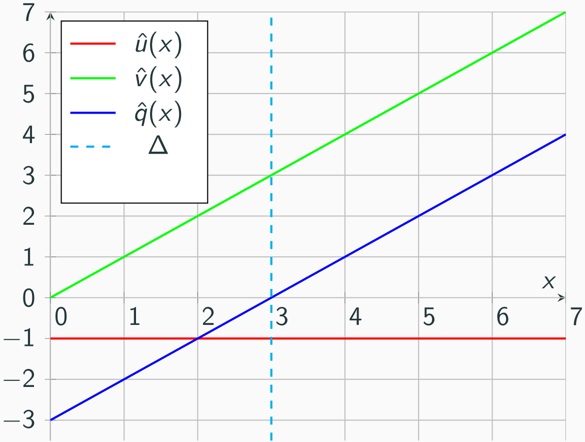

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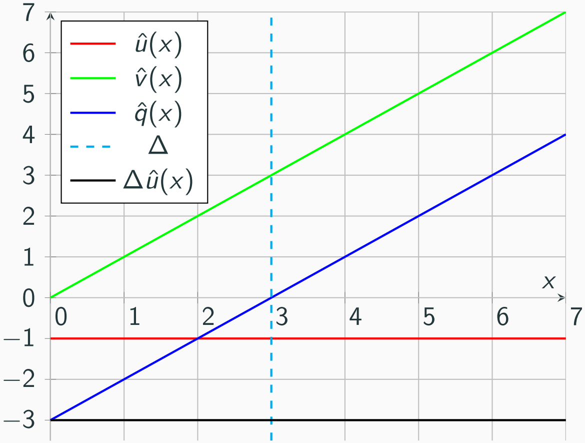
$$\vec{q} - \vec{v} = \sum_{x\in\mathbb{F}_{2^k}} (-\Delta) G(\mathrm{sd}_x) = \Delta \vec{u}$$



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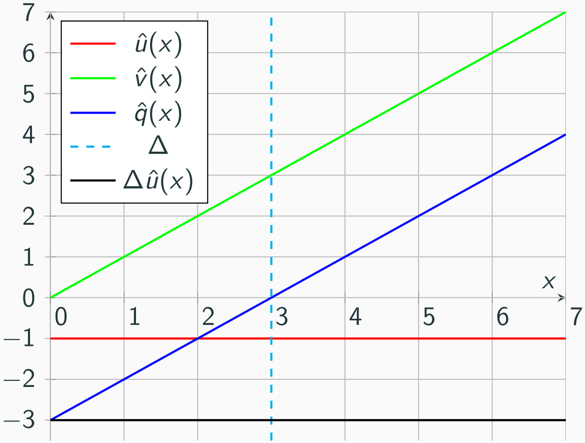

Derandomization: $\mathcal{P}$ sends $\vec{d} = \vec{w} - \vec{u}$. $\mathcal{V}$ updates $\vec{q}' = \vec{q} + \Delta \vec{d}$.

OT-in-the-Head: Commitments

How to get an $\binom{N}{N-1}$-OT for the VOLE?

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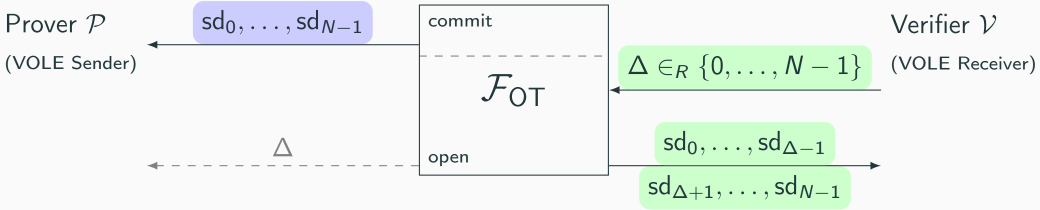
Prover $\mathcal{P}$
(VOLE Sender)
$\mathsf{sd}_0, \ldots, \mathsf{sd}_{N-1}$
Δ
commit
$\mathcal{F}_{\mathsf{OT}}$
open
Verifier $\mathcal{V}$
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$\Delta \in_R \{0, \ldots, N-1\}$
$\mathsf{sd}_0, \ldots, \mathsf{sd}_{\Delta-1}$
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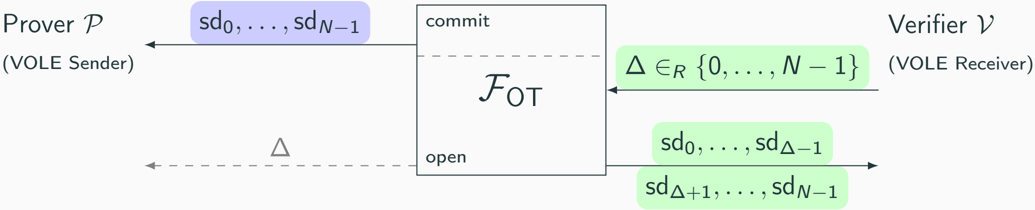
Prover $\mathcal{P}$
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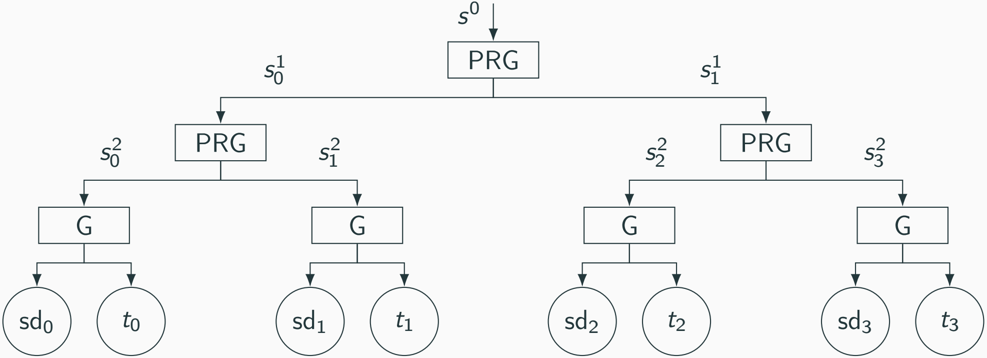
This is just a commitment scheme!

All-but-one Random Vector Commitments



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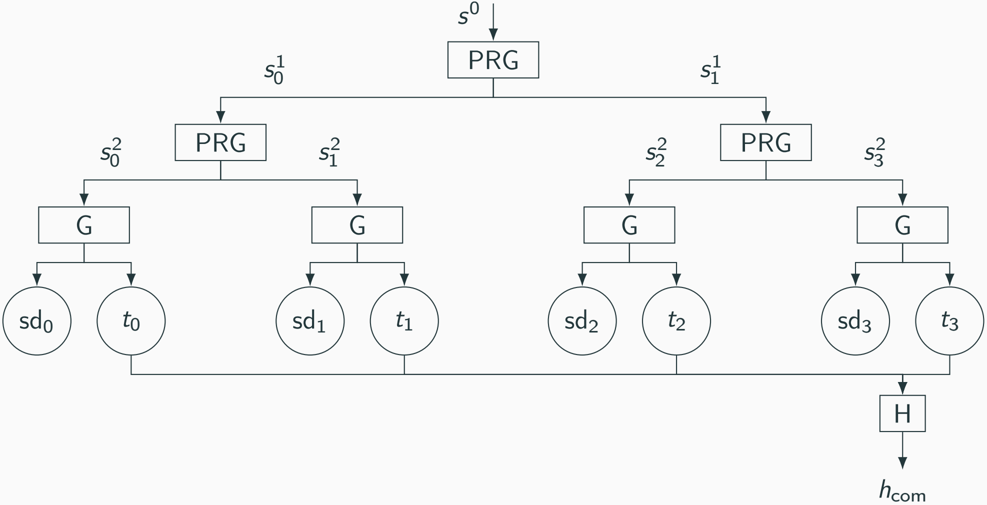
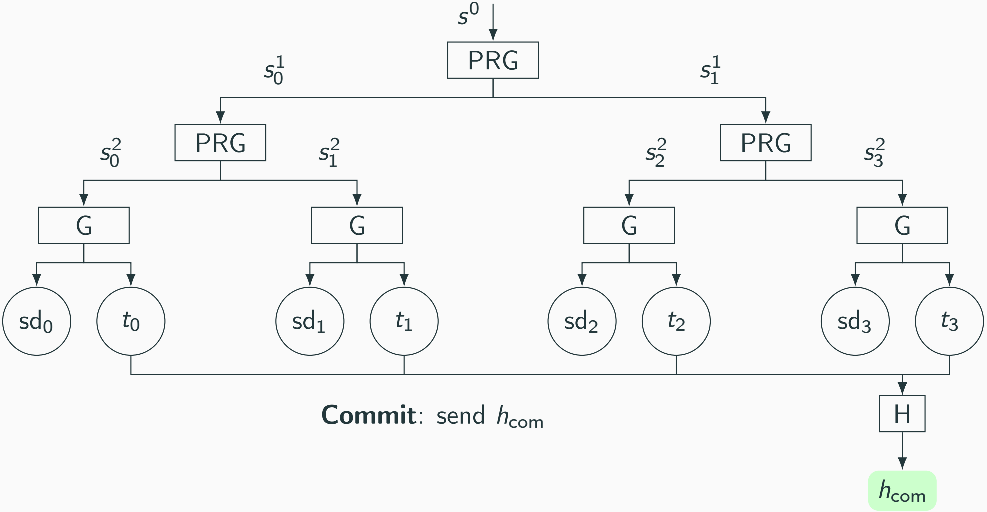


All-but-one Random Vector Commitments



Commit: send h_{com}

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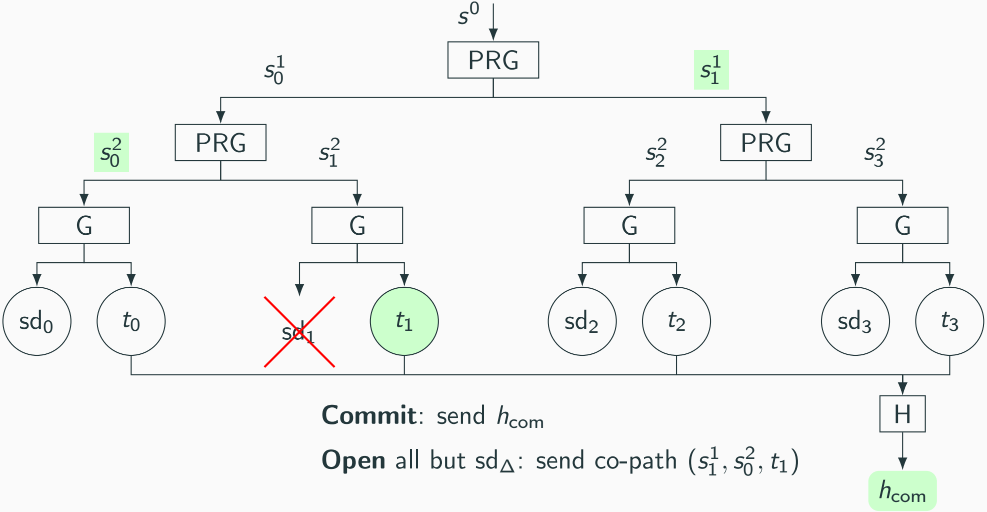

Commit: send h_{com}

Open all but sd_Δ: send co-path (s_1^1, s_0^2, t_1)

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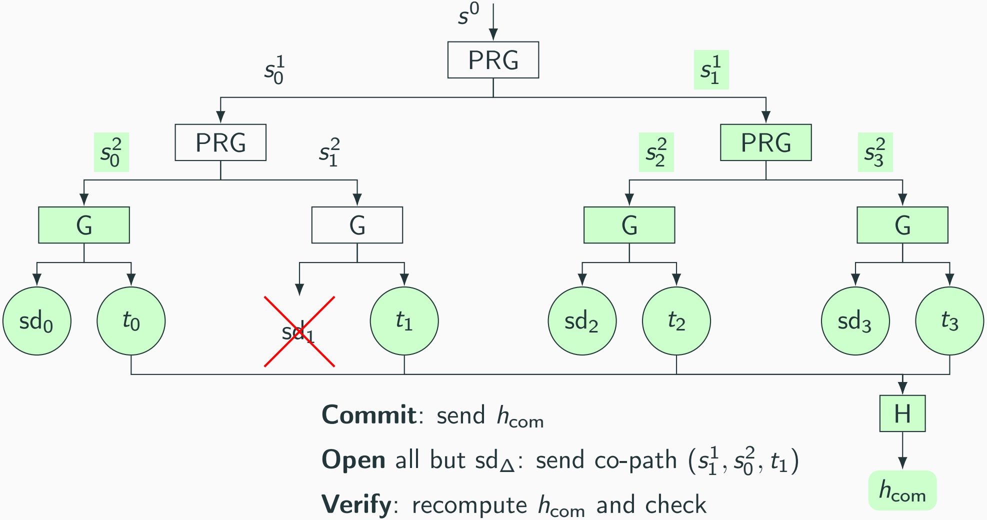

Commit: send h_{com}

Open all but sd_Δ: send co-path (s_1^1, s_0^2, t_1)

Verify: recompute h_{com} and check

Small VOLE to Big VOLE

Small VOLE costs $\mathcal{O}(N)$ work, but gives only soundness $\frac{1}{N}$!

$\implies$ need VOLE over a big field $\mathbb{F}_{2^\lambda}$ and Δ from large set.

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$$\begin{aligned} \vec{q}_0 &= \vec{w} \cdot \Delta_0 + \vec{v}_0 \\ &\vdots \\ \vec{q}_{\tau-1} &= \vec{w} \cdot \Delta_{\tau-1} + \vec{v}_{\tau-1} \end{aligned}$$

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$$\Downarrow$$

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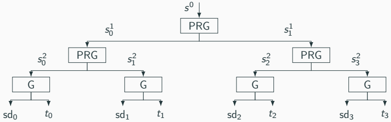
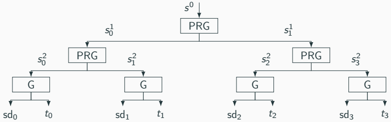
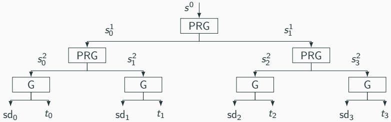
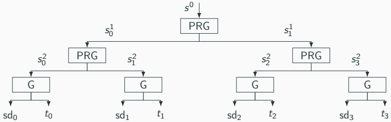
\end{aligned}$$

Use a consistency check to verify that the same $\vec{w}$ was used in every VOLE.

All-but-some Random Vector Commitments

		Field element $x \in [0, 2^k)$				Commitment
		0	1	2	3	
Repetition $i \in [0, \tau)$	0	$\mathrm{sd}_{0,0}$	$\mathrm{sd}_{0,1}$	$\mathrm{sd}_{0,2}$	$\mathrm{sd}_{0,3}$	 s^0 PRG s^1_0 s^1_1 s^2_0 PRG s^2_1 s^2_2 PRG s^2_3 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	1	$\mathrm{sd}_{1,0}$	$\mathrm{sd}_{1,1}$	$\mathrm{sd}_{1,2}$	$\mathrm{sd}_{1,3}$	 s^0 PRG s^1_0 s^1_1 s^2_0 PRG s^2_1 s^2_2 PRG s^2_3 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	2	$\mathrm{sd}_{2,0}$	$\mathrm{sd}_{2,1}$	$\mathrm{sd}_{2,2}$	$\mathrm{sd}_{2,3}$	 s^0 PRG s^1_0 s^1_1 s^2_0 PRG s^2_1 s^2_2 PRG s^2_3 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	3	$\mathrm{sd}_{3,0}$	$\mathrm{sd}_{3,1}$	$\mathrm{sd}_{3,2}$	$\mathrm{sd}_{3,3}$	 s^0 PRG s^1_0 s^1_1 s^2_0 PRG s^2_1 s^2_2 PRG s^2_3 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3

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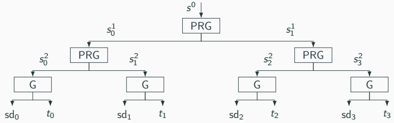
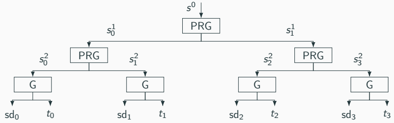
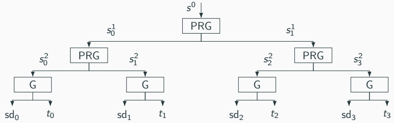
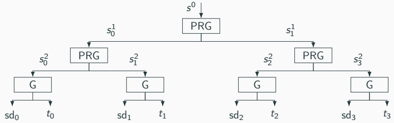
Repetition $i \in [0, \tau)$	0	$\mathrm{sd}_{0,0}$	$\xcancel{\mathrm{sd}_{0,1}}$	$\mathrm{sd}_{0,2}$	$\mathrm{sd}_{0,3}$	 s^0 PRG s_0^1 s_1^1 PRG PRG s_0^2 s_1^2 s_2^2 s_3^2 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	1	$\mathrm{sd}_{1,0}$	$\mathrm{sd}_{1,1}$	$\xcancel{\mathrm{sd}_{1,2}}$	$\mathrm{sd}_{1,3}$	 s^0 PRG s_0^1 s_1^1 PRG PRG s_0^2 s_1^2 s_2^2 s_3^2 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	2	$\xcancel{\mathrm{sd}_{2,0}}$	$\mathrm{sd}_{2,1}$	$\mathrm{sd}_{2,2}$	$\mathrm{sd}_{2,3}$	 s^0 PRG s_0^1 s_1^1 PRG PRG s_0^2 s_1^2 s_2^2 s_3^2 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	3	$\mathrm{sd}_{3,0}$	$\xcancel{\mathrm{sd}_{3,1}}$	$\mathrm{sd}_{3,2}$	$\mathrm{sd}_{3,3}$	 s^0 PRG s_0^1 s_1^1 PRG PRG s_0^2 s_1^2 s_2^2 s_3^2 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3

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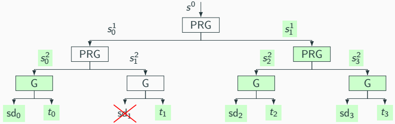
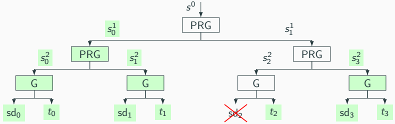
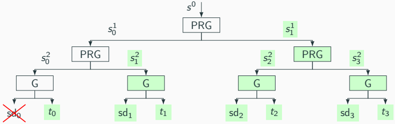
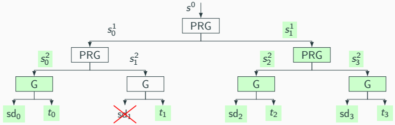
Repetition $i \in [0, \tau)$	0	$\mathsf{sd}_{0,0}$	$\xcancel{\mathsf{sd}_{0,1}}$	$\mathsf{sd}_{0,2}$	$\mathsf{sd}_{0,3}$	 s^0 PRG s^1_0 s^1_1 PRG PRG s^2_0 s^2_1 s^2_2 s^2_3 G G G G sd_0 t_0 $\xcancel{\mathsf{sd}_1}$ t_1 sd_2 t_2 sd_3 t_3
	1	$\mathsf{sd}_{1,0}$	$\mathsf{sd}_{1,1}$	$\xcancel{\mathsf{sd}_{1,2}}$	$\mathsf{sd}_{1,3}$	 s^0 PRG s^1_0 s^1_1 PRG PRG s^2_0 s^2_1 s^2_2 s^2_3 G G G G sd_0 t_0 sd_1 t_1 $\xcancel{\mathsf{sd}_2}$ t_2 sd_3 t_3
	2	$\xcancel{\mathsf{sd}_{2,0}}$	$\mathsf{sd}_{2,1}$	$\mathsf{sd}_{2,2}$	$\mathsf{sd}_{2,3}$	 s^0 PRG s^1_0 s^1_1 PRG PRG s^2_0 s^2_1 s^2_2 s^2_3 G G G G $\xcancel{\mathsf{sd}_0}$ t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	3	$\mathsf{sd}_{3,0}$	$\xcancel{\mathsf{sd}_{3,1}}$	$\mathsf{sd}_{3,2}$	$\mathsf{sd}_{3,3}$	 s^0 PRG s^1_0 s^1_1 PRG PRG s^2_0 s^2_1 s^2_2 s^2_3 G G G G sd_0 t_0 $\xcancel{\mathsf{sd}_1}$ t_1 sd_2 t_2 sd_3 t_3

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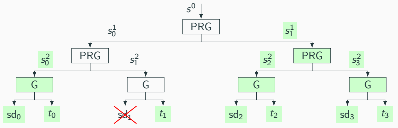
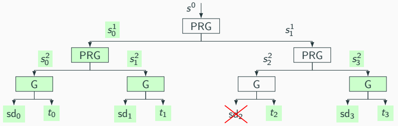
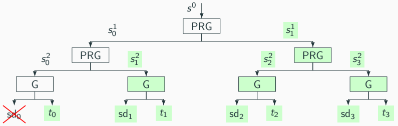
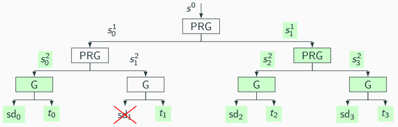
Repetition $i \in [0, \tau)$	0	$\mathsf{sd}_{0,0}$	$\xcancel{\mathsf{sd}_{0,1}}$	$\mathsf{sd}_{0,2}$	$\mathsf{sd}_{0,3}$	 s^0 PRG s^1_0 s^1_1 PRG PRG s^2_0 s^2_1 s^2_2 s^2_3 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	1	$\mathsf{sd}_{1,0}$	$\mathsf{sd}_{1,1}$	$\xcancel{\mathsf{sd}_{1,2}}$	$\mathsf{sd}_{1,3}$	 s^0 PRG s^1_0 s^1_1 PRG PRG s^2_0 s^2_1 s^2_2 s^2_3 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	2	$\xcancel{\mathsf{sd}_{2,0}}$	$\mathsf{sd}_{2,1}$	$\mathsf{sd}_{2,2}$	$\mathsf{sd}_{2,3}$	 s^0 PRG s^1_0 s^1_1 PRG PRG s^2_0 s^2_1 s^2_2 s^2_3 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3
	3	$\mathsf{sd}_{3,0}$	$\xcancel{\mathsf{sd}_{3,1}}$	$\mathsf{sd}_{3,2}$	$\mathsf{sd}_{3,3}$	 s^0 PRG s^1_0 s^1_1 PRG PRG s^2_0 s^2_1 s^2_2 s^2_3 G G G G sd_0 t_0 sd_1 t_1 sd_2 t_2 sd_3 t_3

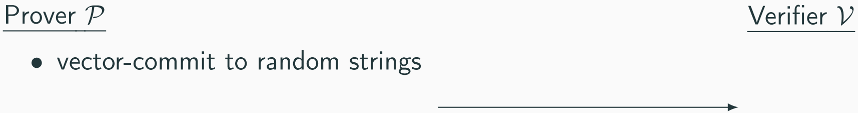
Because $N^\tau = 2^\lambda$, the co-paths always have λ nodes, so opening costs roughly λ^2 bits.

FAEST Rounds

Prover $\mathcal{P}$

Verifier $\mathcal{V}$

FAEST Rounds



Prover $\mathcal{P}$

Verifier $\mathcal{V}$

- vector-commit to random strings

FAEST Rounds

Prover $\mathcal{P}$

- vector-commit to random strings
- expand small VOLEs $\longrightarrow$

Verifier $\mathcal{V}$

FAEST Rounds

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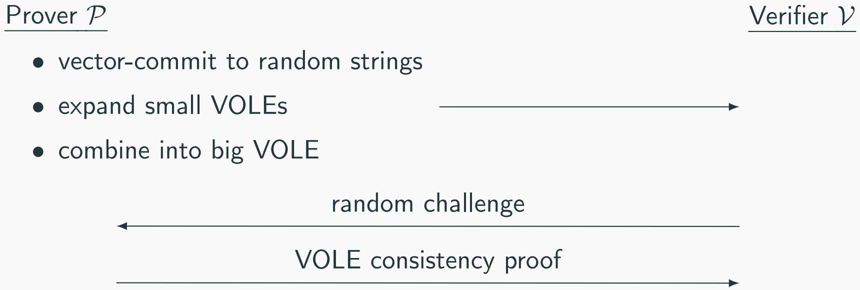
Verifier $\mathcal{V}$

- vector-commit to random strings
- expand small VOLEs
- combine into big VOLE

FAEST Rounds



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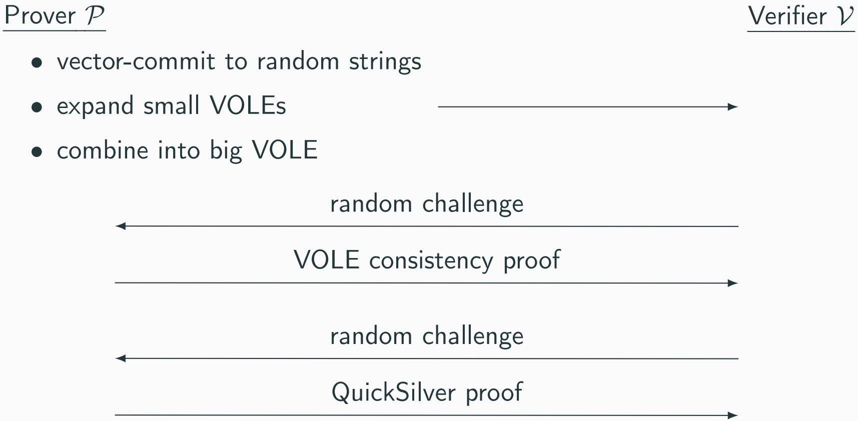
random challenge

VOLE consistency proof

FAEST Rounds



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Verifier $\mathcal{V}$

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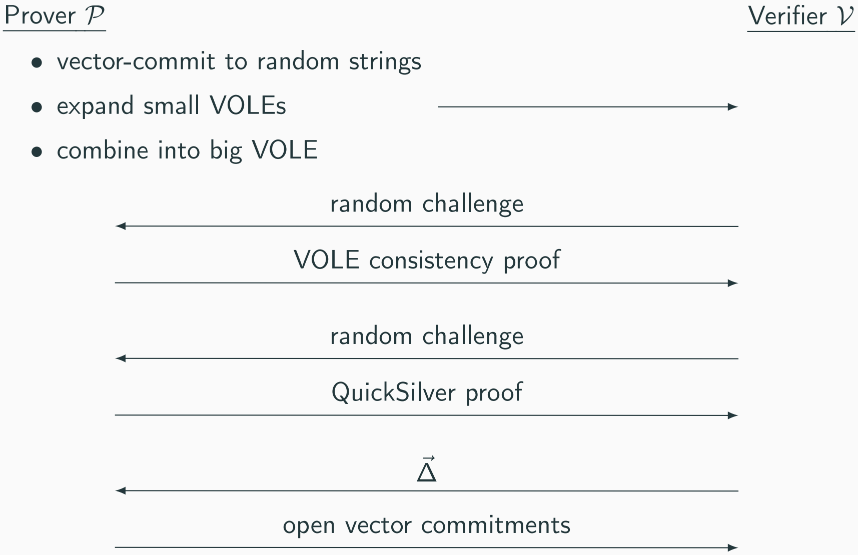
random challenge

VOLE consistency proof

random challenge

QuickSilver proof

FAEST Rounds



Prover $\mathcal{P}$

Verifier $\mathcal{V}$

- vector-commit to random strings
- expand small VOLEs
- combine into big VOLE

random challenge

VOLE consistency proof

random challenge

QuickSilver proof

$\vec{\Delta}$

open vector commitments

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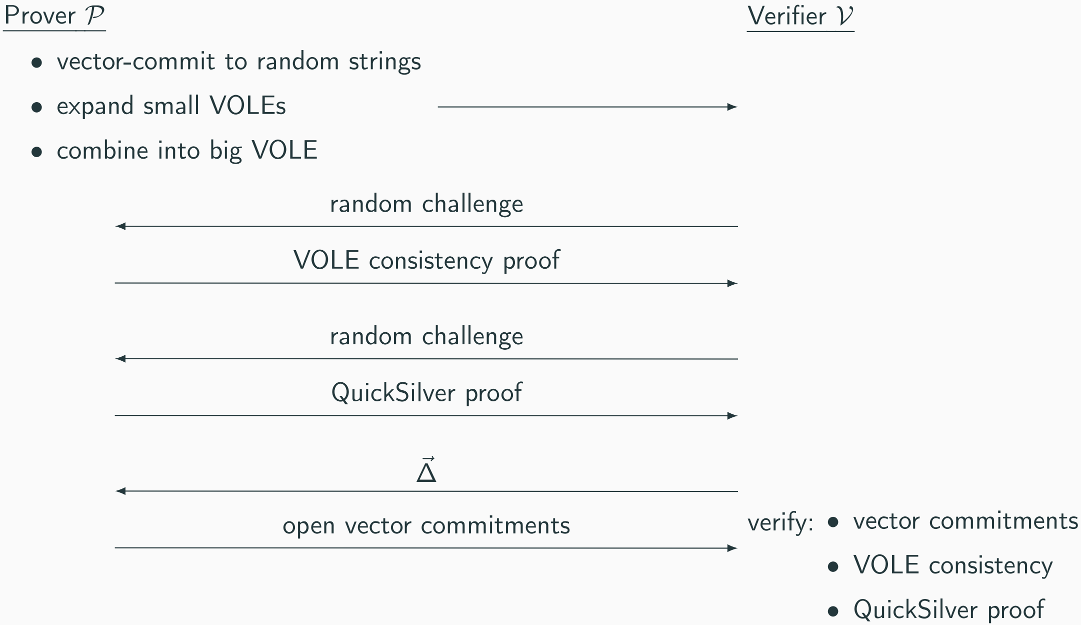

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Grinding

Grinding: Overview

Mismatch: cost of generating a proof $\gg$ per-trial attack cost lowerbound in the security argument.

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Fix: make the Fiat-Shamir hash 2^w times more expensive (Grinding). Only need to target $2^{\lambda - w}$ security level.

- This allows for smaller signatures by reducing τ.
- Counter-intuitively, this can also make signing faster — k can be reduced while preserving security.

Grinding: Correlation

$$\vec{q}_0 = \vec{w} \cdot \Delta_0 + \vec{v}_0$$

$$\vdots$$

$$\vec{q}_{\tau-2} = \vec{w} \cdot \Delta_{\tau-2} + \vec{v}_{\tau-2}$$

$$\vec{q}_{\tau-1} = \vec{w} \cdot \Delta_{\tau-1} + \vec{v}_{\tau-1}$$

$$\Downarrow$$

$$\underbrace{\sum_{i\in[\tau]} \vec{q}_i \cdot X^i}_{\vec{q}\in\mathbb{F}^{\ell}_{q^\tau}} = \vec{w} \cdot \underbrace{\sum_{i\in[\tau]} \Delta_i \cdot X^i}_{\Delta\in\mathbb{F}_{q^\tau}} + \underbrace{\sum_{i\in[\tau]} \vec{v}_i \cdot X^i}_{\vec{v}\in\mathbb{F}^{\ell}_{q^\tau}}$$

Grinding: Correlation

What if $\Delta_{\tau-1} = 0$?

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The last small vole correlation is now trivial,

$$\begin{aligned} \vec{q}_0 &= \vec{w} \cdot \Delta_0 + \vec{v}_0 \\ &\vdots \\ \vec{q}_{\tau-2} &= \vec{w} \cdot \Delta_{\tau-2} + \vec{v}_{\tau-2} \\ 0 &= \vec{w} \cdot 0 + 0 \\ &\Downarrow \end{aligned}$$

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Grinding: Correlation

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The last small vole correlation is now trivial, and can be removed to save communication.

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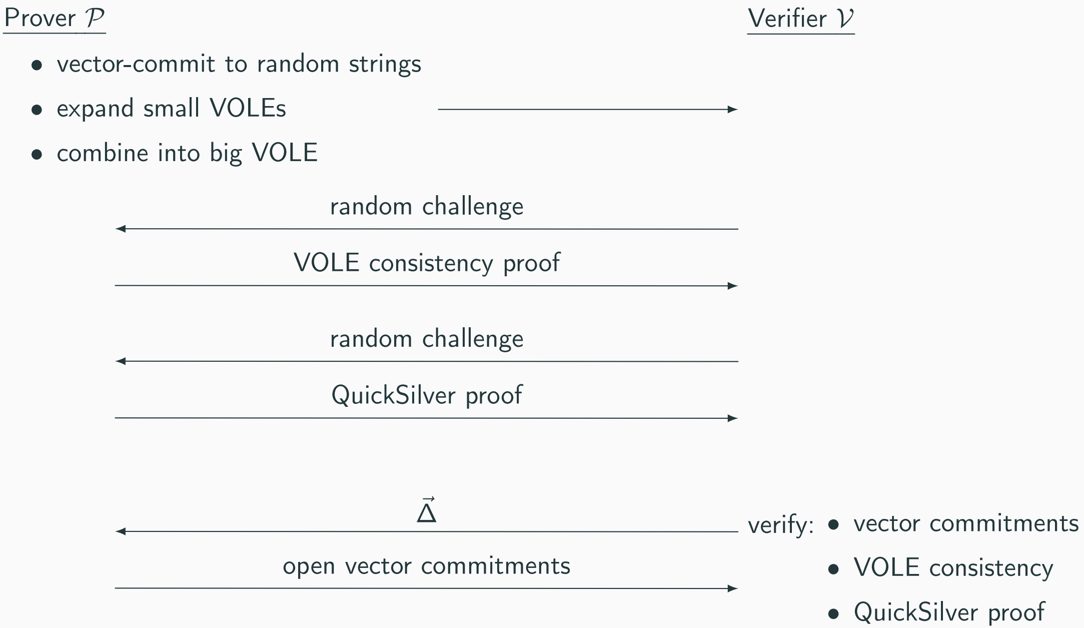

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← random challenge

VOLE consistency proof →

← random challenge

QuickSilver proof →

← $\vec{\Delta}$

open vector commitments →

verify:

- vector commitments
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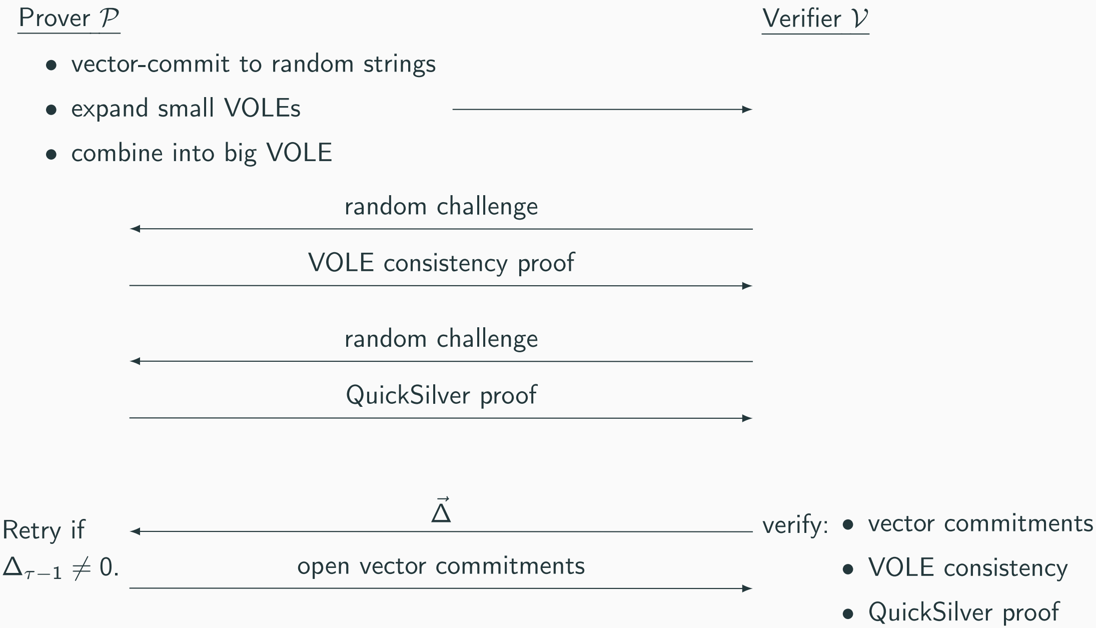

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$\vec{\Delta}$

Retry if $\Delta_{\tau-1} \neq 0$.

open vector commitments

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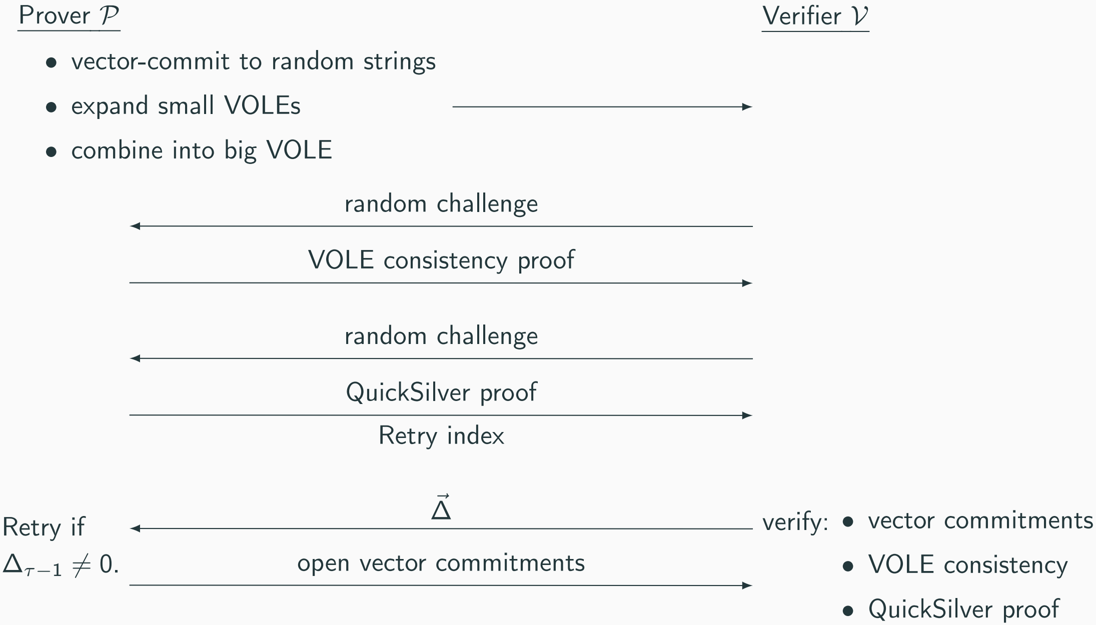

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Verifier $\mathcal{V}$

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random challenge

QuickSilver proof

Retry index

$\vec{\Delta}$

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open vector commitments

verify:
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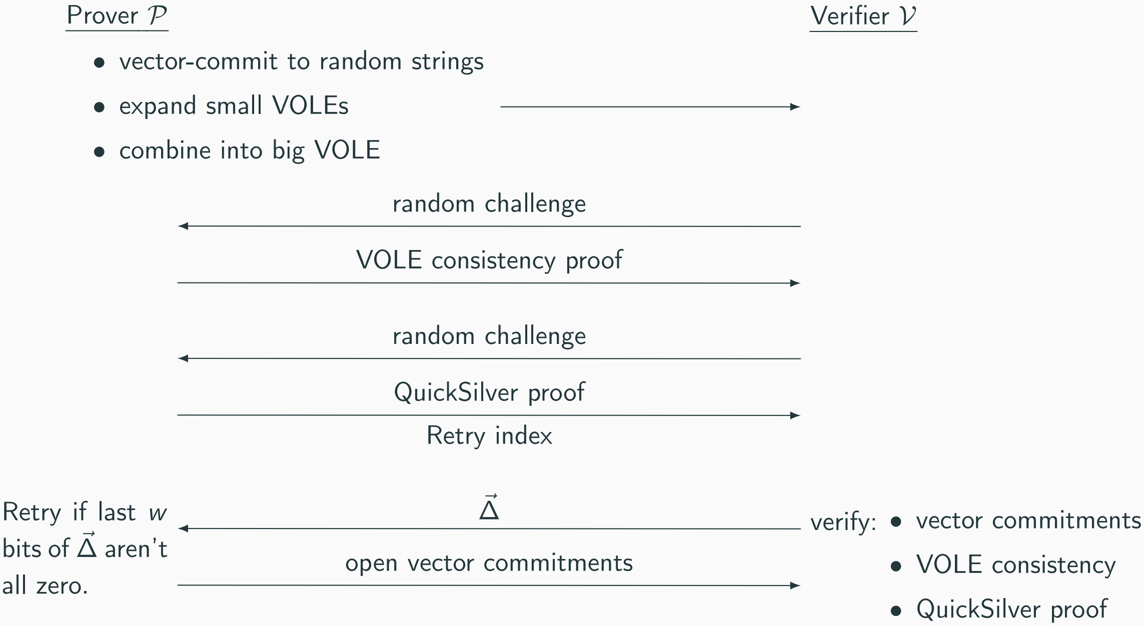

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random challenge

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QuickSilver proof

Retry index

Retry if last w bits of $\vec{\Delta}$ aren't all zero.

$\vec{\Delta}$

open vector commitments

verify:
- vector commitments
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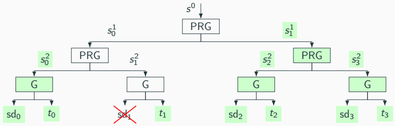
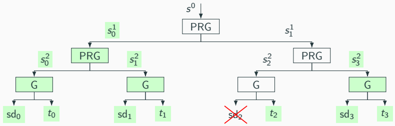
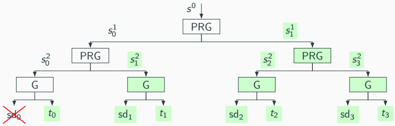
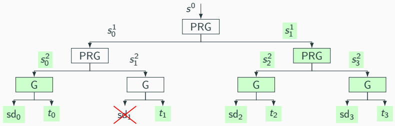
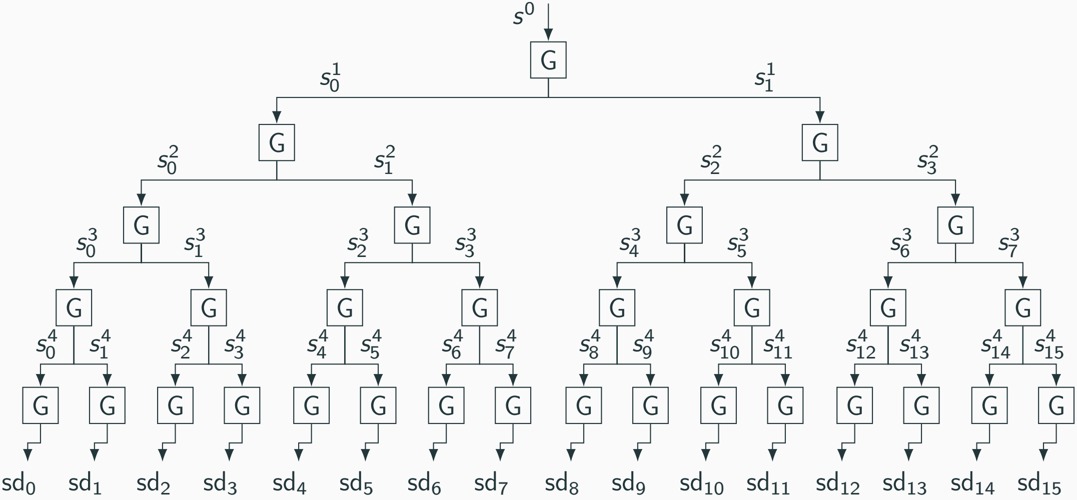
One Tree to Rule Them All

All-but-some Random Vector Commitments

		Field element $x \in [0, 2^k)$				Commitment
		0	1	2	3	
Repetition $i \in [0, \tau)$	0	$\mathsf{sd}_{0,0}$	~~$\mathsf{sd}_{0,1}$~~	$\mathsf{sd}_{0,2}$	$\mathsf{sd}_{0,3}$	
	1	$\mathsf{sd}_{1,0}$	$\mathsf{sd}_{1,1}$	~~$\mathsf{sd}_{1,2}$~~	$\mathsf{sd}_{1,3}$	
	2	~~$\mathsf{sd}_{2,0}$~~	$\mathsf{sd}_{2,1}$	$\mathsf{sd}_{2,2}$	$\mathsf{sd}_{2,3}$	
	3	$\mathsf{sd}_{3,0}$	~~$\mathsf{sd}_{3,1}$~~	$\mathsf{sd}_{3,2}$	$\mathsf{sd}_{3,3}$	

One Tree to Bind Them



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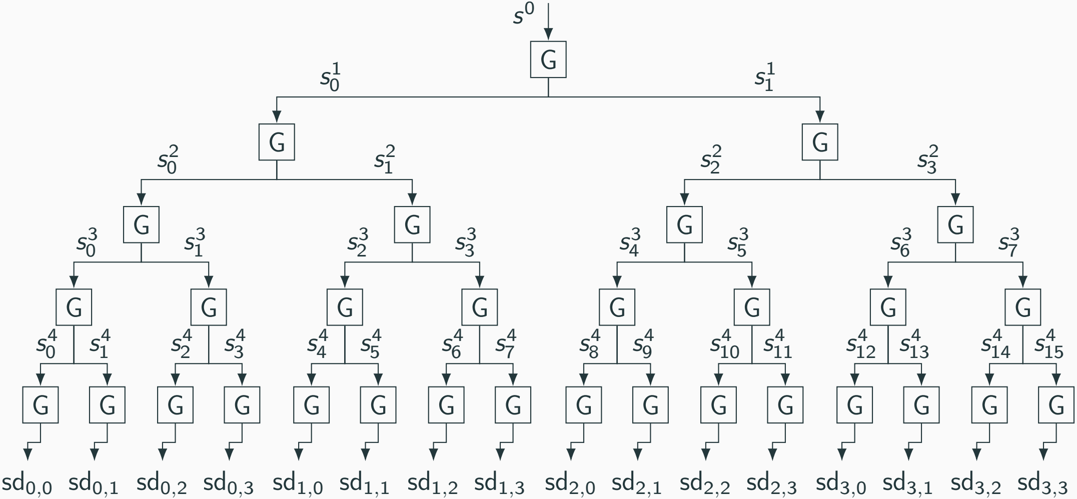

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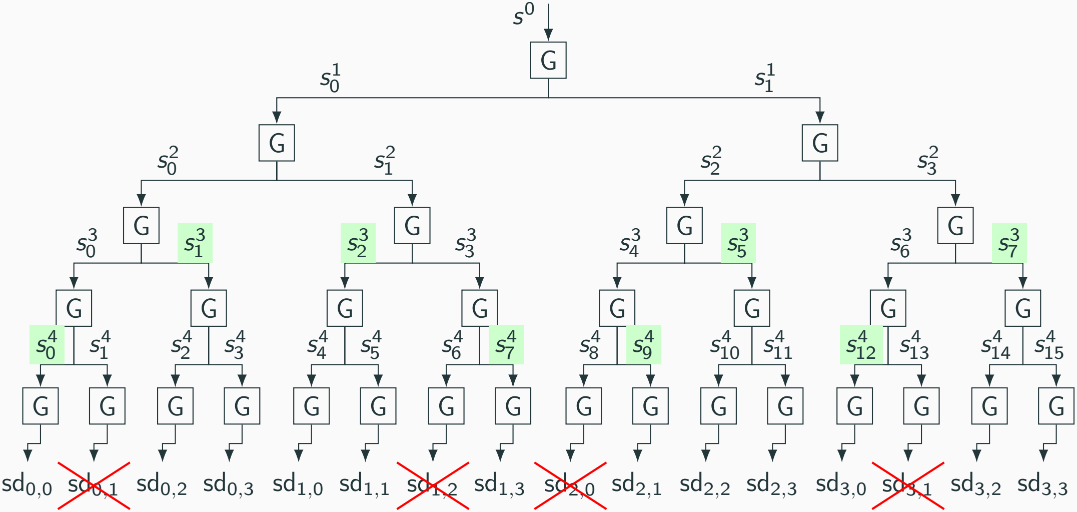

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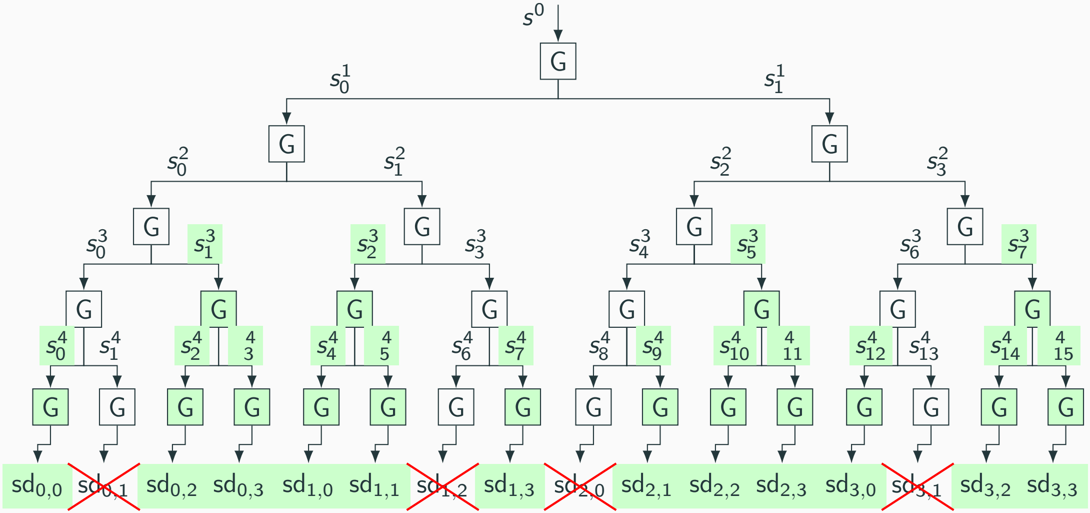

All-but-some Random Vector Commitments

		Field element $x \in [0, 2^k)$			
		0	1	2	3
Repetition $i \in [0, \tau)$	0	$\mathrm{sd}_{0,0}$	$\xcancel{\mathrm{sd}_{0,1}}$	$\mathrm{sd}_{0,2}$	$\mathrm{sd}_{0,3}$
	1	$\mathrm{sd}_{1,0}$	$\mathrm{sd}_{1,1}$	$\xcancel{\mathrm{sd}_{1,2}}$	$\mathrm{sd}_{1,3}$
	2	$\xcancel{\mathrm{sd}_{2,0}}$	$\mathrm{sd}_{2,1}$	$\mathrm{sd}_{2,2}$	$\mathrm{sd}_{2,3}$
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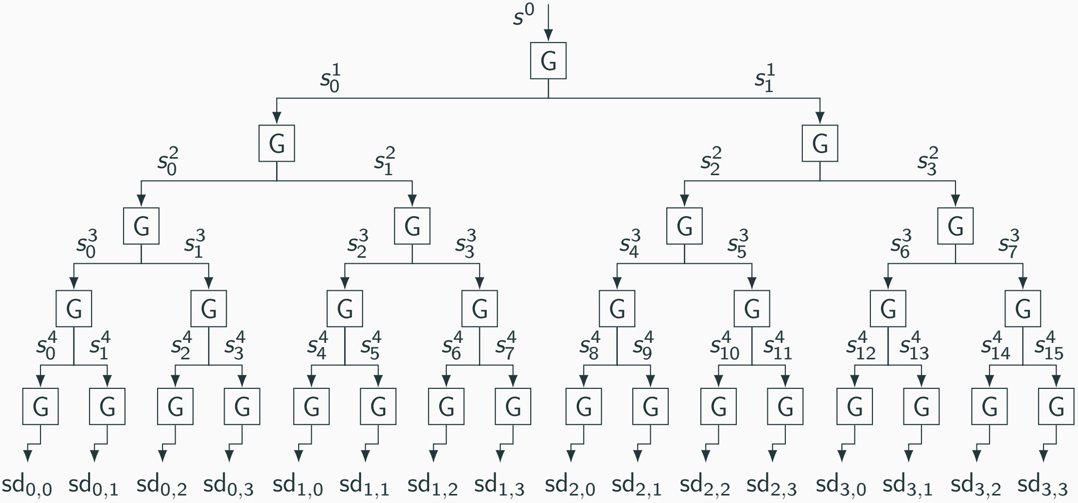

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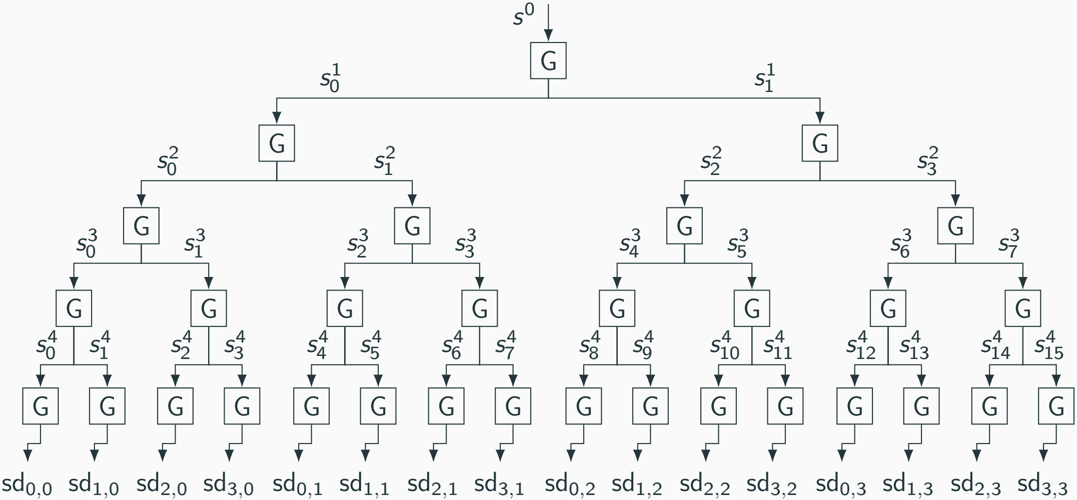

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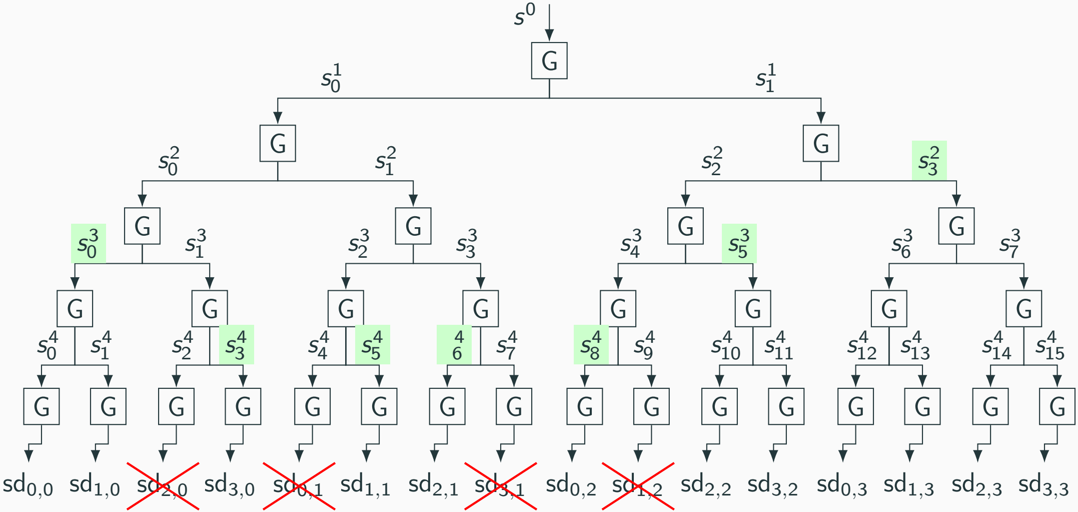

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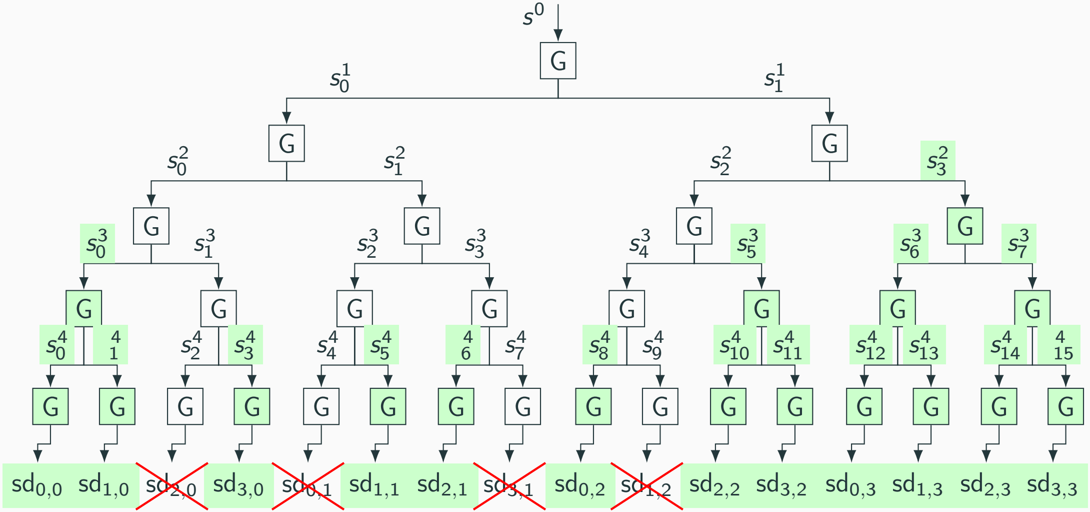

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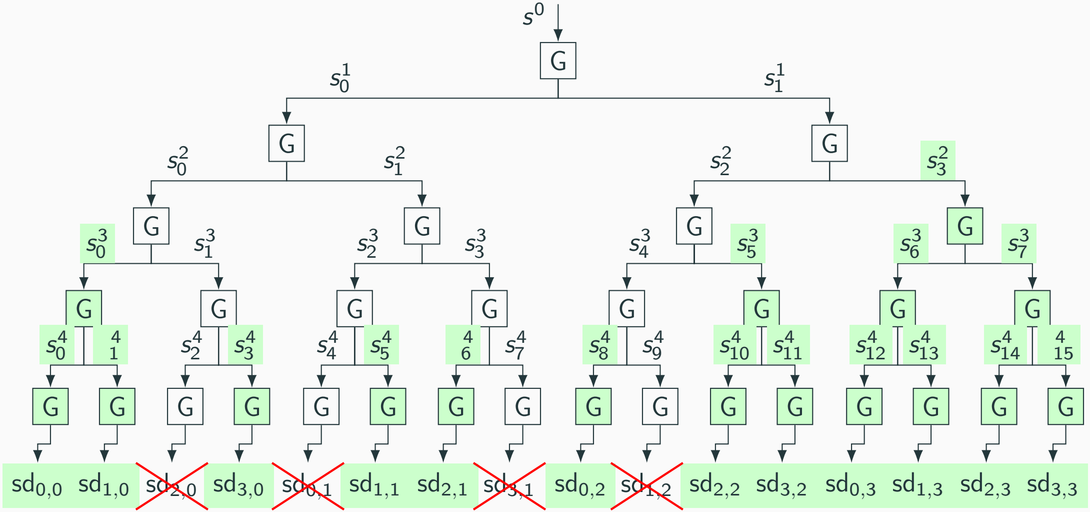

Note: only 7 seeds to open, not 8.

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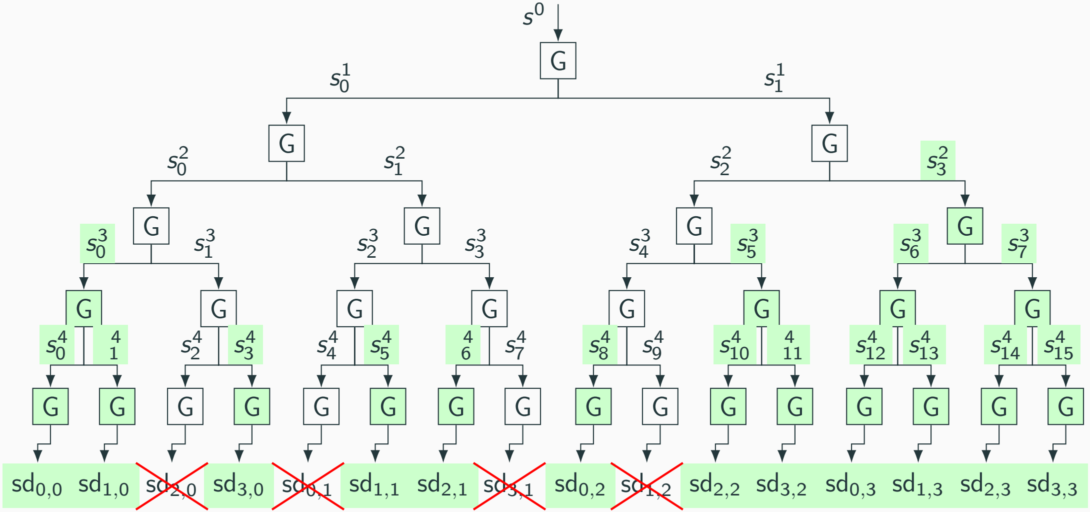


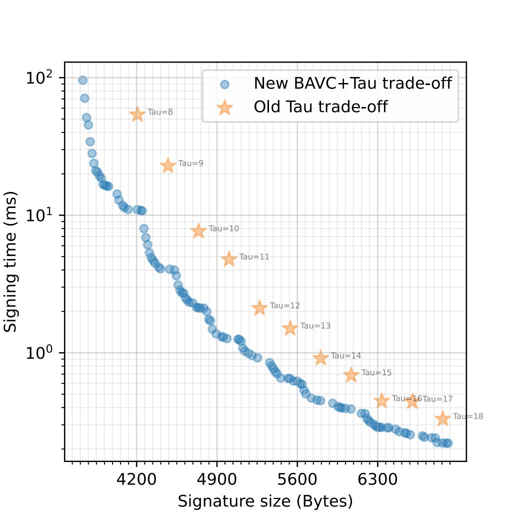
Note: only 7 seeds to open, not 8.

In general, the opening size depends on Δ.

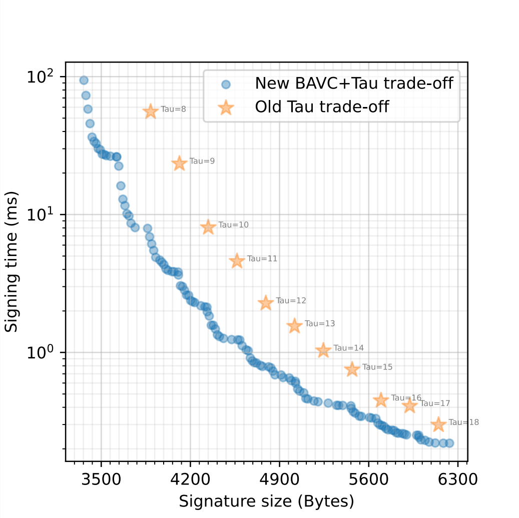
$\rightsquigarrow$ Set a limit T_{open} on seeds in the opening, and retry if it's exceeded.

FAESTER

Size-time Tradeoff



(a) FAESTER-128.



(b) FAESTER-EM-128.

Parameter Choices

Signature Scheme	OWF $E_{sk}(x)$	l	w	$\mathrm{T}_{\mathrm{open}}$	τ	τ_0	τ_1	k_0	k_1	sk size	pk size	sig. size
FAEST-128$_\mathsf{s}$	AES128$_{sk}(x)$	1600	–	–	11	7	4	12	11	16	32	5006
FAEST-128$_\mathsf{f}$	AES128$_{sk}(x)$	1600	–	–	16	0	16	8	8	16	32	6336
FAEST-EM-128$_\mathsf{s}$	AES128$_x(sk) \oplus sk$	1280	–	–	11	7	4	12	11	16	32	4566
FAEST-EM-128$_\mathsf{f}$	AES128$_x(sk) \oplus sk$	1280	–	–	16	0	16	8	8	16	32	5696
FAESTER-128$_\mathsf{s}$	AES128$_{sk}(x)$	1600	7	102	11	0	11	11	11	16	32	4594
FAESTER-128$_\mathsf{f}$	AES128$_{sk}(x)$	1600	8	110	16	8	8	8	7	16	32	6052
FAESTER-EM-128$_\mathsf{s}$	AES128$_x(sk) \oplus sk$	1280	7	103	11	0	11	11	11	16	32	4170
FAESTER-EM-128$_\mathsf{f}$	AES128$_x(sk) \oplus sk$	1280	8	112	16	8	8	8	7	16	32	5444

Performance Comparison

Scheme	Runtime in ms			Size in bytes		
	Keygen	Sign	Verify	sk	pk	Signature
FAEST-128$_s$	0.0006	4.381	4.102	16	32	5006
FAEST-128$_f$	0.0005	0.404	0.395	16	32	6336
FAEST-EM-128$_s$	0.0005	4.151	4.415	16	32	4566
FAEST-EM-128$_f$	0.0005	0.446	0.474	16	32	5696
FAESTER-128$_s$	0.0006	3.282	4.467	16	32	4594
FAESTER-128$_f$	0.0005	0.433	0.610	16	32	6052
FAESTER-EM-128$_s$	0.0005	3.005	4.386	16	32	4170
FAESTER-EM-128$_f$	0.0005	0.422	0.609	16	32	5444

Signing time (ms), verification time (ms), and signature size (bytes).

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Signing time (ms), verification time (ms), and signature size (bytes).

Benchmarking system: AMD Ryzen 9 7900X 12-Core CPU running Ubuntu 22.04.

Zeroes in S-boxes

- AES S-boxes:

$$x \mapsto y = \begin{cases} 0 & \text{if } x = 0 \\ x^{-1} \in \mathbb{F}_{2^8} & \text{otherwise} \end{cases} \tag{$\star$}$$

- Constraint: $x \cdot y = 1$. This requires $x \neq 0$.

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- $(\star) \iff x^2 \cdot y = x \ \wedge \ x \cdot y^2 = y$
  - observe that $x \mapsto x^2$ is $\mathbb{F}_2$-linear.
  - $\rightsquigarrow$ 2 quadratic constraints per S-box.

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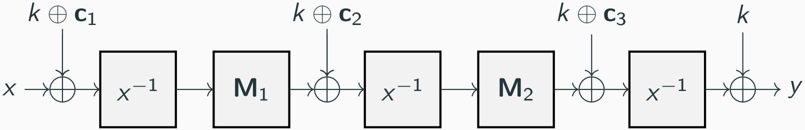
  - $\rightsquigarrow$ 2 quadratic constraints per S-box.

- Can use any AES key! No rejection sampling.

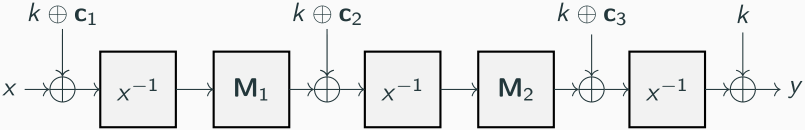
MandaRain

Rain Cipher



3-round Rain.

Rain Cipher



3-round Rain.

- $x, k, y \in \mathbb{F}_{2^\lambda}$.
- M_i is a $\mathbb{F}_2$-linear transformations.

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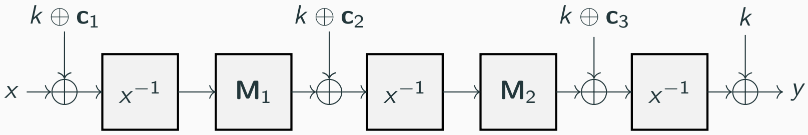
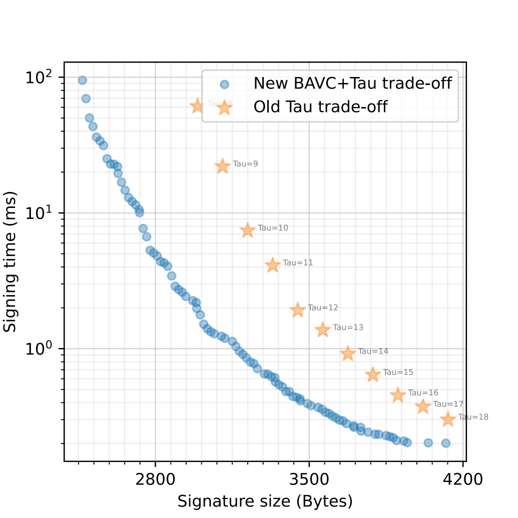


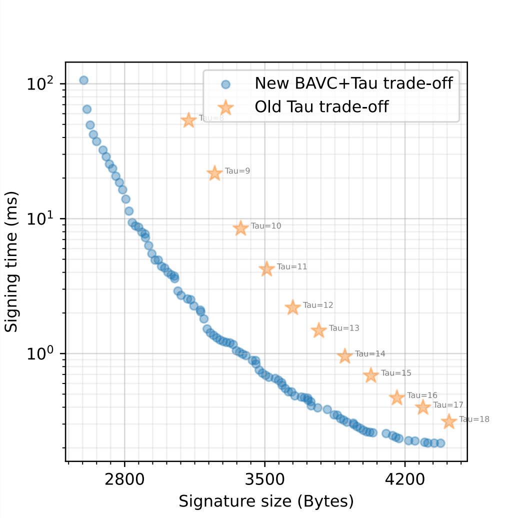
3-round Rain.

- $x, k, y \in \mathbb{F}_{2^\lambda}$.
- M_i is a $\mathbb{F}_2$-linear transformations.
- Fewer rounds $\implies$ smaller witness.

Size-time Tradeoff



(a) MandaRain-3-128.



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FAESTER-EM-128$_f$	0.0005	0.422	0.609	16	32	5444
MandaRain-3-128$_s$	0.0018	2.800	5.895	16	32	2890
MandaRain-3-128$_f$	0.0018	0.346	0.807	16	32	3588
MandaRain-4-128$_s$	0.0026	2.876	6.298	16	32	3052
MandaRain-4-128$_f$	0.0026	0.371	0.817	16	32	3876

Signing time (ms), verification time (ms), and signature size (bytes).

KuMQuat

Unstructured Multivariate-Quadratic

Sample $\mathrm{A}_i \in \mathbb{F}_q^{n \times n}$, $b_i \in \mathbb{F}_q^n$, and $x \in \mathbb{F}_q^n$.
Public key: seeds for A and b, and $y \in \mathbb{F}_q^n$ where

$$y_i = x^{\mathsf{T}} \mathrm{A}_i \, x + b_i^{\mathsf{T}} x_j$$

Unstructured Multivariate-Quadratic

Sample $\mathrm{A}_i \in \mathbb{F}_q^{n \times n}$, $b_i \in \mathbb{F}_q^n$, and $x \in \mathbb{F}_q^n$.
Public key: seeds for A and b, and $y \in \mathbb{F}_q^n$ where

$$y_i = x^{\mathsf{T}} \mathrm{A}_i \, x + b_i^{\mathsf{T}} x_j$$

Witness: $x \in \mathbb{F}_q^n$
Constraints:

$$y_i = \sum_{jk} \mathrm{A}_{ijk} \, x_j x_k + \sum_{j} b_{ij} \, x_j - y_i \quad \forall i \in [n]$$

Unstructured Multivariate-Quadratic

Sample $\mathrm{A}_i \in \mathbb{F}_q^{n \times n}$, $b_i \in \mathbb{F}_q^n$, and $x \in \mathbb{F}_q^n$.
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- Witness size is minimal (assuming only quadratic constraints).

Unstructured Multivariate-Quadratic

Sample $\mathsf{A}_i \in \mathbb{F}_q^{n \times n}$, $b_i \in \mathbb{F}_q^n$, and $x \in \mathbb{F}_q^n$.
Public key: seeds for A and b, and $y \in \mathbb{F}_q^n$ where

$$y_i = x^{\mathsf{T}} \mathsf{A}_i \, x + b_i^{\mathsf{T}} x_j$$

Witness: $x \in \mathbb{F}_q^n$
Constraints:

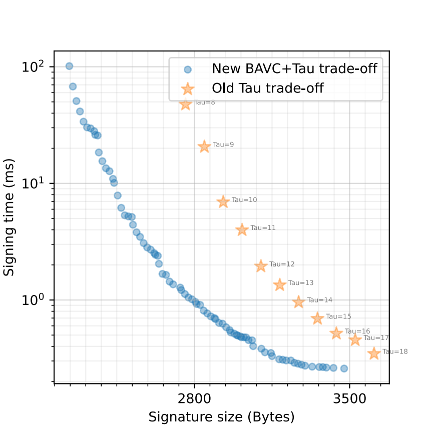
$$y_i = \sum_{jk} \mathsf{A}_{ijk} \, x_j x_k + \sum_j b_{ij} \, x_j - y_i \quad \forall i \in [n]$$

- Witness size is minimal (assuming only quadratic constraints).
- Optimization: pack multiple $\mathbb{F}_q$ constraints together into a $\mathbb{F}_{2^\lambda}$ constraint.

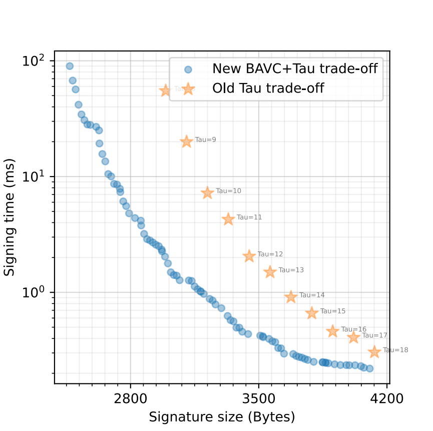
MQ Parameters

Instance	Security Level	$\mathbb{F}_q$	n
$\texttt{MQ-2}^1\texttt{-L1}$	L1	$\mathbb{F}_{2^1}$	152
$\texttt{MQ-2}^8\texttt{-L1}$	L1	$\mathbb{F}_{2^8}$	48
$\texttt{MQ-2}^1\texttt{-L3}$	L3	$\mathbb{F}_{2^1}$	224
$\texttt{MQ-2}^8\texttt{-L3}$	L3	$\mathbb{F}_{2^8}$	72
$\texttt{MQ-2}^1\texttt{-L5}$	L5	$\mathbb{F}_{2^1}$	320
$\texttt{MQ-2}^8\texttt{-L5}$	L5	$\mathbb{F}_{2^8}$	96

Size-time Tradeoff



(a) KuMQuat-2^1-L1.



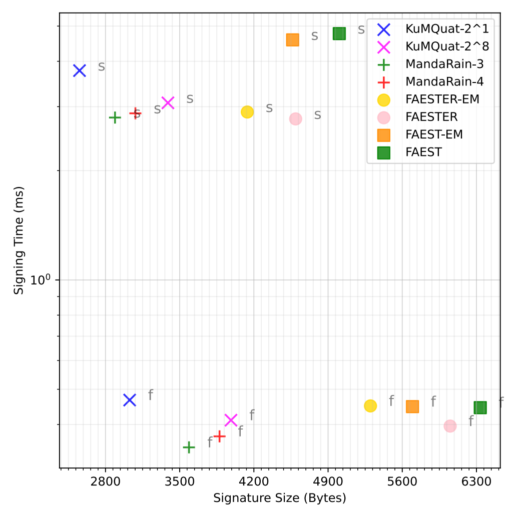
(b) KuMQuat-2^8-L1.

Performance Comparison

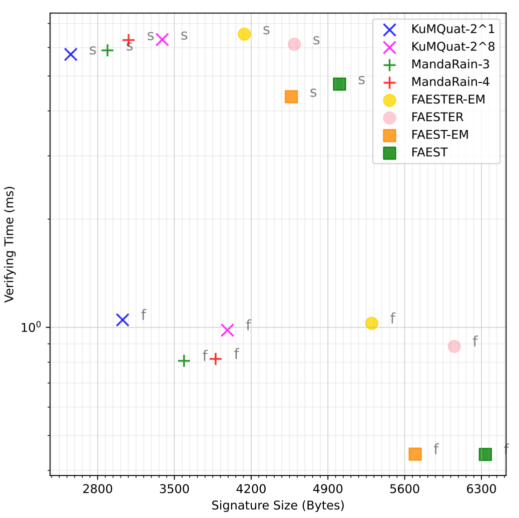
Scheme	Runtime in ms			Size in bytes		
	Keygen	Sign	Verify	sk	pk	Signature
FAEST-128$_s$	0.0006	4.381	4.102	16	32	5006
FAEST-128$_f$	0.0005	0.404	0.395	16	32	6336
FAEST-EM-128$_s$	0.0005	4.151	4.415	16	32	4566
FAEST-EM-128$_f$	0.0005	0.446	0.474	16	32	5696
FAESTER-128$_s$	0.0006	3.282	4.467	16	32	4594
FAESTER-128$_f$	0.0005	0.433	0.610	16	32	6052
FAESTER-EM-128$_s$	0.0005	3.005	4.386	16	32	4170
FAESTER-EM-128$_f$	0.0005	0.422	0.609	16	32	5444
MandaRain-3-128$_s$	0.0018	2.800	5.895	16	32	2890
MandaRain-3-128$_f$	0.0018	0.346	0.807	16	32	3588
MandaRain-4-128$_s$	0.0026	2.876	6.298	16	32	3052
MandaRain-4-128$_f$	0.0026	0.371	0.817	16	32	3876
KuMQuat-2^1-L1$_s$	0.173	4.305	4.107	19	35	2555
KuMQuat-2^1-L1$_f$	0.172	0.539	0.736	19	35	3028
KuMQuat-2^8-L1$_s$	0.174	3.599	4.053	48	64	2890
KuMQuat-2^8-L1$_f$	0.172	0.400	0.623	48	64	3588

Signing time (ms), verification time (ms), and signature size (bytes).

Performance Graph

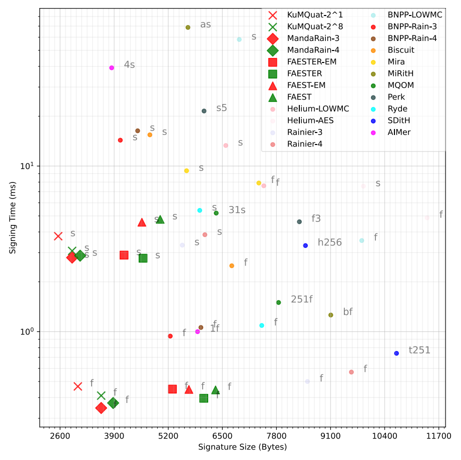


(a) Signing time - signature size trade-off.

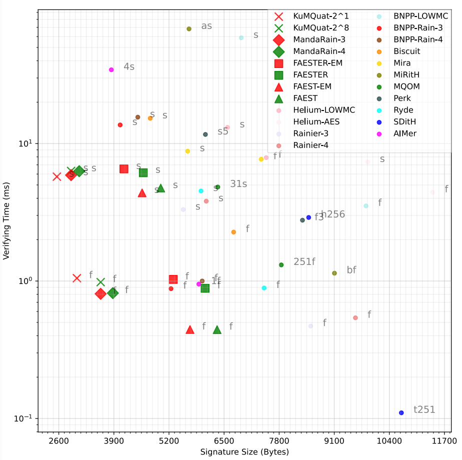


(b) Verification time - signature size trade-off.

Additional Graphs



(a) L1 Signing.



(b) L1 Verify.