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On the practical cost of Grover for AES key recovery

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Aims

- Assess impact of Grover on AES for near-term quantum hardware.
- Estimate logical implementation and parallelisation overheads on any hardware.
  - Logical qubit-cycles.
- Estimate error correction overheads when using planar surface code.
  - Surface code cycles and physical qubit count.

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Grover's algorithm

- Quantum algorithm to solve the unstructured search problem.
- Can be applied to key recovery for AES with key size k.
- Succeeds with high probability after $(\pi/4)\sqrt{2^k}$ quantum AES queries.
  - For AES-128, Grover takes around 2^{64} quantum AES queries compared with 2^{127} classical queries for brute force exhaustion.

Grover's algorithm

- However, the square-root speed-up headline neglects significant details:
  - The cost of quantum AES implementations.
  - The fact that the AES queries must be sequential.
  - The overheads from quantum error correction.

Oracle implementation

- Different implementations optimise for different metrics.
- We use Jang et al. “Quantum analysis of AES”, IACR ePrint 2022/683:
  - Minimises (circuit depth)^2 x (number of qubits).

AES Key Size	Depth	Qubits	Depth^2 x Qubits
128	731	3428	$2^{30.8}$
192	874	3748	$2^{31.4}$
256	1025	4036	$2^{32.0}$

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Maximum depth

Max depth	Cycle time 1µs	Cycle time 200ns	Cycle time 1ns
2^{40}	12.7 days	2.55 days	18.3 mins
2^{48}	8.92 years	1.78 years	3.26 days
2^{56}	2,280 years	457 years	2.28 years
2^{64}	585,000 years	117,000 years	585 years

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Parallelisation

- Limiting maximum depth limits number of iterations that can be performed.
- Reducing number of iterations by a factor of S reduces success probability by S^2.
- Alternatively, we can split the search space into subsets of size N/S^2.
- Either way, S^2 quantum processors are needed to cover the same search space.
- Overall costs (compute cost x time taken) have increased by a factor of *S*.

Costing Methodology – When Parallelisation Is Required

1. Calculate number of AES iterations per run from the implementation depth and MAX DEPTH choice.

$$N_{iter} = \frac{D_{max}}{D_{AES}}$$

2. Calculate the number of quantum processors needed, i.e. find S such that.

$$N_{iter} = \left(\frac{\pi}{4}\right)\frac{2^{k/2}}{\sqrt{S}}$$

3. Calculate the total number of logical qubits required.

$$W_{tot} = SW_{AES}$$

4. Calculate the cost in terms of number of logical qubit cycles.

$$C_{tot} = W_{tot}D_{max} = SW_{AES}D_{max} = \left(\frac{4}{2^{k/2}\pi}N_{iter}\right)^{-2} W_{AES}D_{max} = \boxed{2^k\left(\frac{\pi}{4}\right)^2\frac{D_{AES}^2 W_{AES}}{D_{max}}}$$

AES-128 logical costs

- Using logical qubit-cycles accounts for the non-trivial cost of idle qubits.

Max depth	Grover iterations	Parallel instances	Logical qubits	Logical qubit-cycles
2^{40}	$2^{30.5}$	$2^{66.3}$	$2^{78.1}$	$2^{118.1}$
2^{48}	$2^{38.5}$	$2^{50.3}$	$2^{62.1}$	$2^{110.1}$
2^{56}	$2^{46.5}$	$2^{34.3}$	$2^{46.1}$	$2^{102.1}$
2^{64}	$2^{54.5}$	$2^{18.3}$	$2^{30.1}$	$2^{94.1}$
∞	$2^{63.7}$	1	$2^{12.7}$	$2^{85.9}$

Quantum error correction

- Important to distinguish between perfect logical qubits and noisy physical qubits.
- Logical qubits are built from many physical qubits using quantum error correction.
- The planar surface code is currently the best studied QEC scheme.
  - Exponentially suppresses errors as code distance *d* increase.
  - Uses $2d^2 - 1$ physical qubits to produce one logical qubit.

Quantum error correction

- All error correction schemes have quantum gates that cannot be applied directly.
- These can instead be applied by producing “magic states”, which can be combined with basic gates to produce the desired non-basic gate.
- Creating high accuracy magic states will be done via magic state distillation, which creates them by combining many lower accuracy states.
- Magic state distillation requires additional quantum hardware, known as magic state factories or distilleries.

AES-128 surface code costs

Maximum depth	10^{-4} physical error		10^{-6} physical error	
	Physical qubits	Surface code cycles	Physical qubits	Surface code cycles
2^{40}	$2^{97.1}$	$2^{128.7}$	$2^{91.6}$	$2^{125.0}$
2^{48}	$2^{81.7}$	$2^{120.9}$	$2^{76.7}$	$2^{117.4}$
2^{56}	$2^{66.3}$	$2^{112.8}$	$2^{62.9}$	$2^{111.5}$
2^{64}	$2^{51.1}$	$2^{105.3}$	$2^{48.1}$	$2^{104.2}$

AES-128 overheads

- Logical implementation: 31 bits
- Parallelisation: 8 - 32 bits (depending on maximum depth)
- Error correction: 6 - 10 bits (depending on physical error rate)
  - *Distillation: 1 - 3 bits (included in error correction overhead)*

These are not entirely independent: less parallelisation needs more error correction.

Potential cost reductions

- Smaller AES implementations.
- Faster cycle times.
- Better physical error rates.
- More efficient error correcting codes.

Conclusions

- The practical security impact of Grover with existing techniques on plausible near-term quantum hardware is limited.
  - Bounding the length of time an adversary is prepared to wait introduces unavoidable overheads from parallelisation.
  - Error correction adds further overheads, but these are less significant.
  - Early post-quantum migration efforts should focus on traditional public-key algorithms.

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Thank you.

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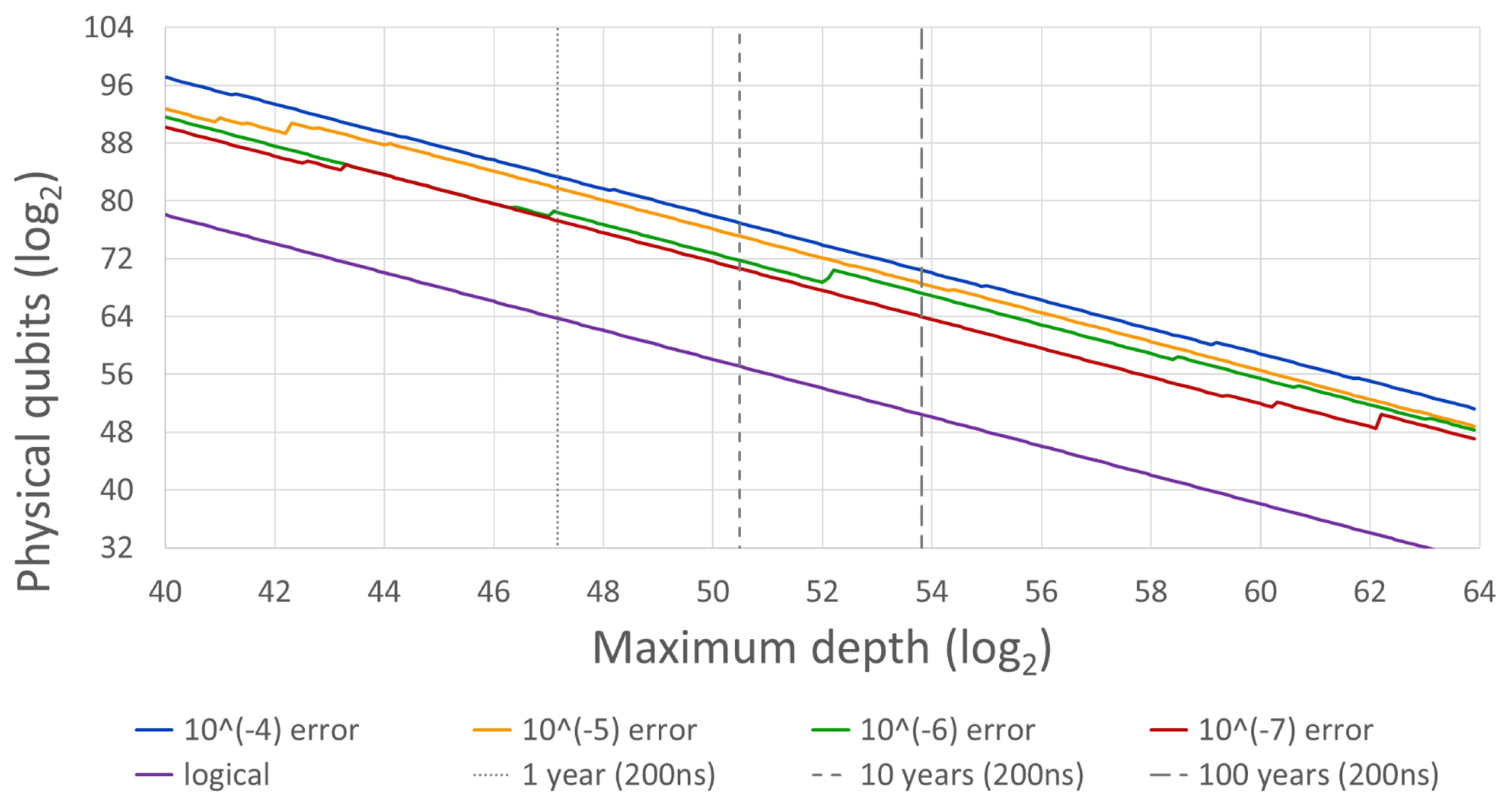
AES-128: Physical qubits



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AES-128: Surface code cycles