



# CryptographicEstimators

## A Software Library for Cryptographic Hardness Estimation

Andre Esser, Javier Verbel, Floyd Zweydinger and Emanuele Bellini

@NIST PQC Seminar, Nov. 2023

# Contents

## 1. Introduction

- Hardness Estimation
- CryptographicEstimators

## 2. Theoretical Considerations

- Time and Memory
- Memory Access Costs

## 3. Technical Design

## 4. Usage

## 5. Contributing

## 6. NIST PQC Signatures

## 7. Future Developments

# Introduction



# Cryptographic Hardness Estimation

# Cryptographic Hardness Estimation

## Estimation of required time to solve a (cryptographic) problem

- Security guarantees
- Parameter selection
  - Example: RSA keysize recommendations
- Estimates change over time: adaptive process

# The Case of Classical Cryptography

- Established methodology

# The Case of Classical Cryptography

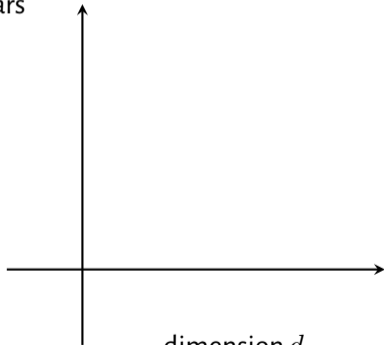
- Established methodology
  - 1 Theory
  - 2 Experiments
  - 3 Extrapolate

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CPU Years



runtime formula  $T_d$

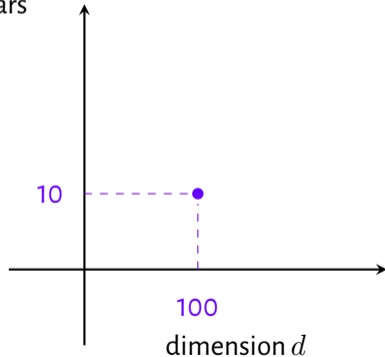


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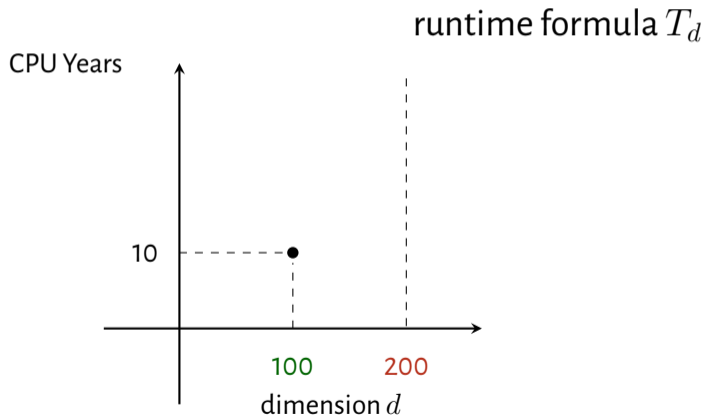


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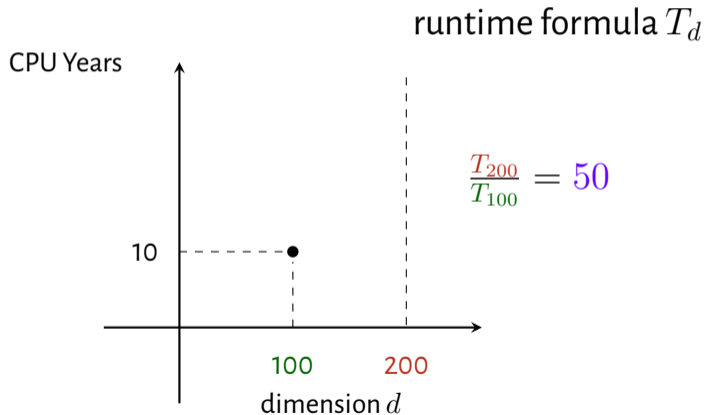
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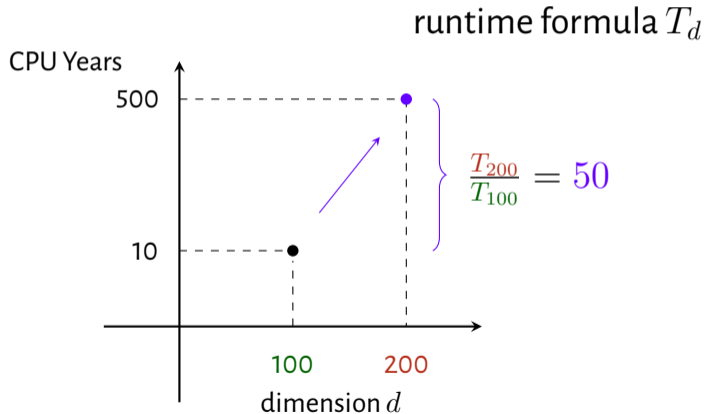
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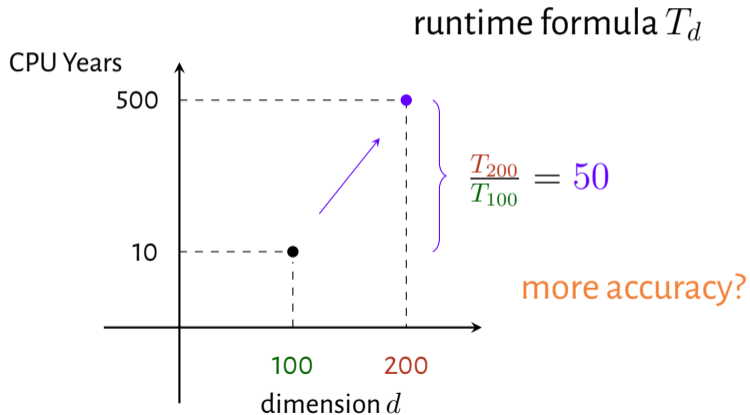
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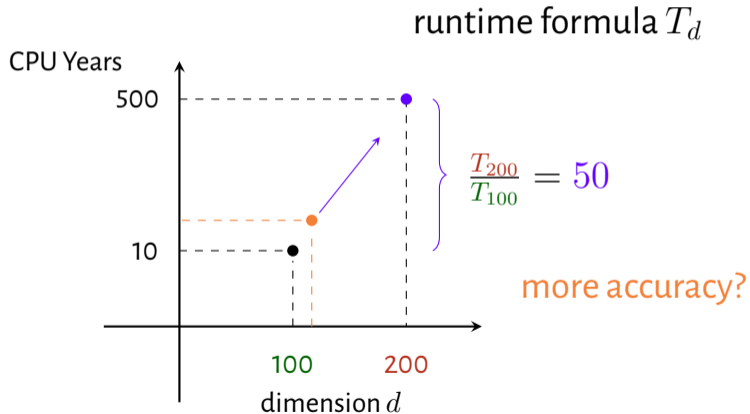
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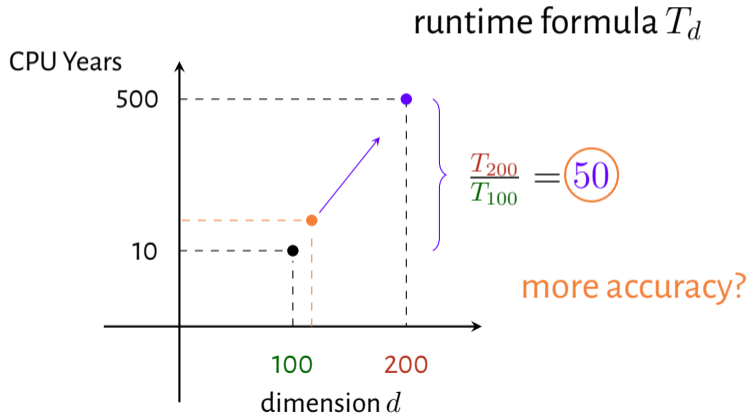
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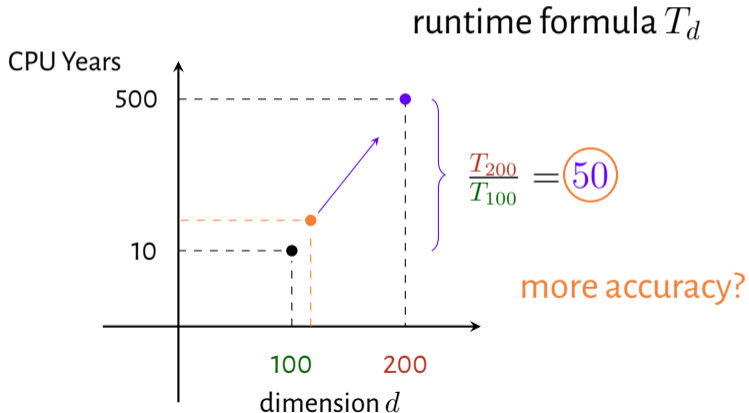


# The Case of Classical Cryptography

Assumption: Scalability

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## Python / Sage library for estimations of cryptographic problems

- Main goals
  - State-of-the-art estimations
  - Centralization of estimation efforts
  - Community-driven open-source project
  - Easy accessibility
- Current State: 6 Estimators, 32 Algorithms
  - Multivariate Quadratic (MQ)
  - Binary Syndrome Decoding (SD)
  - Syndrome Decoding over  $\mathbb{F}_q$  (SDFq)
  - Permutation Equivalence (PE)
  - Linear Equivalence (LE)
  - Permuted Kernel (PK)

## Theoretical Considerations



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  - Example MQ-Problem :  $op$ :  $\mathbb{F}_q$ -multiplication and  $e1$ :  $\mathbb{F}_q$ -element
  - Example binary SD-Problem:  $op$ :  $\mathbb{F}_2^n$ -vector addition and  $e1$ :  $\mathbb{F}_2^n$ -vector

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- Problem defines  $op / e1$  to bit (operation) conversion



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  - Real cost  $C$  of the full algorithm:  $T \leq C \leq T \cdot f(M)$

# Technical Design

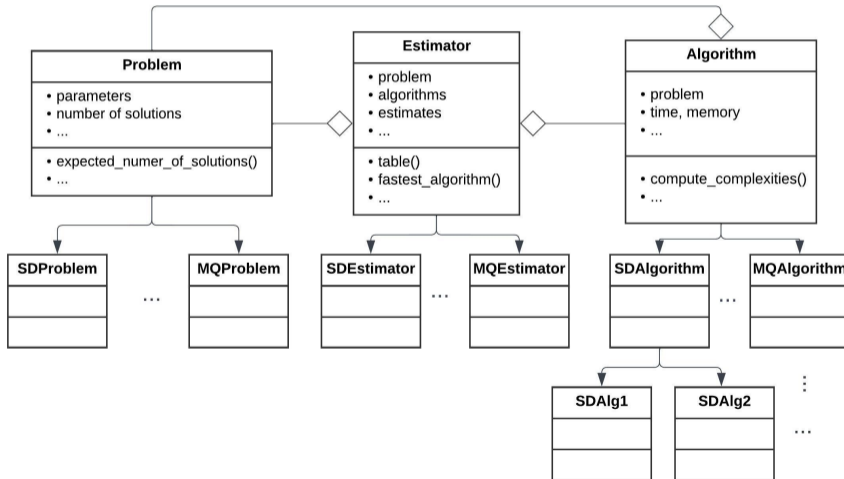


# Class Design



# Class Design

## CryptographicEstimators: An object-oriented Python library



Usage



## Usage

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- From web application <sup>2</sup>:

---

<sup>1</sup>Docker: <https://www.docker.com>

<sup>2</sup>Webapp: <https://estimators.cryptotii.ae>

# Usage

- From command line:

- Install → sage →

```
1 from cryptographic_estimators.SDEstimator import SDEstimator
2 A = SDEstimator(n=500, k=250, w=50)
3 A.table()
```

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|---------------|----------|--------|
| algorithm     | time     | memory |
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| BJMMdw        | 64.0     | 25.2   |
| BJMMpdw       | 63.9     | 28.0   |
| BJMM          | 63.6     | 29.4   |
| BJMM_plus     | 63.5     | 26.5   |
| BothMay       | 63.2     | 25.2   |
| Dumer         | 63.9     | 28.5   |
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The screenshot shows the web application interface for CryptographicEstimators. At the top, there is a purple header with the TII logo (Technology Innovation Institute) and the text "CryptographicEstimators". Below the header is a search bar containing "Binary Syndrome Decoding" with a dropdown arrow and a close button (X). The main content area is light purple and contains several input fields for parameters:

- Code length \*** (with a help icon): Input field containing "500".
- Code dimension \*** (with a help icon): Input field containing "250".
- Error weight \*** (with a help icon): Input field containing "50".
- Memory limit** (with a help icon): Input field containing "Insert value". Below it is the text "Leave empty if no limit is desired".
- Number of solutions** (with a help icon): Input field containing "Insert value". Below it is the text "Leave empty to take expected amount of solutions".

At the bottom of the interface, there are two blue links with right-pointing chevrons: [Optional parameters](#) and [Estimators parameters](#).



# Usage

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## Results

Algorithms

BallCollision, BJM...

Estimate

Time, Memory, Par...

| Algorithm     | Time | Memory | Estimate | Parameters                        | + |
|---------------|------|--------|----------|-----------------------------------|---|
| BallCollision | 64   | 22     |          | r: 6 p: 2 pl: 0 l: 12             | + |
| BJMM          | 63   | 29     |          | r: 6 depth: 2 p: 4 pl: 2 l: 26    | + |
| BJMM_plus     | 63   | 26     |          | r: 6 p: 4 pl: 2 l: 26 ll: 9       | + |
| BothMay       | 63   | 25     |          | r: 6 p: 4 w1: 0 w2: 0 pl: 2 l: 10 | + |
| Dumer         | 63   | 28     |          | r: 6 l: 19 p: 3                   | + |
| MayOzerov     | 62   | 33     |          | r: 6 depth: 2 p: 6 pl: 4 l: 25    | + |
| Prange        | 77   | 17     |          | r: 6                              | + |
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Export configuration

Download table.tex

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complexity of crossbred →

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70.8336959616846
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{'D': 9, 'd': 1, 'k': 11}
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A full user guide is [available](#).

Contributing



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  - 1 Check the code and raise issues.
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## NIST PQC Signatures



## Coverage

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- Best known attacks of 8 / 30 (remaining) submissions fall into this scope

## Estimation of NIST Candidates

- Estimates for NIST Category I parameter sets

| Scheme     | Hardness Assumption | Est. Time | Est. Memory |
|------------|---------------------|-----------|-------------|
| SDitH      | SDFq                | 147.0     | 26.9        |
| LESS       | LE                  | 136.6     | 39.0        |
| PERK       | PK                  | 155.5     | 154.4       |
| MQOM       | MQ                  | 142.8     | 51.8        |
| TUOV / UOV | MQ                  | 144.5     | 59.6        |
| VOX        | MQ                  | 153.0     | 59.8        |
| PROV       | MQ                  | 150.1     | 62.3        |

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