LESS: Digital Signatures from Linear Code Equivalence

NIST PQC Seminars

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- Background
- Code-based Signatures
- Group Actions
- LESS
- Considerations
Roadmap

► Background

► Code-based Signatures

► Group Actions

► LESS

► Considerations
$[n, k]$ Linear Code over $\mathbb{F}_q$

A subspace of dimension $k$ of $\mathbb{F}_q^n$. Value $n$ is called length.
[\mathbf{n}, \mathbf{k}] \textbf{ Linear Code over } \mathbb{F}_q

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**Hamming Metric**

\[\text{wt}(x) = |\{i : x_i \neq 0, 1 \leq i \leq n\}|, d(x, y) = \text{wt}(x - y)\]

Minimum distance (of \(\mathcal{C}\)):

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\min\{d(x, y) : x, y \in \mathcal{C}\}.
\]
## Error-Correcting Codes

### 1 Background

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\( G \in \mathbb{F}_q^{k \times n} \) defines the code as: \( x \in \mathcal{C} \iff x = uG \) for \( u \in \mathbb{F}_q^k \).

Not unique: \( SG, S \in GL(k, q) \); Systematic form: \( (I_k | M) \).
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**$w$-error correcting**: $\exists$ algorithm that corrects up to $w$ errors.
Example: Goppa Codes

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Select \( g(X) \in \mathbb{F}_{q^m}[X] \) and non-zero \( \alpha_1, \ldots, \alpha_n \in \mathbb{F}_{q^m} \) with \( g(\alpha_i) \neq 0 \).

Parity-check given by \( H = \{H_{ij}\} = \{\alpha_j^{i-1}/g(\alpha_j)\} \). Codewords over \( \mathbb{F}_q \).

Let noisy codeword be \( y = x + e, x \in C, \text{wt}(e) \leq w \).

For Goppa codes, \( w = r/2 \) (or \( w = r \) if binary), where \( r = \deg(g) \).
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To decode:

1. Compute syndrome $s = Hy^T = (s_0, \ldots, s_{r-1})$.
2. Obtain error locator poly $\sigma(X)$ and error evaluator poly $\omega(X)$ by solving key equation
   \[
   \frac{\omega(X)}{\sigma(X)} \equiv s(X) \mod X^r.
   \]
3. Find roots; error positions are reciprocals (values from $\omega(X)$).
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Decoding Problems

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For a given finite field $\mathbb{F}_q$ and integers $n, k$, the Gilbert-Varshamov (GV) distance is the largest integer $d_0$ such that

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Very well-studied, solid security understanding (ISD).
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Let $M$ be a matrix defining a code. Then $M$ is indistinguishable from a randomly generated matrix of the same size.
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Hardness of assumption depends on chosen code family.
Roadmap

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- Code-based Signatures

- Group Actions

- LESS

- Considerations
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Recent renditions show great improvements, but still exhibit similar features.
(Debris-Alazard, Sendrier, Tillich, 2018)
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Idea 2: Zero-Knowledge Protocols

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- Group Actions
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Let $\mathcal{X}$ be a set and $(\mathcal{G}, \cdot)$ be a group. A \textit{group action} is a mapping
\[ * : \mathcal{G} \times \mathcal{X} \rightarrow \mathcal{X} \]
\[ (g, x) \mapsto g \ast x \]

such that, for all $x \in \mathcal{X}$ and $g_1, g_2 \in \mathcal{G}$,
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**Group Action Vectorization Problem**

Given the pair $x_1, x_2 \in \mathcal{X}$, find, if any, $g \in \mathcal{G}$ such that $g \star x_1 = x_2$. 
Famous Examples

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A huge amount of cryptography has been built using this simple, but very special group action!
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What about group actions from coding theory?
Isometries in the Hamming Metric

3 Group Actions

Three types:

- **Permutations**: \( \pi \left( (a_1, a_2, \ldots, a_n) \right) = (a_{\pi(1)}, a_{\pi(2)}, \ldots, a_{\pi(n)}) \).
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**Monomial matrix**: permutation \( \times \) diagonal.
Isometries in the Hamming Metric

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- **Monomials + field automorphism**.
Isometries in the Hamming Metric

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  Monomial matrix: permutation \( \times \) diagonal.

- **Monomials + field automorphism.**

Two codes are **equivalent** if they are connected by an isometry.
Isometries in the Hamming Metric

3 Group Actions

Three types:

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We talk about **permutation**, **linear** and **semilinear** equivalence, respectively.
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\[ \star : \mathcal{G} \times \mathcal{X} \rightarrow \mathcal{X} \]
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In practice, we consider simply $RREF(G_0 Q)$.  

**Code-based Group Action**

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**Permutation Equivalence Problem (PEP)**

Given $\mathcal{C}_0, \mathcal{C}_1 \subseteq \mathbb{F}_q^n$, find a permutation $\pi$ such that $\pi(\mathcal{C}_0) = \mathcal{C}_1$. Equivalently, given generators $G_0, G_1 \in \mathbb{F}_q^{k \times n}$, find $P \in S_n$ such that

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For practical applications, we are not interested in the semilinear version of the problem.
Roadmap

► Background

► Code-based Signatures

► Group Actions

► LESS

► Considerations
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Other applications (e.g. ring signatures) will not be discussed in this talk.

(Barenghi, Biasse, Ngo, P., Santini, 2022)
LESS ZK Identification Scheme

Public data: system params, hash function $Hash$, code $\mathcal{C}$ with generator $G_0$. 
LESS ZK Identification Scheme

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- Choose random monomial matrix $\tilde{Q} \in M_n(q)$.
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- If $ch = 0$ verify that $Hash(RREF(G_0 \cdot rsp)) = cmt$.
- If $ch = 1$ verify that $Hash(RREF(G_1 \cdot rsp)) = cmt$.
LESS Signatures
4 LESS

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Both modifications do not affect security, only require small tweaks in proofs.
Key Generation

Input: system params, code $\mathcal{C}$ with generator $G_0$. 
Input: system params, code \( \mathcal{C} \) with generator \( G_0 \).

1. Set \( SK_0 = I_n \) and \( PK_0 = G_0 \).
2. Choose random seed \( seed_{sk} \in \{0, 1\}^\lambda \).
3. Generate \( Q_1, \ldots, Q_{s-1} \) from \( seed_{sk} \).
4. for \( i := 1 \) to \( s - 1 \)
5. Set \( SK(i) = Q_i \) and \( PK(i) = RREF(G_0Q_i) \).
6. Output \( SK = (SK_0, \ldots, SK_{s-1}) \) and \( PK = (PK_0, \ldots, PK_{s-1}) \).
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Private key can be easily compressed to a single seed.
Input: system params, hash function $Hash$, private key $SK$, message $msg$. 

---

The expand function is obtained via application of a PRNG, sampling uniformly at random from the target set.
Input: system params, hash function $Hash$, private key $SK$, message $msg$.

<table>
<thead>
<tr>
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| 1. Choose random master seed $mseed \in \{0, 1\}^\lambda$.
| 2. Generate $seed_0, \ldots, seed_{t-1}$ from $mseed$.
| 3. for $i := 1$ to $t - 1$
| 4. Generate $\tilde{Q}_i$ from $seed_i$.
| 5. Compute $\tilde{G}_i = RREF(G_0\tilde{Q}_i)$.
| 6. Set $d = Hash(\tilde{G}_0|| \ldots ||\tilde{G}_{t-1}||msg)$.
| 7. Expand $d$ to string $(x_0, \ldots, x_{t-1})$ with $\omega$ non-zero elements from $[0; s - 1]$.
| 8. for $i := 0$ to $t - 1$
| 9. Set $rsp_i$ to either $seed_i$ (if $x_i = 0$) or $Q_{x_i}^{-1}\tilde{Q}_i$ (otherwise).
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The $\text{expand}$ function (7.) is obtained via application of a PRNG, sampling uniformly at random from the target set.
Verify

Input: system params, hash function $Hash$, public key $PK$, message $msg$, signature $sigma$. 
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Verify

1. Expand d to string \((x_0, \ldots, x_{t-1})\) with \(\omega\) non-zero elements from \([0; s - 1]\).
2. for \(i := 1\) to \(t - 1\)
3. Recover \(\overline{Q}_i\) from \(rsp_i\).
4. Compute \(\overline{G}_i = RREF(G_{x_i} \overline{Q}_i)\).
5. Set \(d' = Hash(\overline{G}_0 || \ldots || \overline{G}_{t-1} || msg)\).
6. Output true if \(d = d'\), or false otherwise.
Input: system params, hash function $Hash$, public key $PK$, message $msg$, signature $sigma$.

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The `recover` function (3.) compactly describes: $rsp$ is either already a monomial, or a matrix can be obtained expanding a seed.
Roadmap

- Background
- Code-based Signatures
- Group Actions
- LESS
- Considerations
PEP is *not NP-complete*, unless the polynomial hierarchy collapses.

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As a consequence, most solvers for PEP can be easily adapted to solve LEP as well.
Exploit a variety of properties, give rise to (potentially) most efficient solvers.
Attack Strategy 1: Weak Instances

5 Considerations

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- Algebraic approaches of different nature, for example:
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- Support Splitting Algorithm (SSA) looks for invariants to distinguish equivalent codes. (Sendrier, 2000)

Weight Enumerator Function (WEF) is one, but too expensive; compute on hull.

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\mathcal{H}(\mathcal{C}) = \mathcal{C} \cap \mathcal{C}^\perp
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If \( \mathcal{C}_1 = \pi(\mathcal{C}_0) \), then \( \mathcal{H}(\mathcal{C}_1) = \pi(\mathcal{H}(\mathcal{C}_0)) \); running in \( \mathcal{O}(q^h) \).

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* To solve LEP, need to target closure of the code; these are always self-dual for \( q \geq 5 \).

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  * Set up a system of equations, solve via Gröbner basis. (Saeed-Taha, 2017)
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These are only efficient (or applicable in the first place) if hull is trivial.
Attack Strategy 2: Codeword Search

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Can obtain small improvement by carefully matching 2-dimensional subcodes instead. (Barenghi, Biasse, P., Santini, 2023)
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Gain from advanced techniques deteriorates quickly for increasing values of $q$. (Meurer, 2013)
Design Considerations

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We parametrize using latter type of attacks, following conservative criterion. Namely, we pick $n, k, q$ so that, for any $d$ and any $w$, we have:

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We compactly generate and transmit seeds using a \textit{seed tree} structure.
Sizes and Timings
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Protocol parameters $(t, \omega, s)$ infer performance profile:

<table>
<thead>
<tr>
<th></th>
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<td>NIST Parameter Code</td>
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</tr>
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<td>Cat. Set</td>
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<td>27.5</td>
<td>525 57 2</td>
<td>62.1</td>
<td>49.7</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>506 253 509</td>
<td>2300 18 64</td>
<td>4447.9</td>
<td>14.6</td>
<td>116 28 64</td>
<td>4447.9</td>
<td>19.3</td>
</tr>
</tbody>
</table>

Runtime is dominated by RREF computation, for both Keygen and Sign/Verify.

This yields timings with contrasting behavior. For our reference code:

- **Balanced**, Cat. 1: Keygen \(\approx 8\) Mcycles, Sign/Verify \(\approx 834\) Mcycles
Sizes and Timings
5 Considerations

Protocol parameters \((t, \omega, s)\) infer performance profile:

<table>
<thead>
<tr>
<th>NIST Cat.</th>
<th>Parameter Set</th>
<th>Code Params. ((n, k, q))</th>
<th>Prot. Params. ((t, \omega, s))</th>
<th>PK (kB)</th>
<th>Sig (kB)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Balanced</td>
<td>252 126 127</td>
<td>1053 18 2</td>
<td>13.7</td>
<td>6.1</td>
<td>247 30 2</td>
<td>13.7</td>
<td>10.8</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>252 126 127</td>
<td>1263 9 64</td>
<td>862.4</td>
<td>3.3</td>
<td>46 15 64</td>
<td>862.4</td>
<td>4.2</td>
</tr>
<tr>
<td>3</td>
<td>Balanced</td>
<td>468 234 31</td>
<td>1776 26 2</td>
<td>33.7</td>
<td>14.8</td>
<td>377 44 2</td>
<td>33.7</td>
<td>26.5</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>400 200 127</td>
<td>1297 14 64</td>
<td>2167.2</td>
<td>8</td>
<td>72 22 64</td>
<td>2167.2</td>
<td>10.3</td>
</tr>
<tr>
<td>5</td>
<td>Balanced</td>
<td>636 318 31</td>
<td>2518 34 2</td>
<td>62.1</td>
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Optimized implementations (e.g. ARM, possibly hardware) are also a target for June.
Thank you for listening!
Any questions?