Entropy Estimation on the Basis of a Stochastic Model

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Introduction

- Motivation and Background
- Stochastic model
  - Definition and objective
  - Illustrating examples
  - Health tests (online tests)
- Experiences with the AIS 31
- Conclusion

NIST SP 800-90B [4]

- Entropy estimation is the most critical part of a security evaluation of a physical RNG.
- Among others [4], Subsection 3.2.2, demands that the documentation *... shall include a description of how the noise source works and rationale about why the noise source provides acceptable entropy output,...*

Entropy estimation

- Unfortunately, entropy cannot be measured like voltage and temperature.
- Instead, entropy is a property of random variables.
- In the following we interpret random numbers as realizations of (i.e. as values taken on by) random variables.
- We present a field-tested method for the estimation of the entropy of physical RNGs.

Notation

- In the following we use the terminology of SP 800-90B [4]. In particular,
  - digitized data = data after the digitization of the analog signals
  - raw data = data after (non-cryptographic) postprocessing
- NOTE: In the literature also other definitions are widespread. In particular,
  - raw random numbers (or digitized analog signals) = data after digitization
  - internal random numbers = data after (non-cryptographic / cryptographic) postprocessing

What is a stochastic model?

- Ideally, a stochastic model specifies a family of probability distributions, which contains the true (but unknown) distribution of the raw data (interpreted as realizations of random variables).
- In a second step therefrom the (average gain of) entropy per raw data bit is estimated.
- In most cases it is yet easier to develop and to verify a stochastic model for the digitized data (or, alternatively, for suitable 'auxiliary random variables').
  $\rightarrow$ entropy(digitized data) $\rightarrow$ entropy(raw data)

Example 1: Coin tossing

- A coin is tossed N times ('head'$\simeq 1$, 'tail'$\simeq 0$').
- We interpret the observed outcome $x_1, \ldots, x_N$ (= digitized data) of N coin tosses as realizations of random variables $X_1, \ldots, X_N$.
- The random variables $X_1, \ldots, X_N$ are assumed to be iid (independent and identically distributed).
  Justification: A coin has no memory.
- $p := \mathrm{Prob}(X_j = 1) \in [0, 1]$ with unknown parameter p.

Example 1: Entropy estimation

- $X_1, \ldots, X_N$ are iid $\Longrightarrow H(X_1, \ldots, X_N)/N = H(X_1)$ (= (average) entropy per coin toss) where

  $H(X_1) = -(p \log_2(p) + (1-p) \log_2(1-p))$ (Shannon entropy)

- Equivalently, $H_{min}(X_1, \ldots, X_N)/N = H_{min}(X_1)$ with

  $H_{min}(X_1) = min\{-log_2(p), -\log_2(1-p)\}$ (min entropy)

- $$\widetilde{p} := \frac{x_1 + \cdots + x_N}{N} \quad \text{(estimator for } p\text{)}$$

- Substituting $\widetilde{p}$ into the above formulae provides estimators for the Shannon entropy and for the min entropy per coin toss.

Example 1: Stochastic model

- A stochastic model is not a physical model. In Example 1 a physical model would consider the impact of the start conditions and the mass distribution within the coin etc. on the trajectory.
- It is much easier to develop and to verify a stochastic model than a physical model.
- In our coin tossing example the stochastic model defines a 1-parameter family of probability distributions.

Real world RNGs

- For real world physical RNGs the derivation of the stochastic model is more complicated. The stochastic model should be confirmed by engineering arguments and experiments.
- Typically, a stochastic model specifies a 1-, 2- or a 3-parameter family of distributions.
- If the digitized data are not iid *the increase of entropy* per random bit has to be considered.
- During the life cycle of the RNG the true distribution shall remain in the specified family of probability distributions, also if the quality of the random numbers goes down ($\rightarrow$ health tests).

Example 2: Killmann, Schindler (CHES 2008)[6]

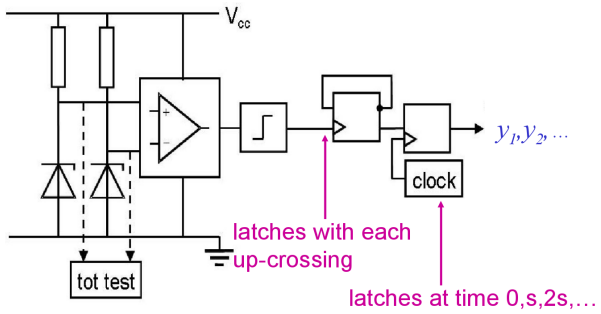


Abbildung: RNG with two noisy diodes, c.f. Fig. 1 in [6]

Stochastic model (for $y_1, y_2, \ldots$)

- t_n: time between the $(n-1)^{th}$ and the n^{th} upcrossing
- $T_1, T_2, \ldots$ is stationary (mild assumption) $\rightsquigarrow \ldots \quad \ldots \rightsquigarrow$ $Y_1, Y_2, \ldots$ is stationary
- 2-parameter family of distributions (depends on the expectation and the generalized variance of T_1)
- details: see [6]

Example 3: Haddad, Fischer, Bernard, Nicolai [5]

- Source of randomness: transient effect ring oscillator (TERO)
- Thorough analysis of the electric processes in the TERO structure
- $\rightarrow$ stochastic model of the TERO
- $\rightarrow$ stochastic model of the complete RNG
- Implementation of the RNG design on a 28 nm CMOS ASIC

Health tests (online tests)

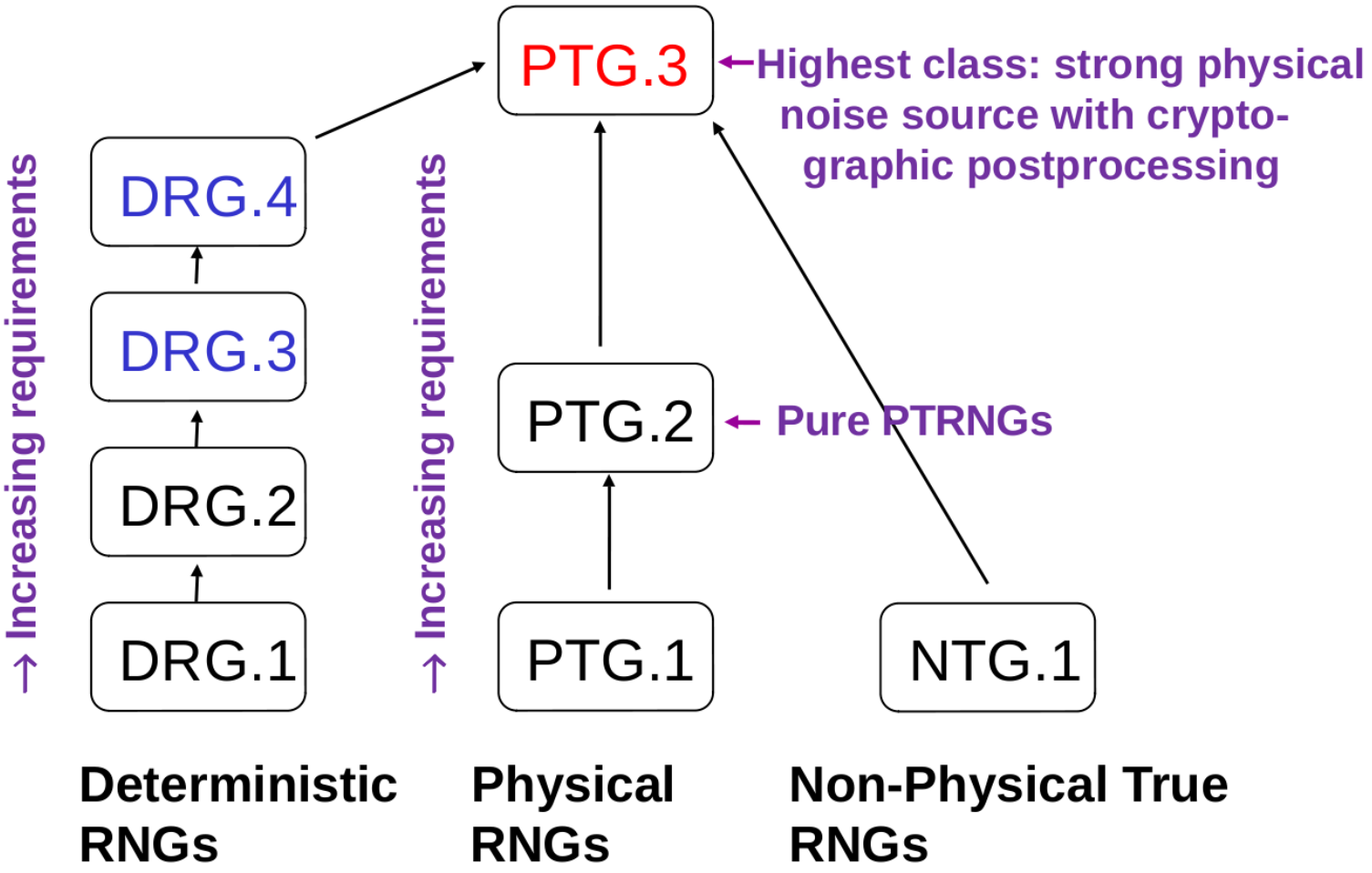
- Health tests, which are universally effective for any RNG design, do not exist.
- The health test (online test) should be tailored to the stochastic model. The health test should detect non-tolerable deficiencies of the random numbers sufficiently soon.
- Example 1: A monobit test would be suitable.
  If # '1's deviates significantly from sample size / 2
  $\rightsquigarrow$ indicator that p is (no longer) acceptable.

AIS 20 [1] / AIS 31 [2]

- In the German evaluation and certification scheme the evaluation guidance documents
  - AIS 20: Functionality Classes and Evaluation Methodology for Deterministic Random Number Generators
  - AIS 31: Functionality Classes and Evaluation Methodology for Physical Random Number Generators

  have been effective since 1999, resp. since 2001.
- NOTE: The mathematical-technical reference [3] was updated in 2011.

Functionality classes



Miscellaneous

- The AIS 20 and the AIS 31 are technically neutral.
- For physical RNGs (PTG.2, PTG.3) a stochastic model is mandatory. The digitized data shall be stationary distributed.
- The applicant for a certificate and the security lab have to give evidence that the RNG meets the class-specific requirements.
- Further documents support the tasks of the developer and the evaluator.
- For sensitive applications the BSI prefers RNGs, which belong to the functionality classes PTG.3, DRG.4 or DRG.3.

The functionality classes PTG.3, DRG.4, DRG.3

- PTG.3 (highest class):
  - strong physical RNG (possibly with mathematical postprocessing), effective online test and total failure test
  - DRG.3-conformant postprocessing algorithm with memory; output rate(postprocessing) $\leq$ input rate(postprocessing)

  information theoretical security + computational security
- DRG.4:
  - DRG.3-conformant deterministic RNG
  - the internal state can be updated / reseeded (time-dependent, event-driven or on demand)

  substantially only computational security.
- DRG.3: deterministic RNG (backward secrecy, forward secrecy, enhanced backward secrecy)

Conclusion

- A sound stochastic model of a physical RNG allows to derive a reliable lower bound for the entropy per raw data bit.
- We explained the concept of a stochastic model by an elementary example.
- Elaborated stochastic models of real world RNGs can be found in the literature.
- In the German certification scheme (Common Criteria) the concept of stochastic models has proved successful for many years.

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[4] NIST Special Publication 800-90B (Second Draft): Recommendation for the Entropy Sources Used for Random Bit Generation.

[5] P. Haddad, V. Fischer, F. Bernard, J. Nicolai: A Physical Approach for Stochastic Modeling of TERO-Based TRNG. In: CHES 2015, Springer, LNCS 9293, 357–372

[6] W. Killmann, W. Schindler: A Design for a Physical RNG with Robust Entropy Estimators. In: CHES 2008, Springer, LNCS 5154, 146–163.