CWI Centrum Wiskunde & Informatica

Secure Multiparty Computation (MPC)

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Meeting on Privacy-Enhancing Cryptography

December 8 & 9, 2011

Outline

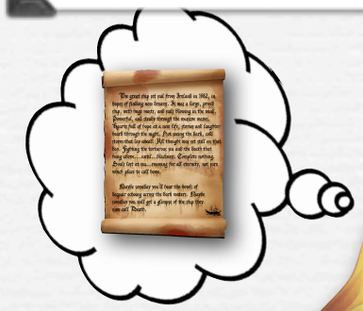
- Intro and problem description
- Possibility result
- High-level idea

Encryption and more

ALICE

BOB

Encryption and more



ALICE

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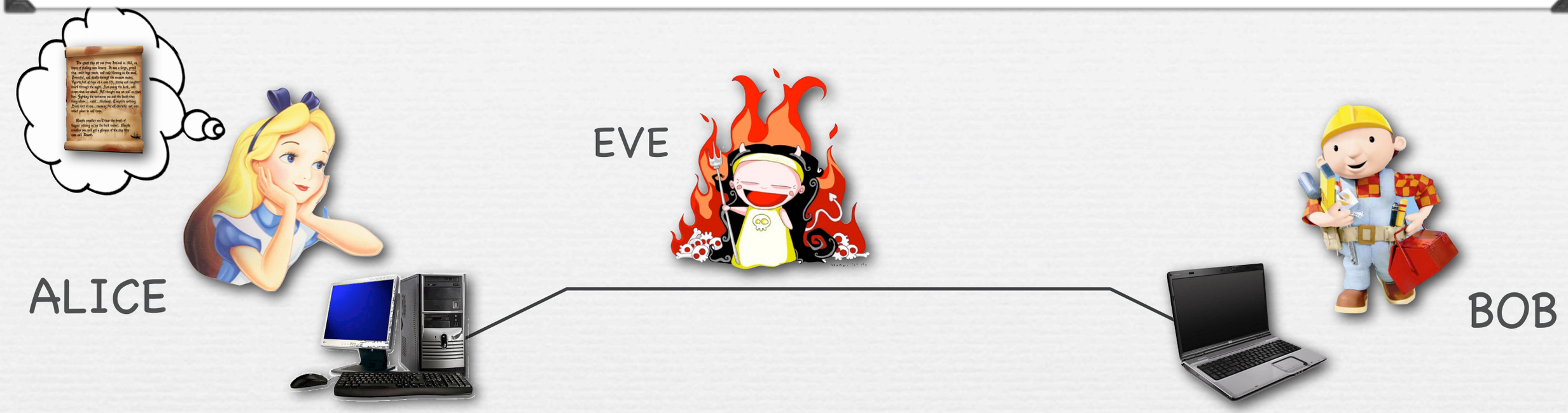
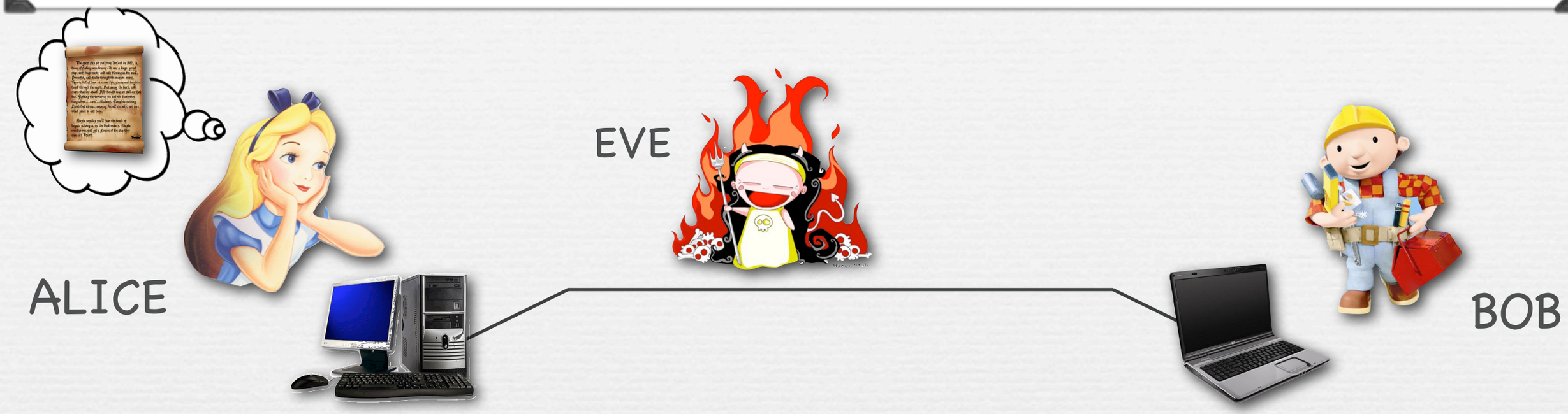


Eve can:

- eavesdrop the communication
  -> use encryption (symmetric or public-key)

Encryption and more



Eve can:

- eavesdrop the communication
  -> use encryption (symmetric or public-key)
- modify (or insert/delete) messages
  -> use authentication or digital signatures

Encryption and more

ALICE

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BOB

Eve co

- eav
  ->
- moc
  -> use authentication or digital signatures

Distinguishing features

- clear distinction between good and bad
- know whom to trust
- reveal all-or-nothing

Encryption and more

ALICE

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BOB

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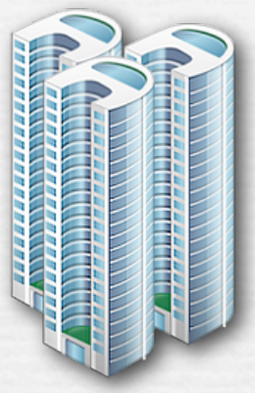
But:

The world is not just **black** and 

Examples

Company A 

 Company B

- A and B want to compare their performance

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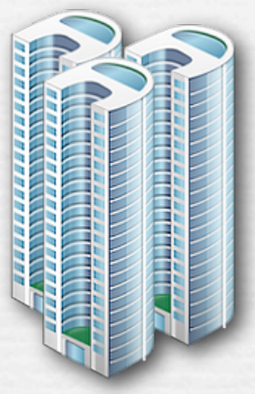 Company B

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- Neither is willing to reveal its detailed performance data

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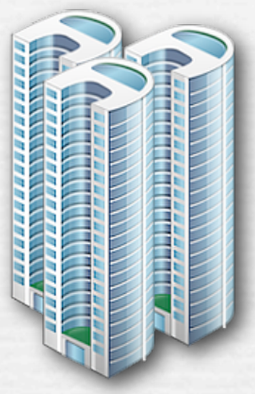 Company B

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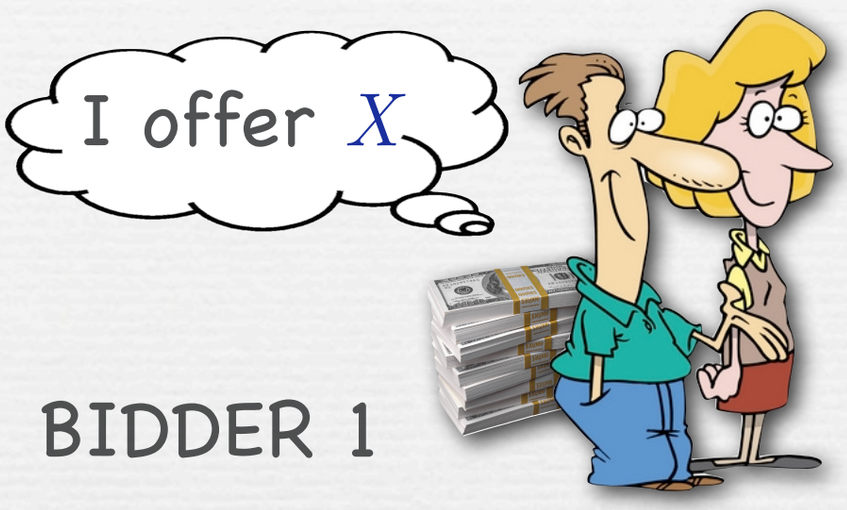
OR

- A and B want to find the overlap in customers
- Neither is willing to reveal its own customer list

Examples



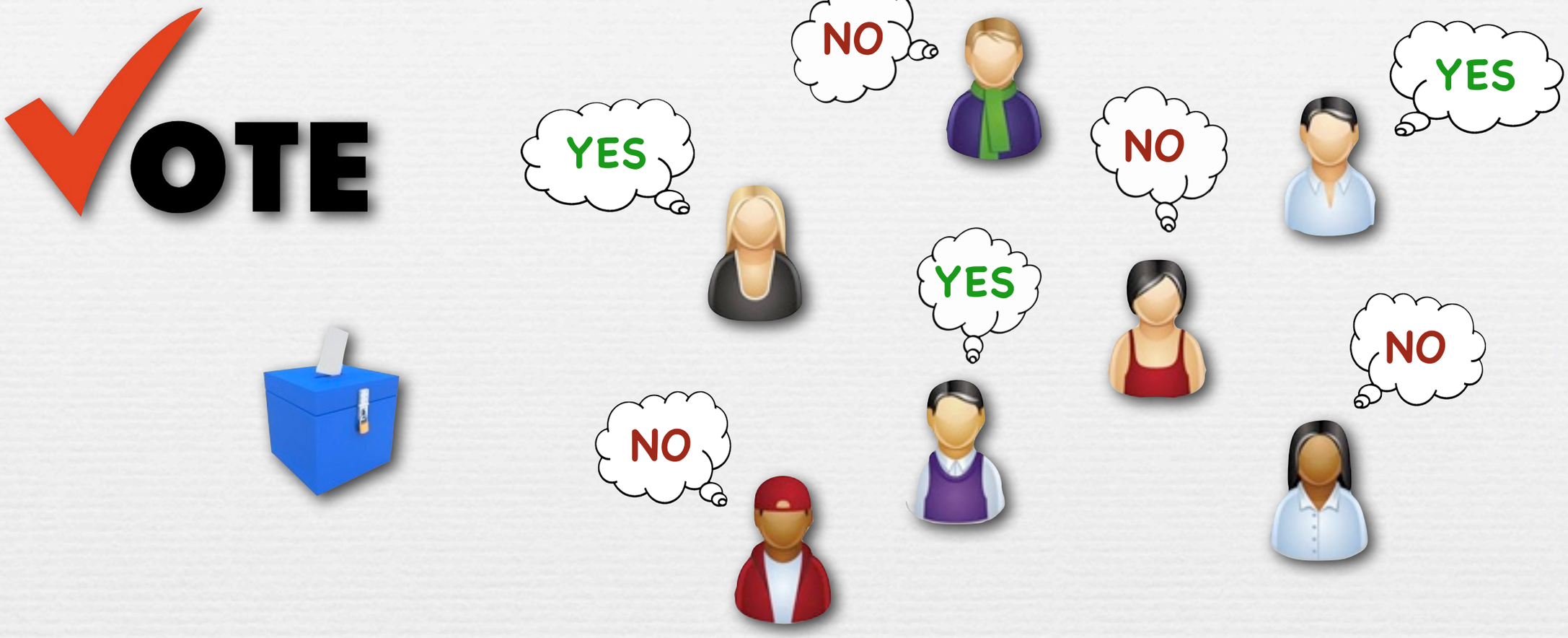
BIDDER 1

BIDDER 2

SELLER

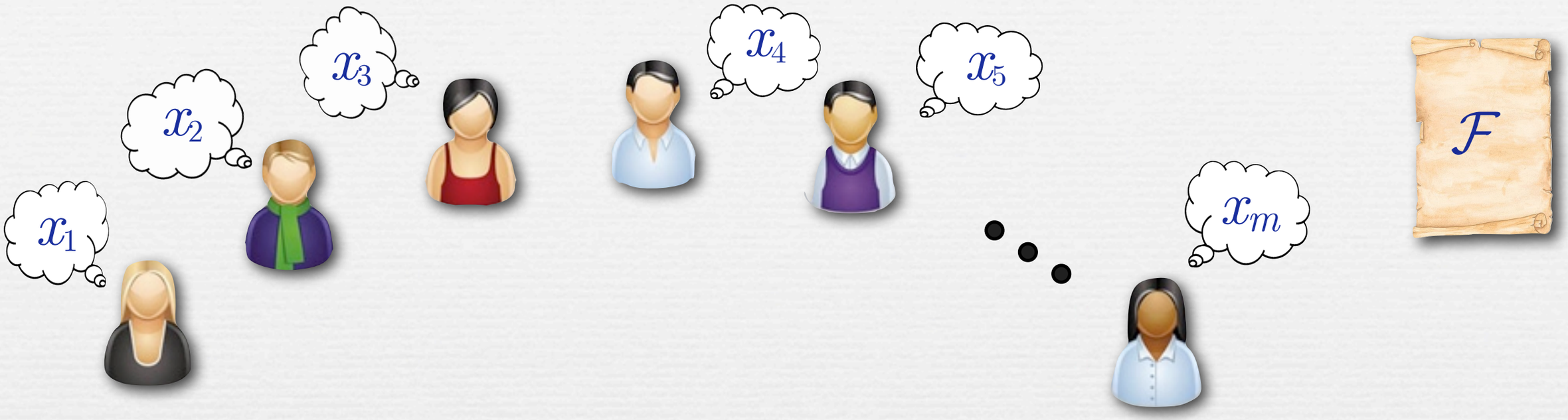
- Want to find out if bids are sufficient and who bids more, and e.g. agree on $\max\{W, \min\{X, Y\}+1\}$ as price.
- No one is willing to reveal his upper/lower bound.

Examples



- Voters want to find out outcome of the vote.
- None is willing to reveal his individual vote.

The General Problem



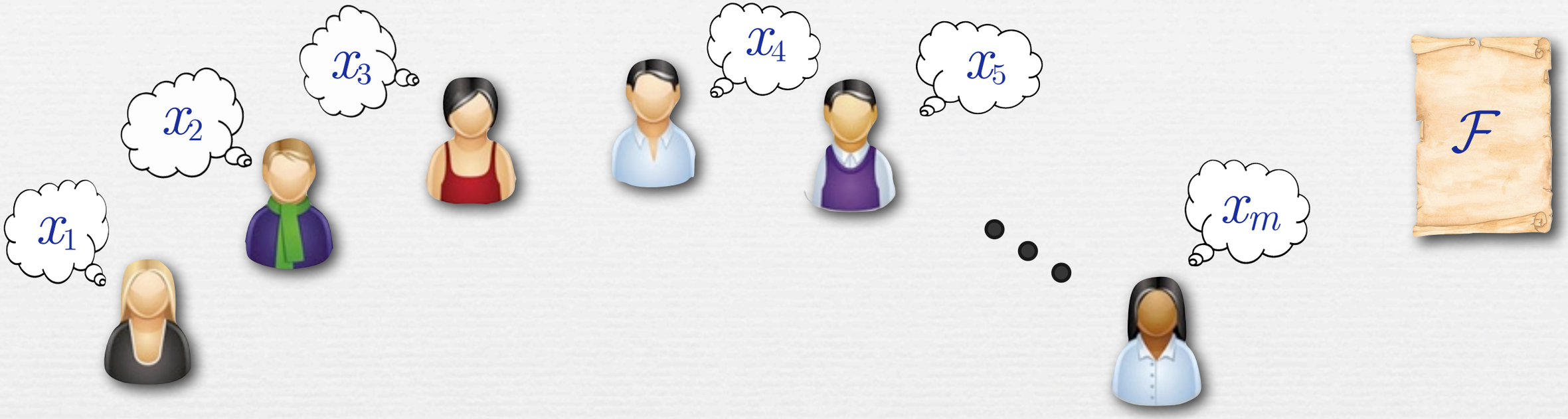
- Every user U_i has a private input x_i .

The General Problem

$$\mathcal{F}(x_1, x_2, x_3, \ldots, x_m) = ?$$

- Every user U_i has a private input x_i.
- Users want to learn $\mathcal{F}(x_1, x_2, x_3, \ldots, x_1)$.
  Variation: Different users learn different functions.
- Private inputs should remain private.

An Ideal Solution



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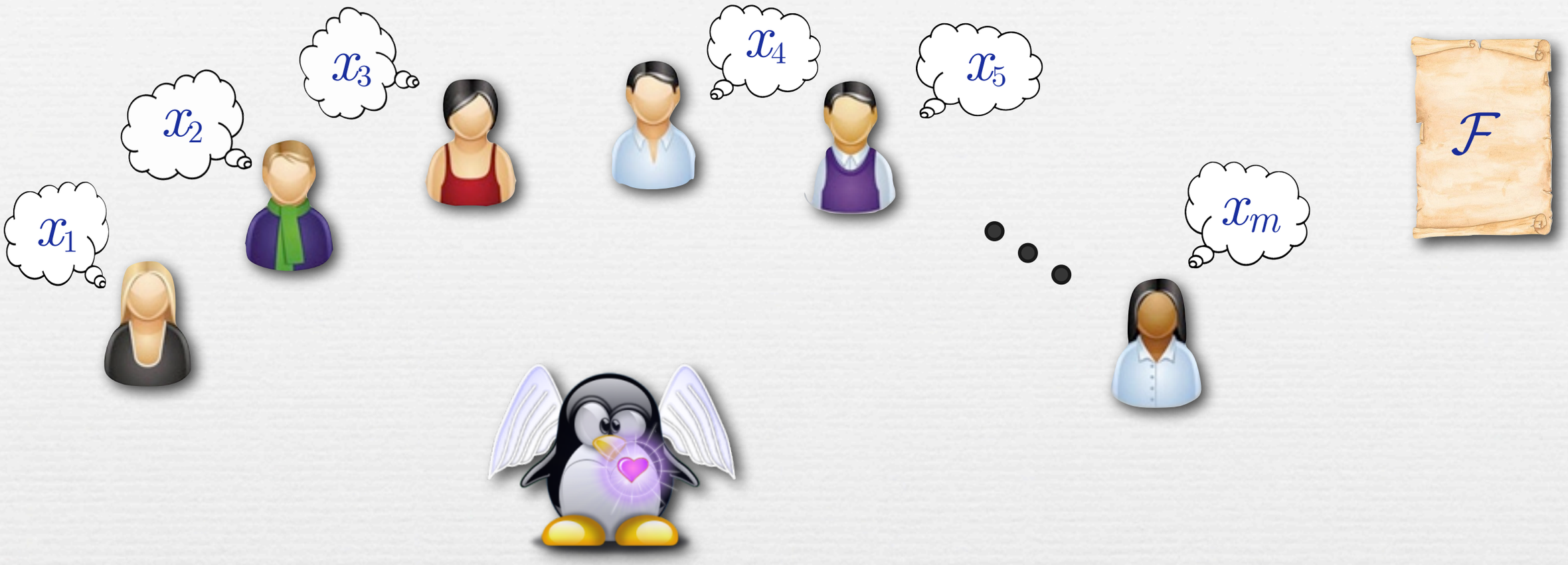

trusted authority TA

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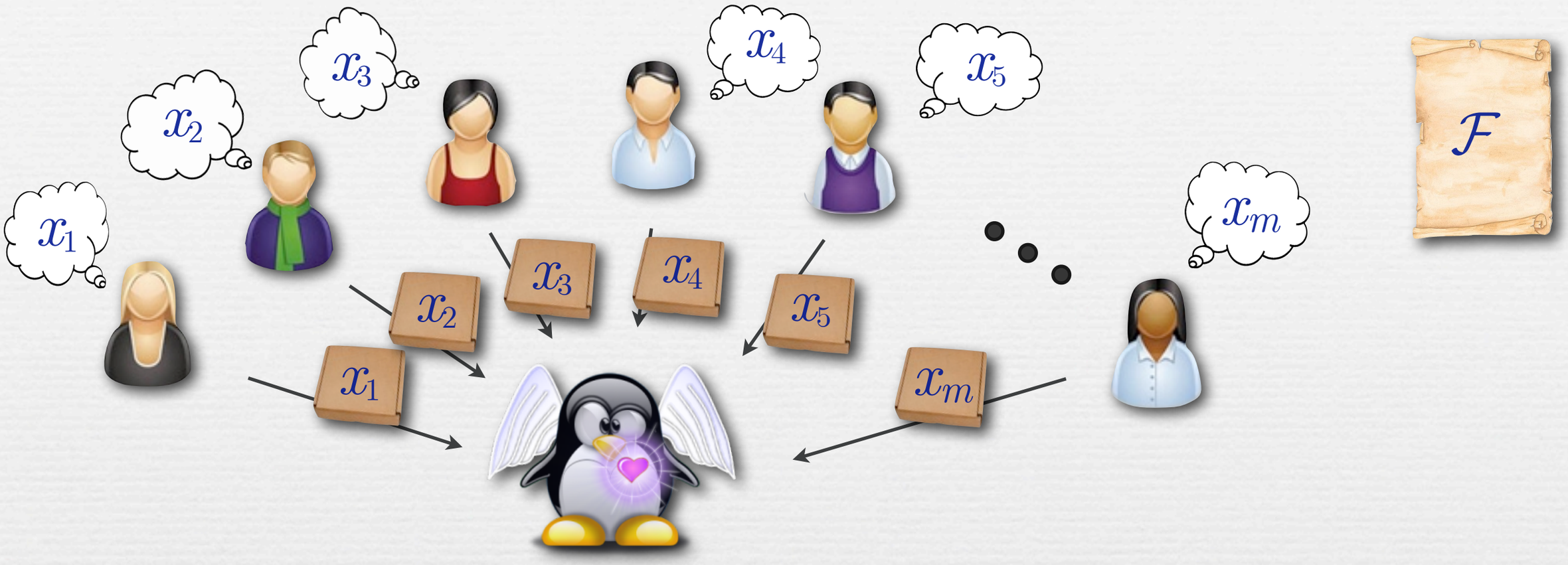

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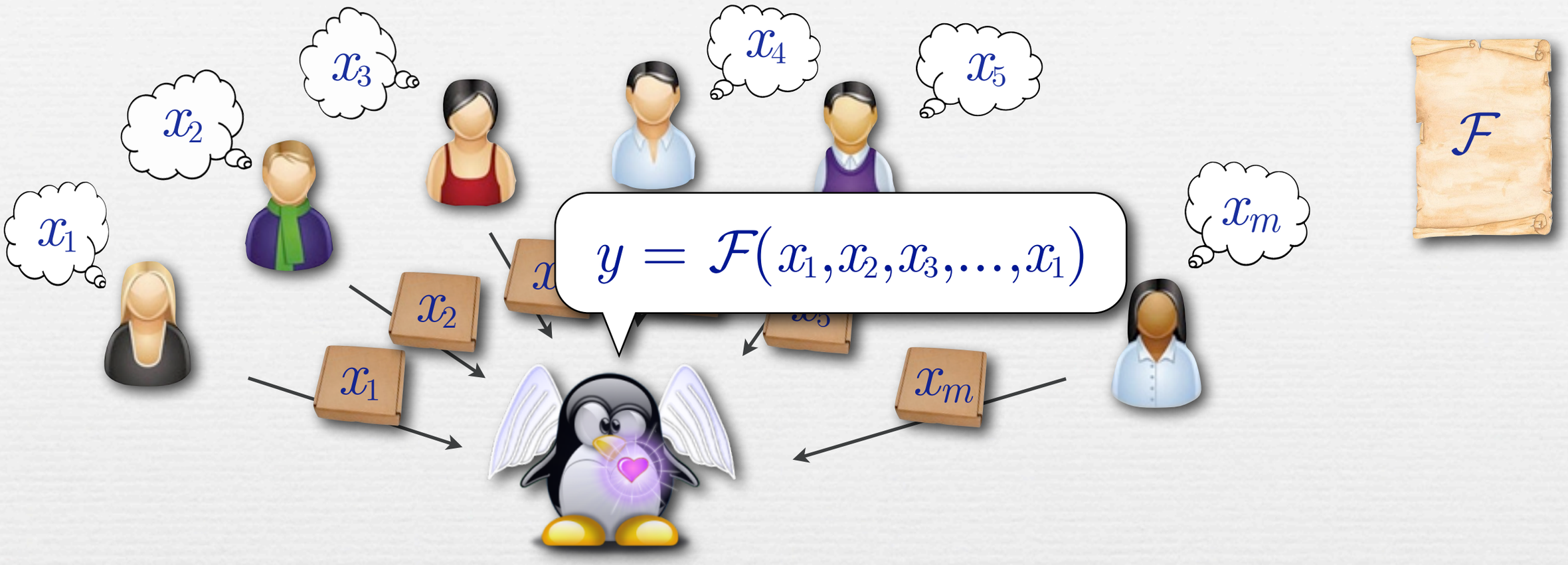
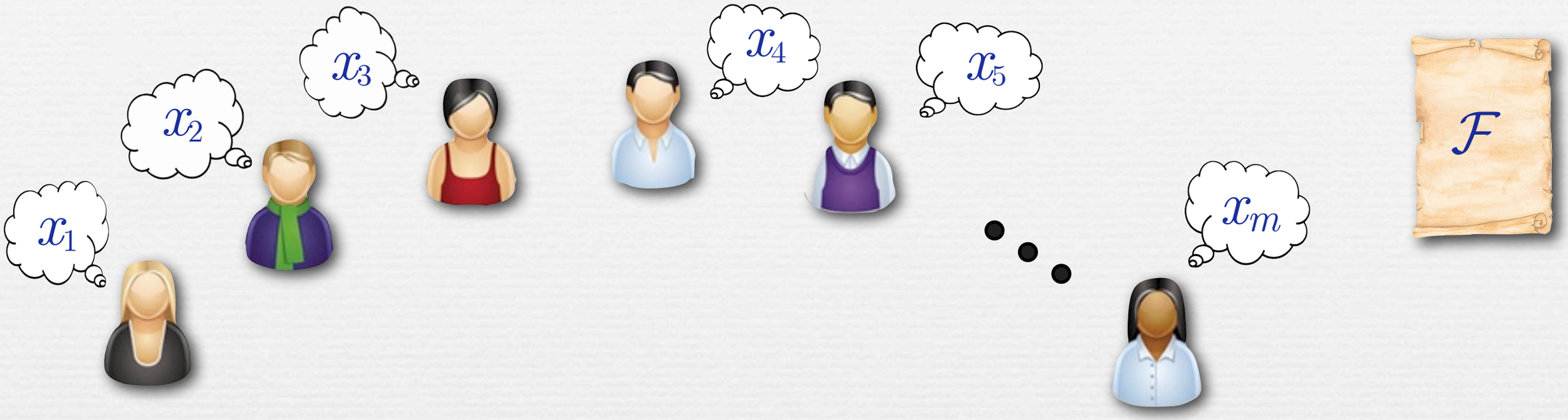


- Every user U_i sends his x_i to trusted authority TA.
- TA computes $y = \mathcal{F}(x_1, x_2, x_3, \ldots, x_1)$, and
- announces y to everyone.

MPC: Removing the Trusted Authority



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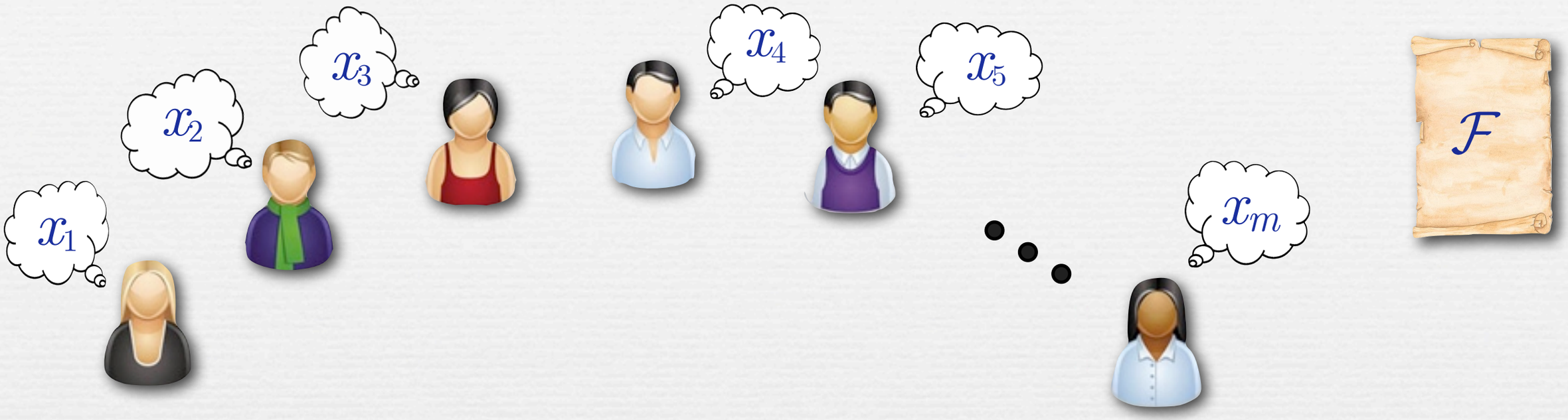

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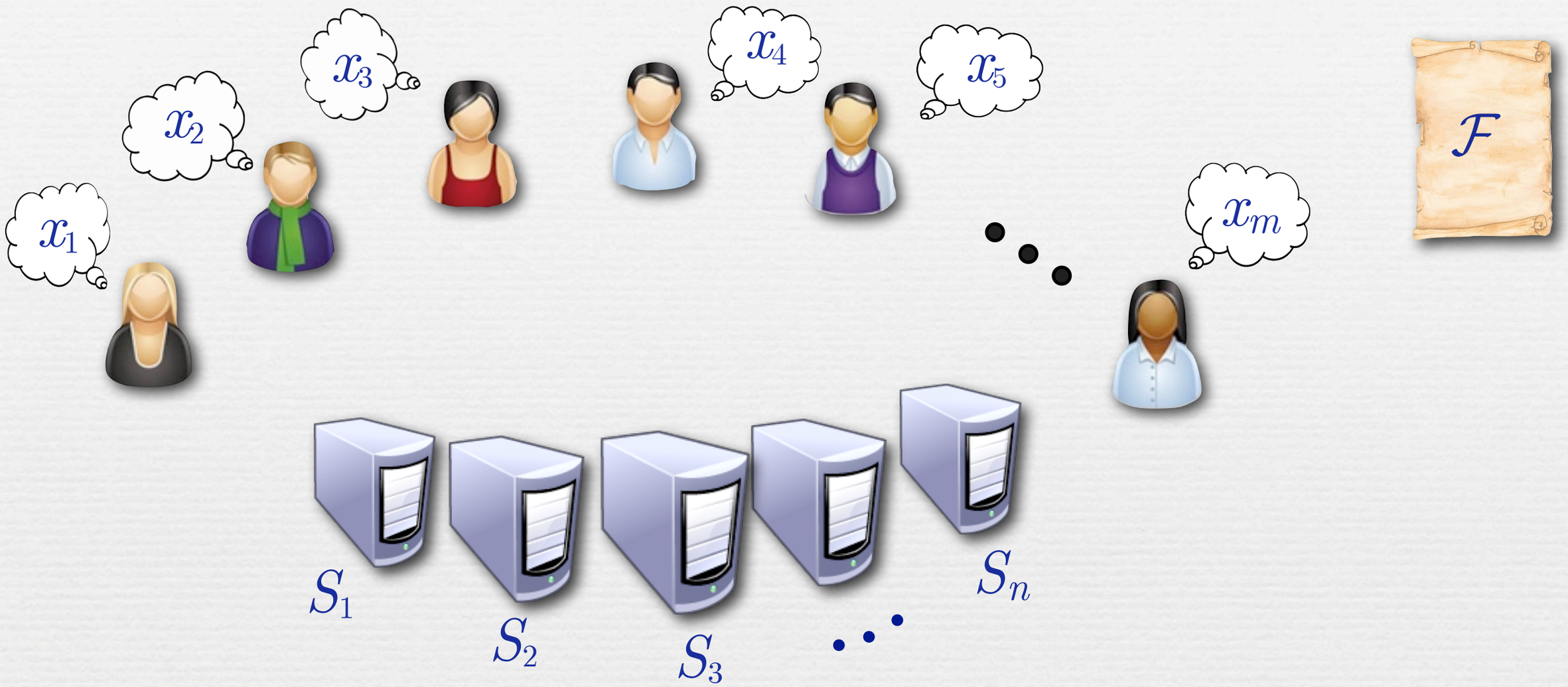

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- Perform computation by a group of servers.
- Some of the servers may be malicious.

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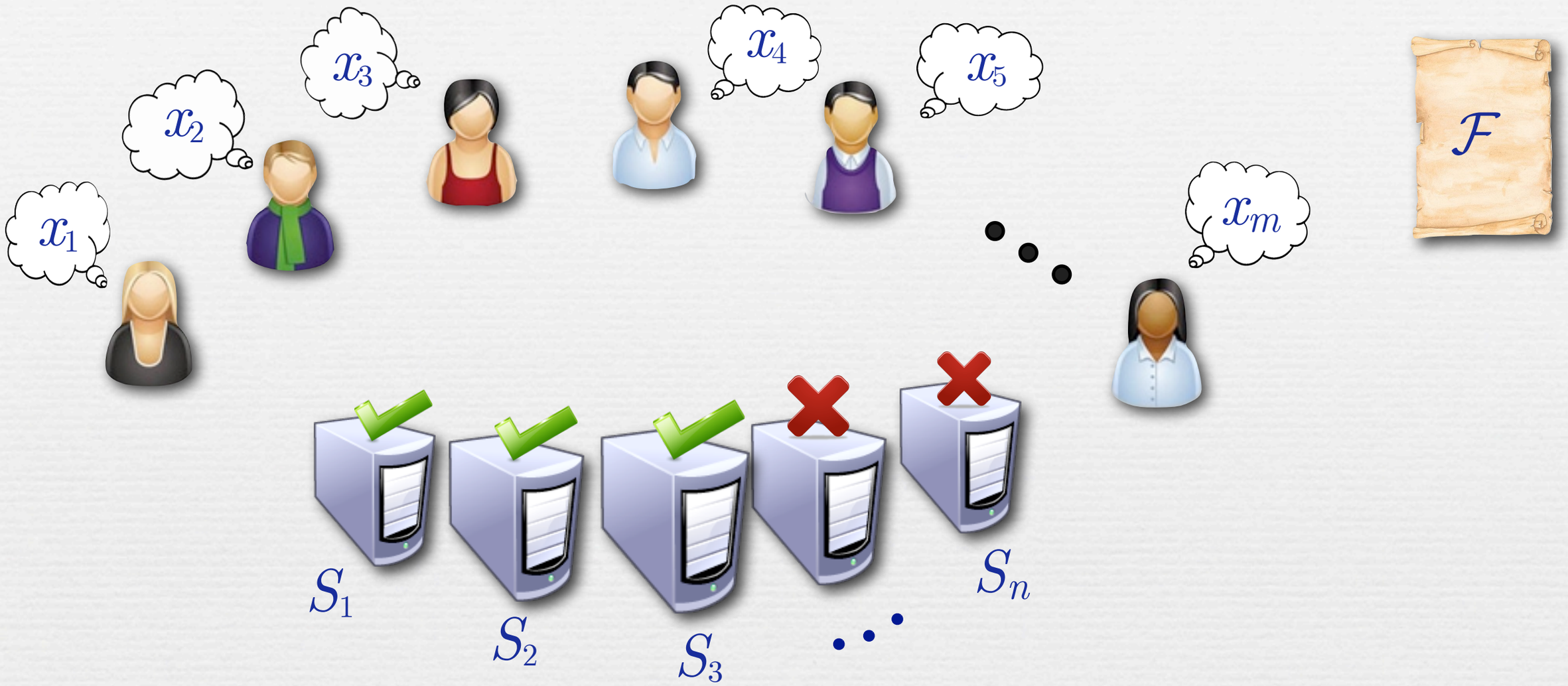

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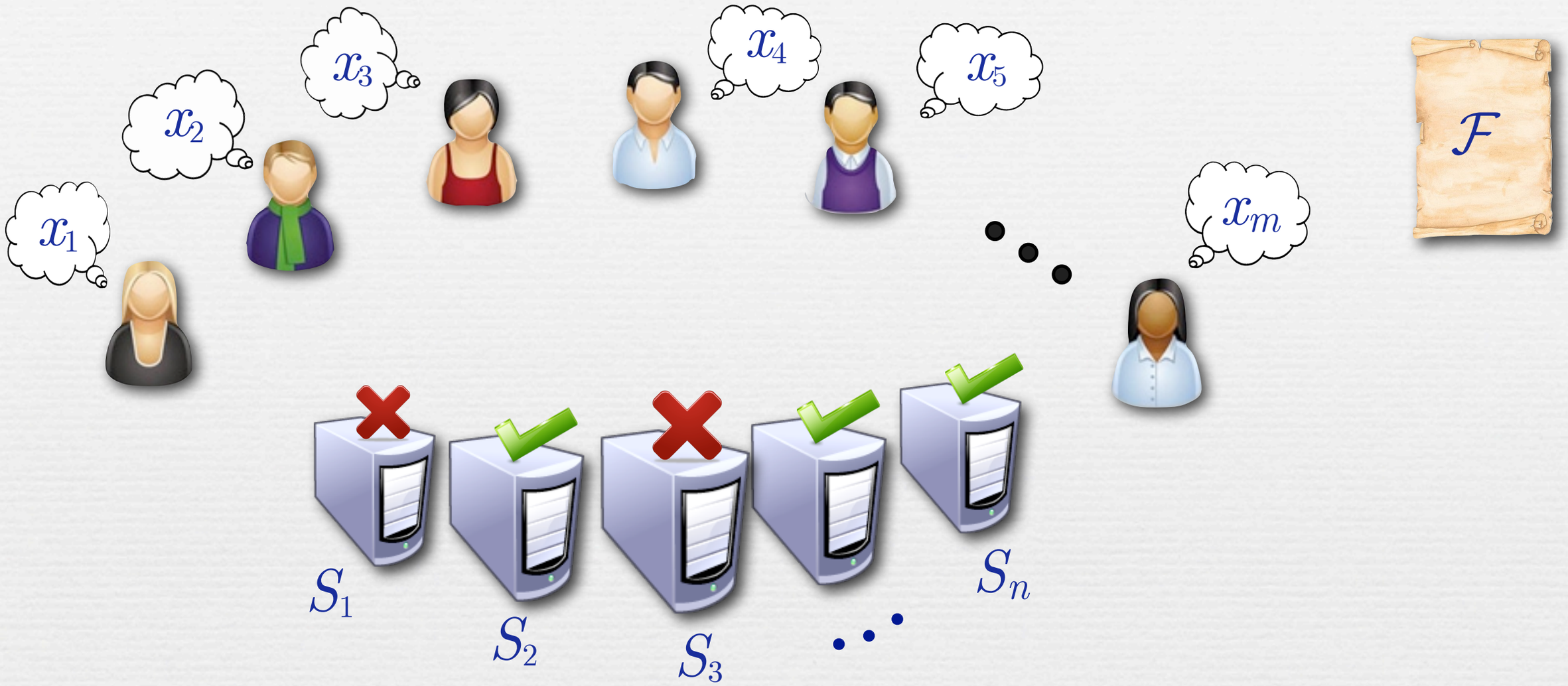

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- Malicious servers jointly should not learn any input.
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- No need to know whom to trust.
- Different users may trust different servers.
- No single point of failure

Only requirement:

- sufficiently many servers are honest.

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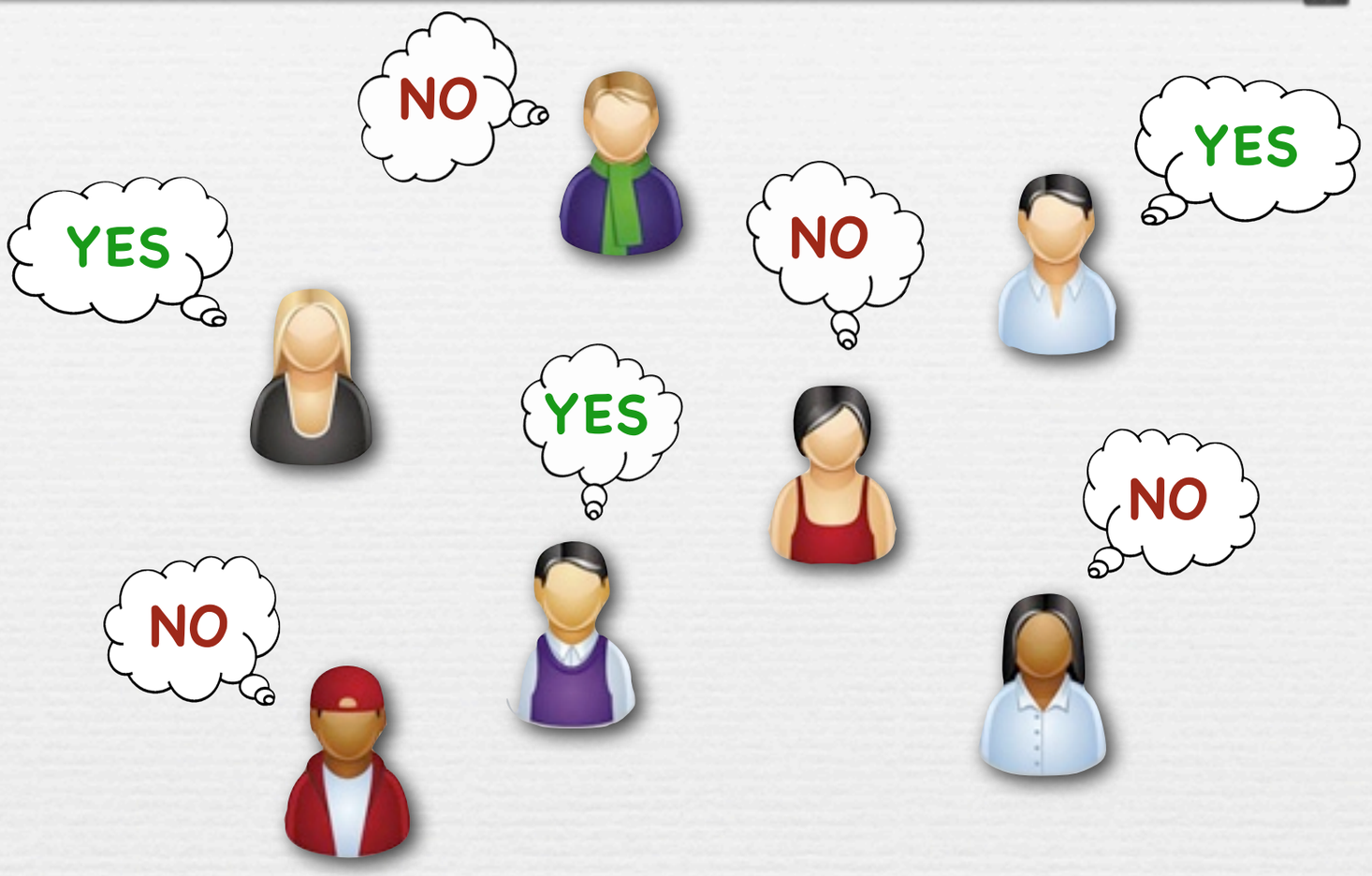
Possibility of MPC

Under reasonable set-up assumptions (e.g. PKI),
general secure MPC is possible if (and only if)
a **majority** of the servers are honest,
i.e., $t < n/2$ of the n servers are malicious.

Exist many different variants which differ in:

- flavors of security
- set-up assumptions
- complexity
- # of malicious servers
- communication model
- etc.

Example



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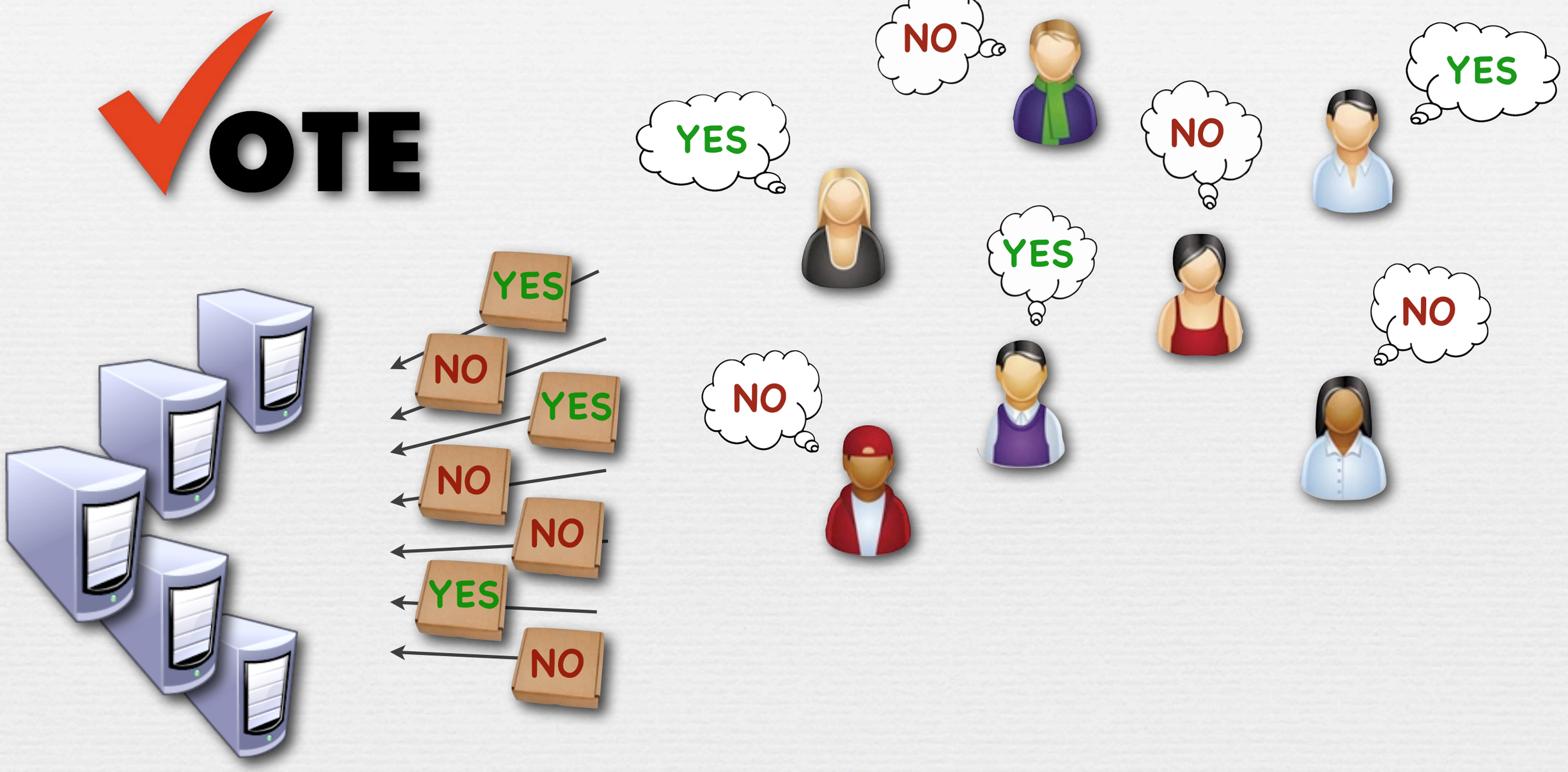

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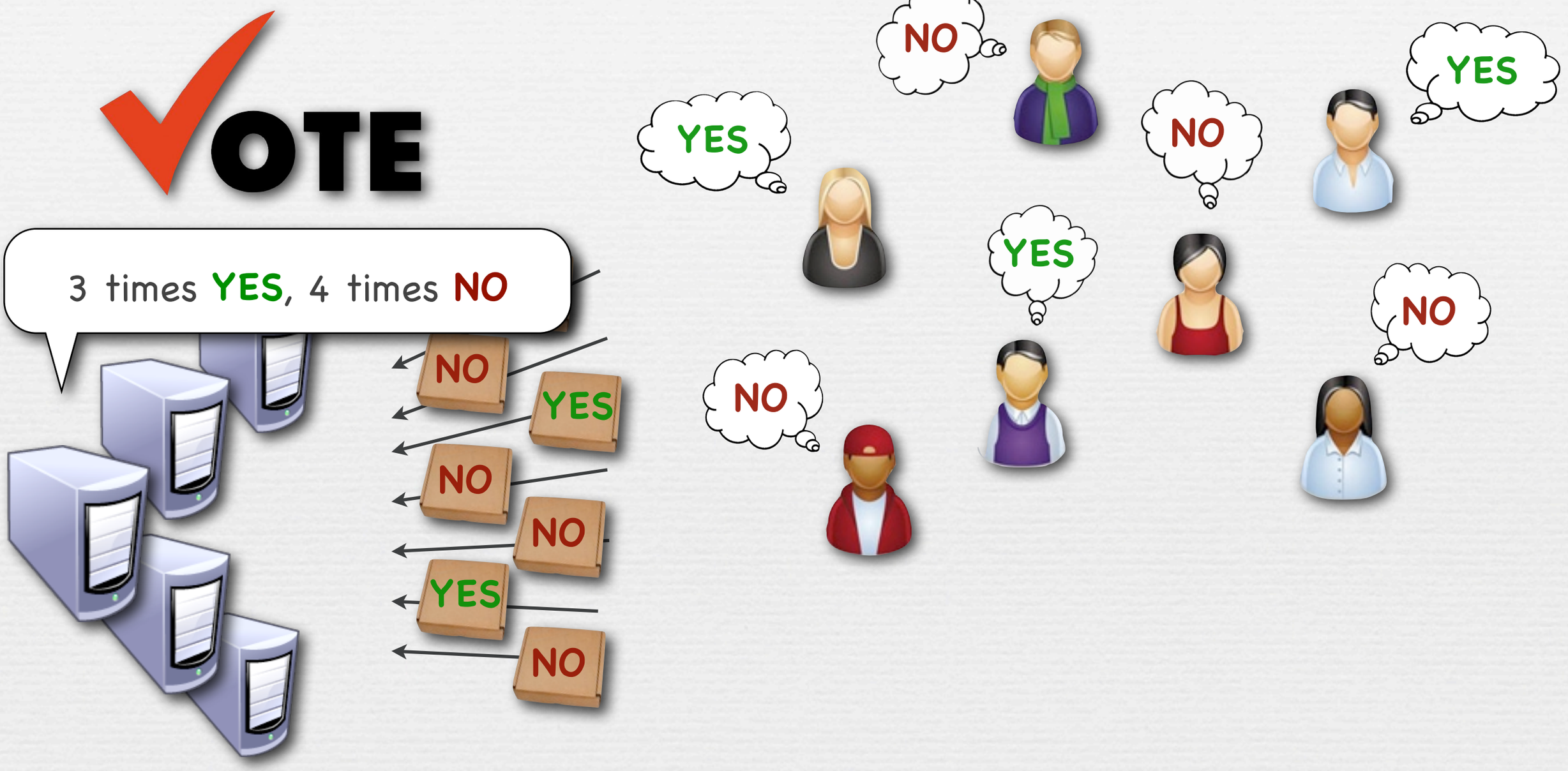

Promise:

- Votes remain private and tally is guaranteed correct
- If a majority of servers is honest.

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Public-key encryption scheme with special properties

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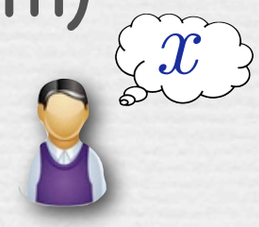

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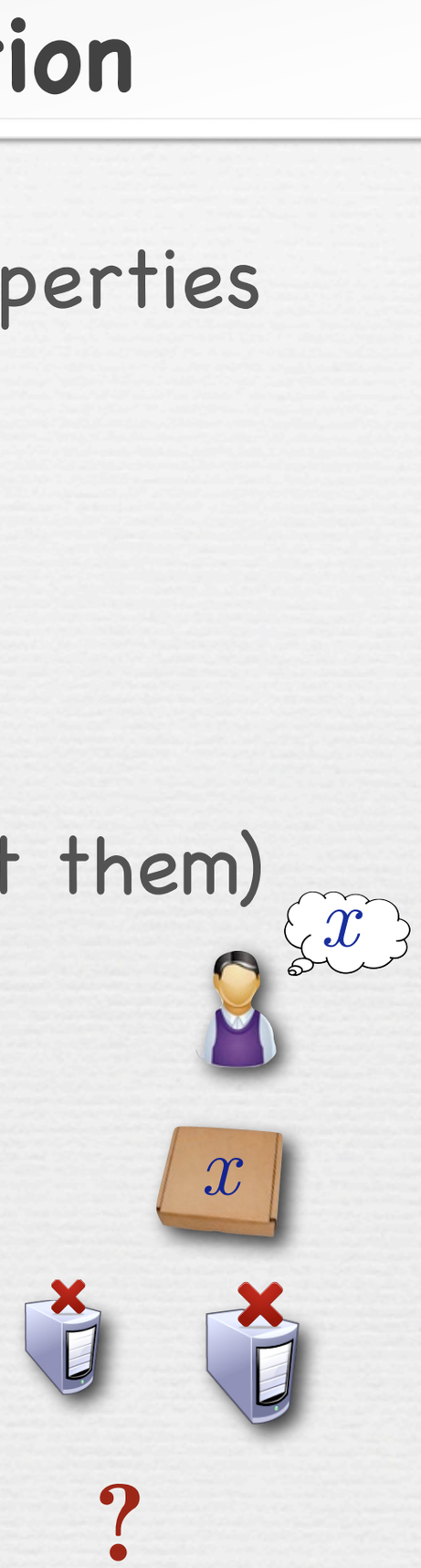

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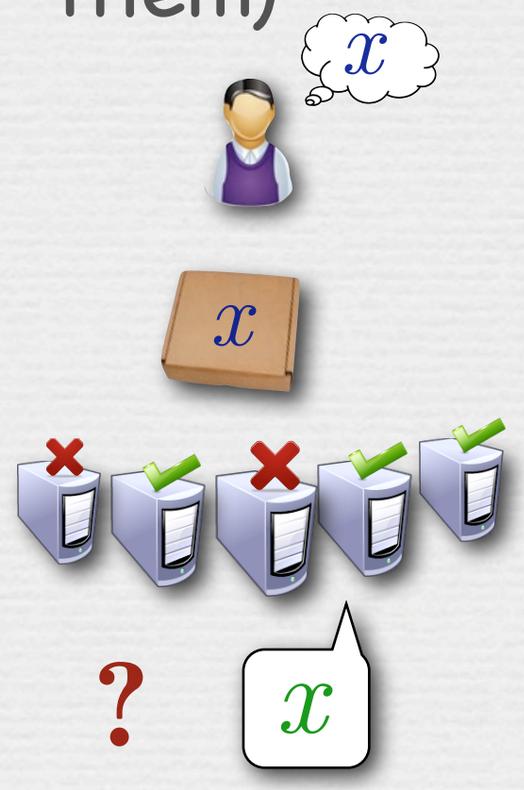

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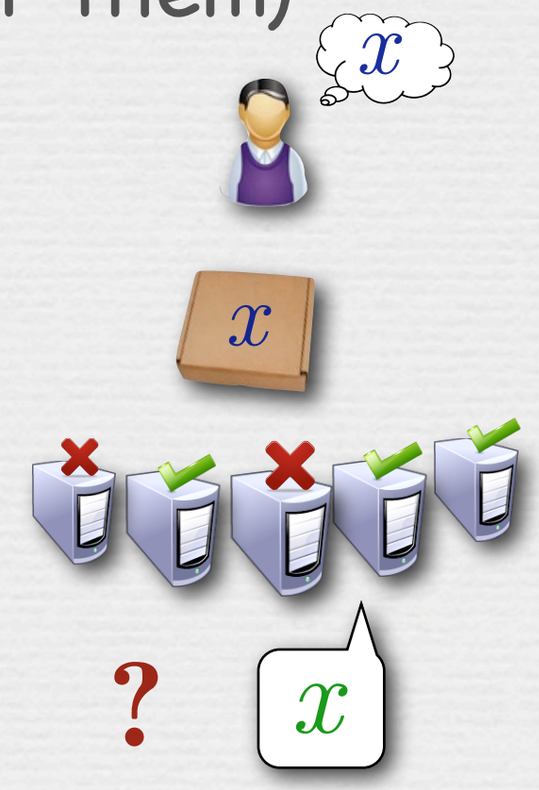

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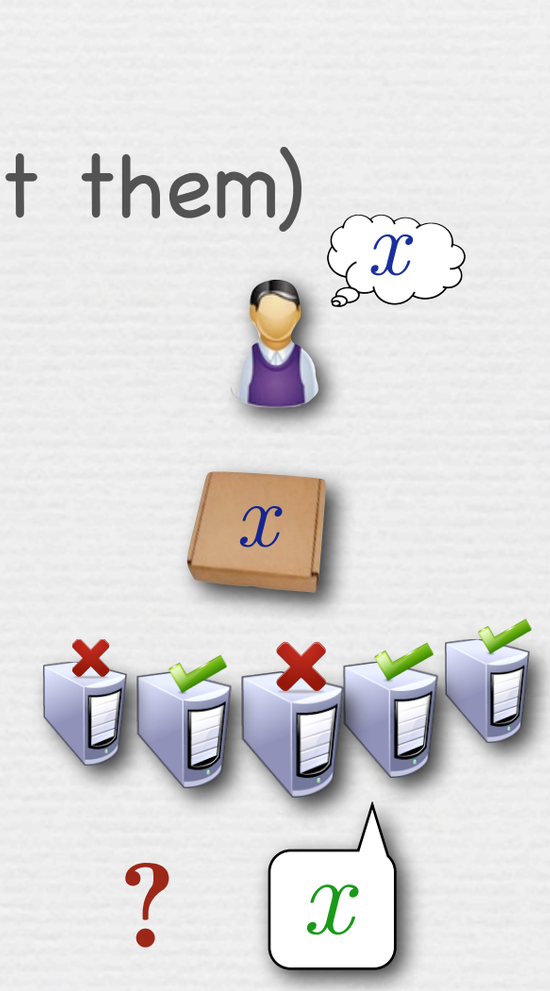

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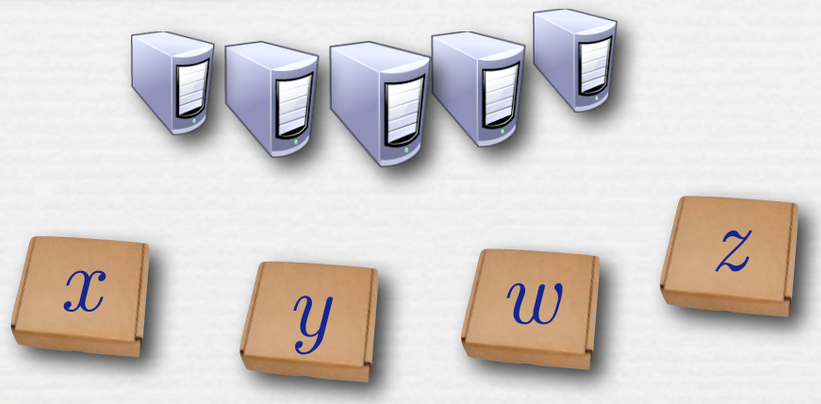

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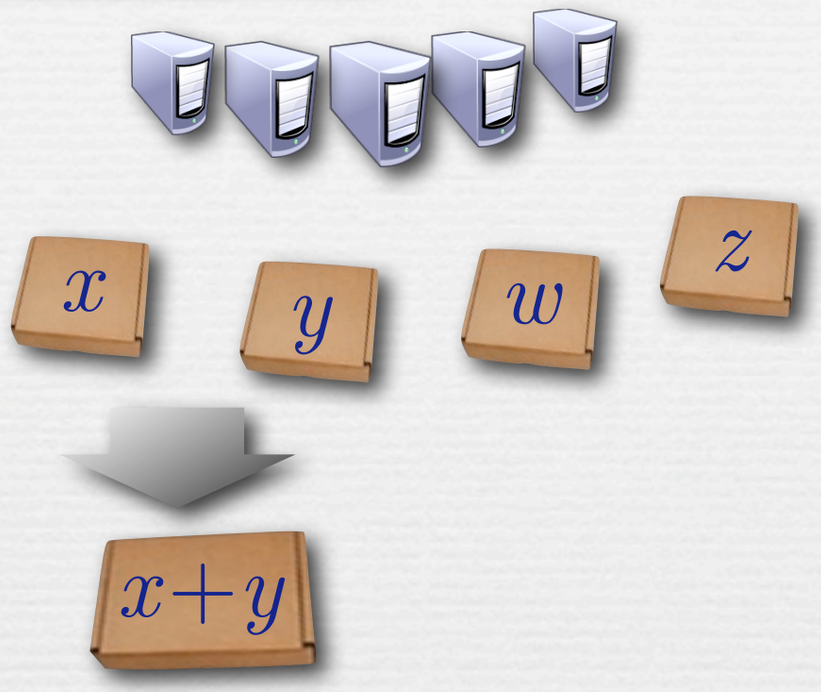

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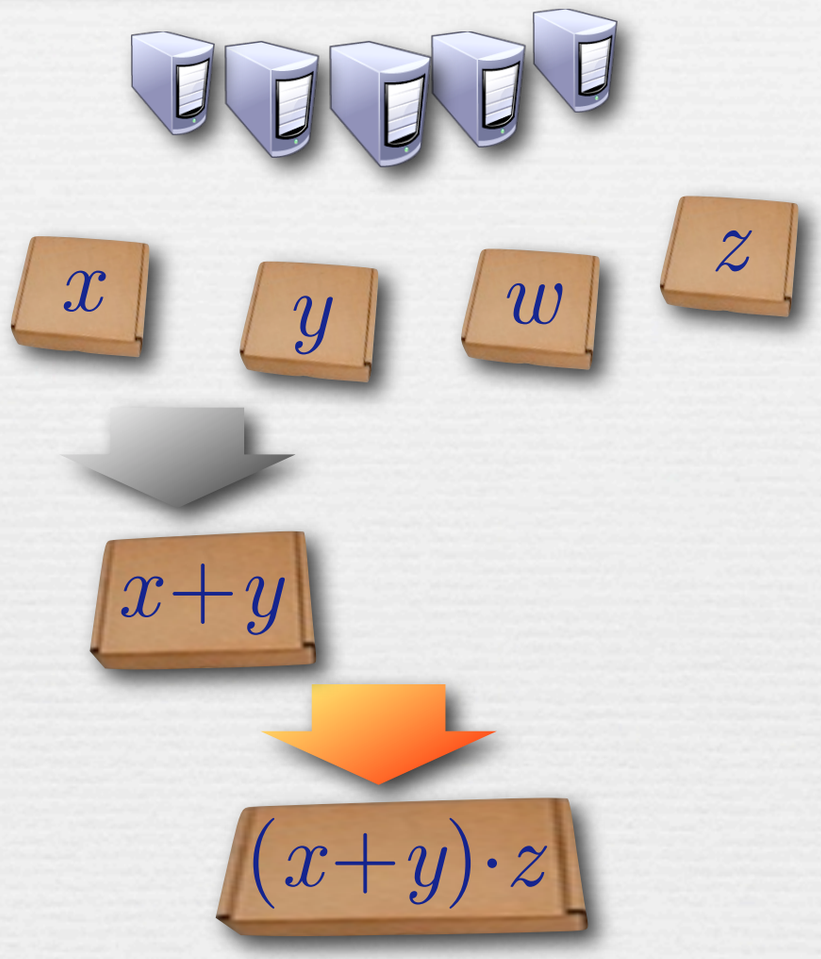

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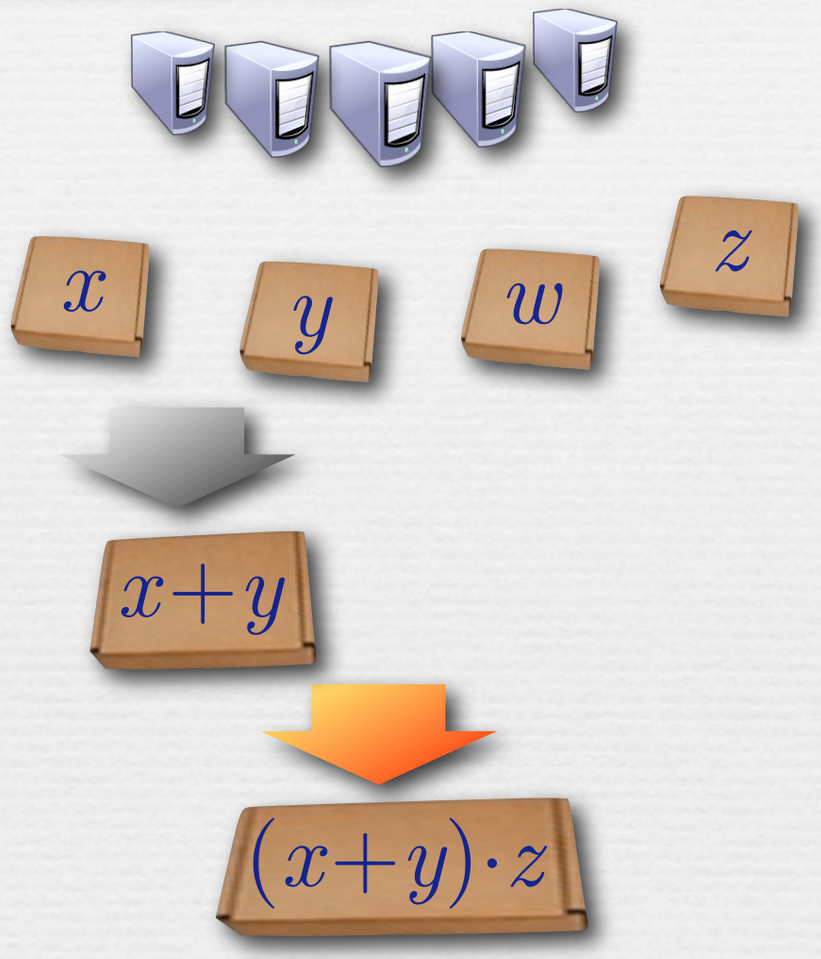

homomorphic property

complex subprotocol, involving
communication among the servers

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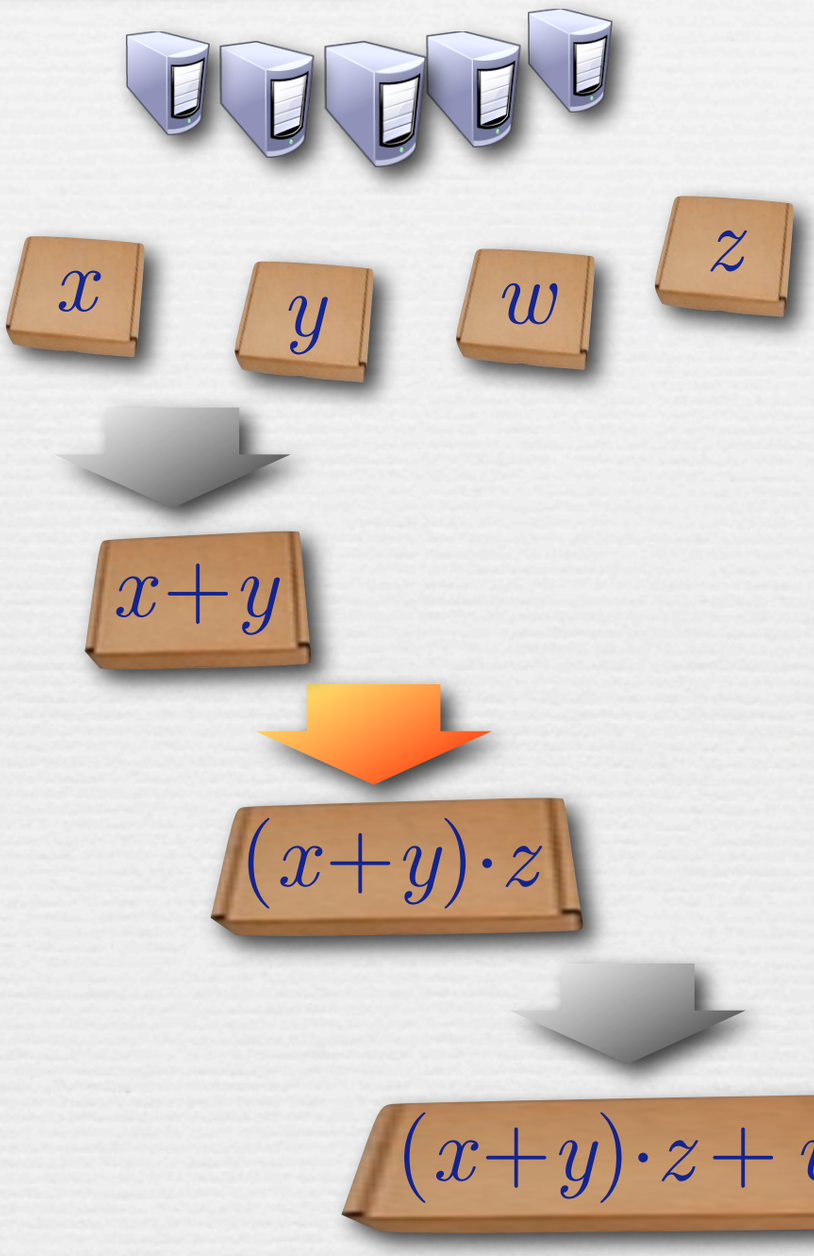

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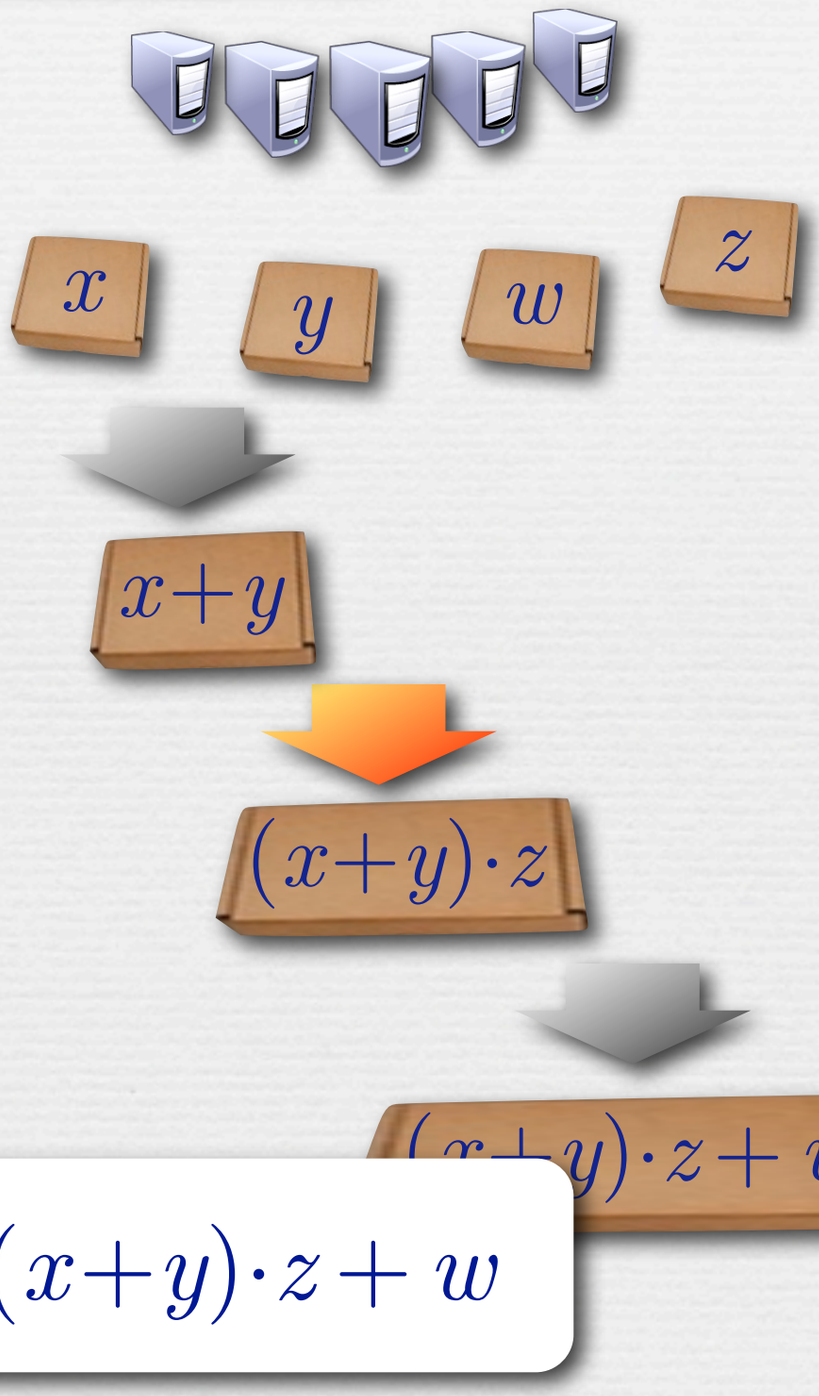

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But: special purpose solutions can be reasonably efficient
(see next talk by Tomas Toft)

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