X9.82 Part 3 Number Theoretic DRBGs

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WHY?

- Asymmetric key operations are about <u>**100 times slower**</u> than symmetric key or hash operations
- Why have 2 DRBGs based on hard problems in number theory?
- Certainly <u>**not**</u> expected to be chosen for performance reasons!

Some Possible Reasons

- Do not need lots of random bits, but want the potentially **increased assurance**
- Already using an asymmetric key algorithm and want to limit the number of algorithms that IF broken will break my system
- Have an asymmetric algorithm accelerator in the design already

Performance Versus Assurance

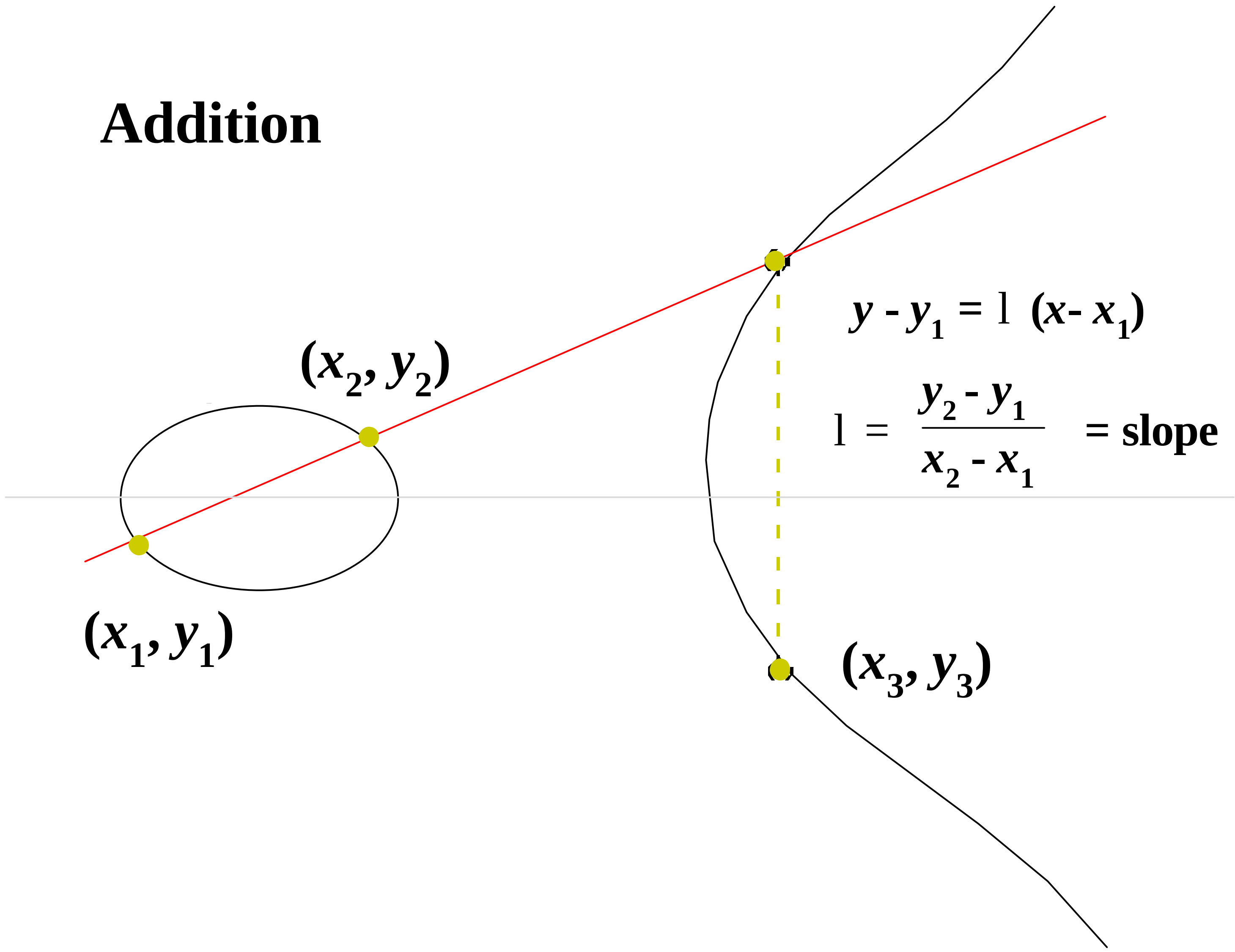
- As performance is not likely THE reason an NT DRBG is included in a product
- Make the problem needing to be broken as **hard as possible**, within reason
- This increases the assurance that the DRBG will not be broken in the future, up to its security level

Quick Elliptic Curve Review

- An elliptic curve is a **cubic equation** in 2 variables X and Y which are elements of a field. If the field is finite, then the elliptic curve is finite
- Point addition is defined to form a group
- ECDLP Hard problem: given P = nG, find n where G is generator of EC group and G has order of 160 bits or more

Elliptic Curve $y^2 = x^3 + ax + b$

Addition



Toy Example: The Field Z_{23}

- The field $\mathbf{Z_{23}}$ has **<u>23 elements</u>** from 0 to 22
- The "+" operation is addition modulo 23
- The "*" operation is multiplication mod 23
- As 23 is a prime this is a field (acts like rational numbers except it is finite)

The Group Z^*_{23}

- Z^*_{23} consists of the 22 elements of Z_{23} excluding 0

$5^0 = 1$	$5^8 = 16$	$5^{16} = 3$
$5^1 = 5$	$5^9 = 11$	$5^{17} = 15$
$5^2 = 2$	$5^{10} = 9$	$5^{18} = 6$
$5^3 = 10$	$5^{11} = 22$	$5^{19} = 7$
$5^4 = 4$	$5^{12} = 18$	$5^{20} = 12$
$5^5 = 20$	$5^{13} = 21$	$5^{21} = 14$
$5^6 = 8$	$5^{14} = 13$	And return
$5^7 = 17$	$5^{15} = 19$	$5^{22} = 1$

- The element 5 is called a generator
- The “group operation” is modular multiplication

Solutions to $y^2 = x^3 + x + 1$ Over Z_{23}

(0, 1)	(6, 4)	(12, 19)
(0, 22)	(6, 19)	(13, 7)
(1, 7)	(7, 11)	(13, 16)
(1, 16)	(7, 12)	(17, 3)
(3, 10)	(9, 7)	(17, 20)
(3, 13)	(9, 16)	(18, 3)
(4, 0)	(11, 3)	(18, 20)
(5, 4)	(11, 20)	(19, 5)
(5, 19)	(12, 4)	(19, 18)
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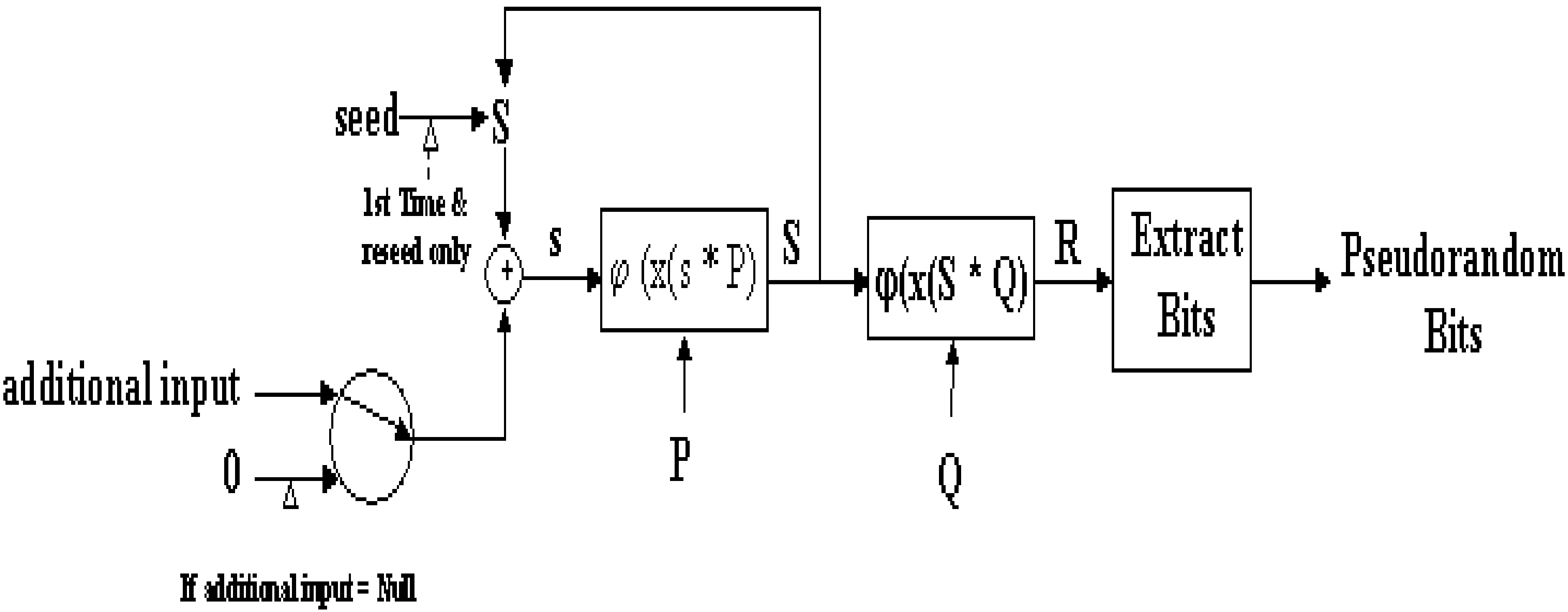
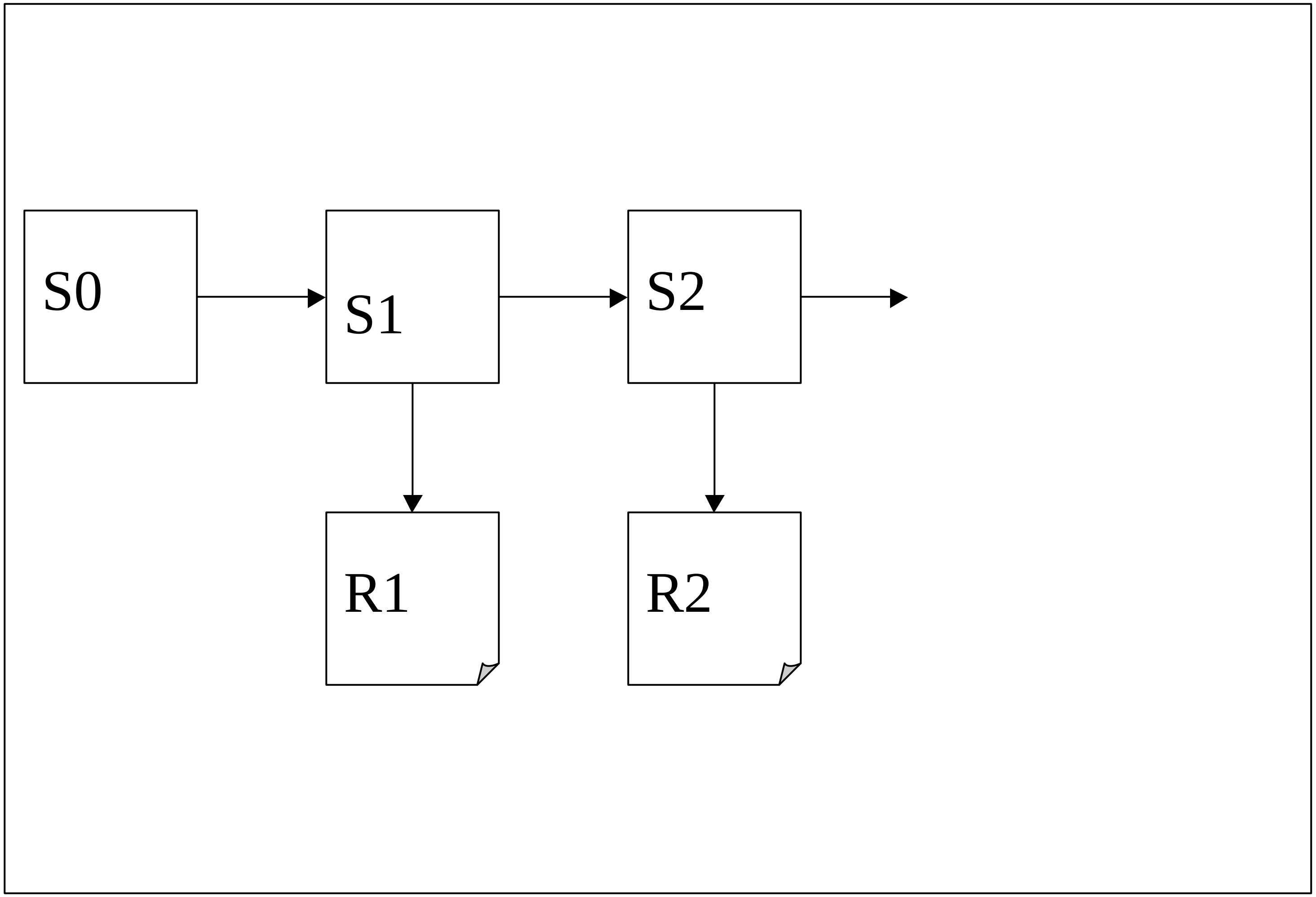
There are <u>28 points</u> on this toy elliptic curve

ECC DRBG Flowchart



Unlooped Flowchart



3 Facts and a Question

1. Randomness implies next bit unpredictability
2. The number of points on a curve is approximately the number of field elements
3. All points (X, Y) have a inverse (X, -Y) and at most 3 points are of form (X, 0)

Q: Can I use the X-coordinate of a **random** point as **random** bits?

X-Coordinate Not Random

No, I cannot use a **raw** X-coordinate!

As most X-coordinates are associated with 2 different Y-coordinates, about half the X values have **NO** point on the curve,

Such X gaps can be considered randomly distributed on X-axis

Look at toy example to see what is going on

Toy Example of X Gaps

Possible X coordinate values: 0 to 22

X values appearing once: 4

Twice: 0, 1, 3, 5, 6, 7, 9, 11, 12, 13, 17, 18, 19

None: 2, 8, 10, 14, 15, 16, 20, 21, 22

An X coordinate in bits from 00000 to 10110

If I get first 4 bits of X value of 0100*a*, I know *a* must be a 1, as 9 exists but 8 does not

1-bit Predictability

- If output 4 bits as a random number, the next bit is **completely predictable!**
- This property also holds for 2-bit gaps, 3-bit gaps, etc. with decreasing frequency.
- **Bad luck is not an excuse** for an RBG to be predictable!
- The solution: **Truncate** the X-coordinate. Do not give all that info out. How much?

X Coordinate Truncation Table

Prime field	Truncate at least 13 leftmost bits of x coordinate
Binary Field, cofactor = 2	Truncate at least 14 leftmost bits of x coordinate
Binary Field, cofactor = 4	Truncate at least 15 leftmost bits of x coordinate

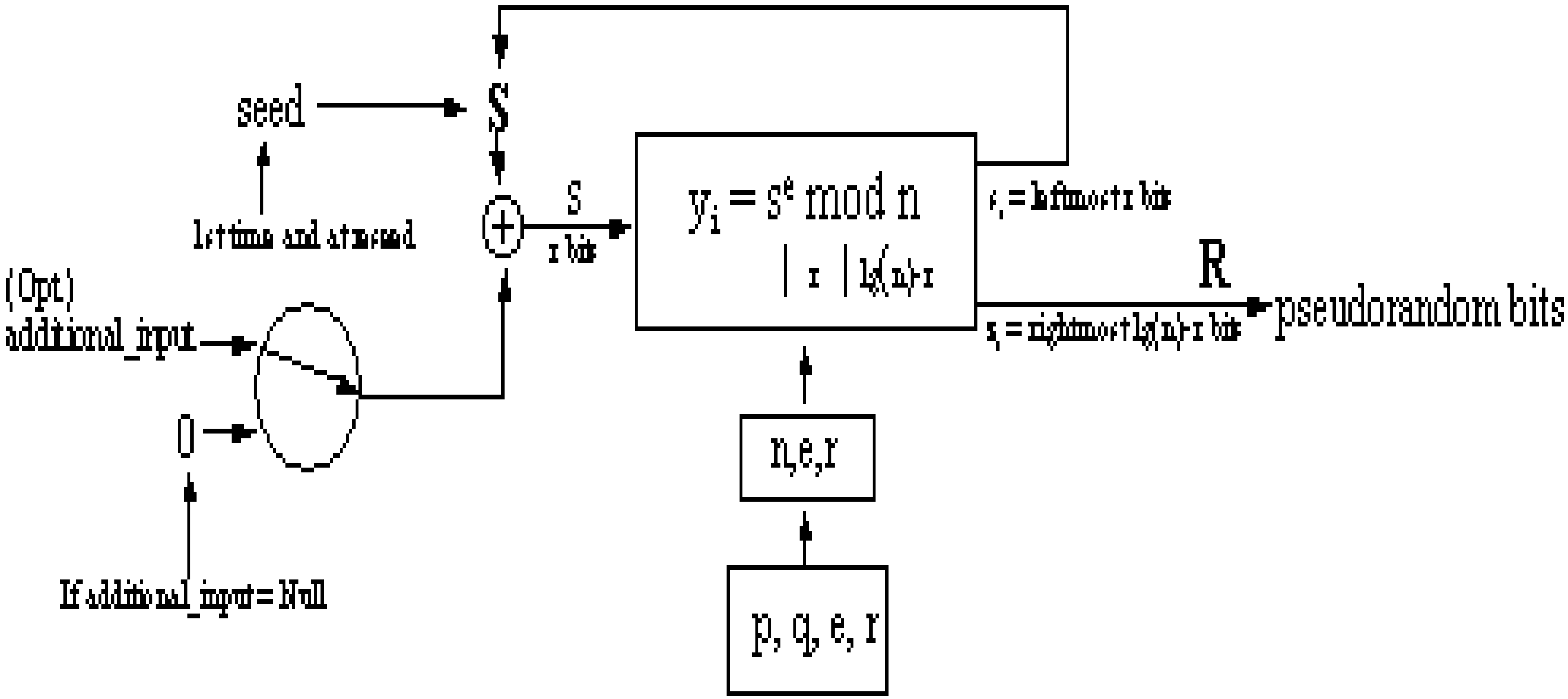
Truncation

- This truncation will ensure no bias greater than 2**-44
- Reseed every 10,000 iterations so bias effect is negligible
- To work with bytes, round up so remainder of X-coordinate is a multiple of 8 bits, this truncates from 16 to 19 bits

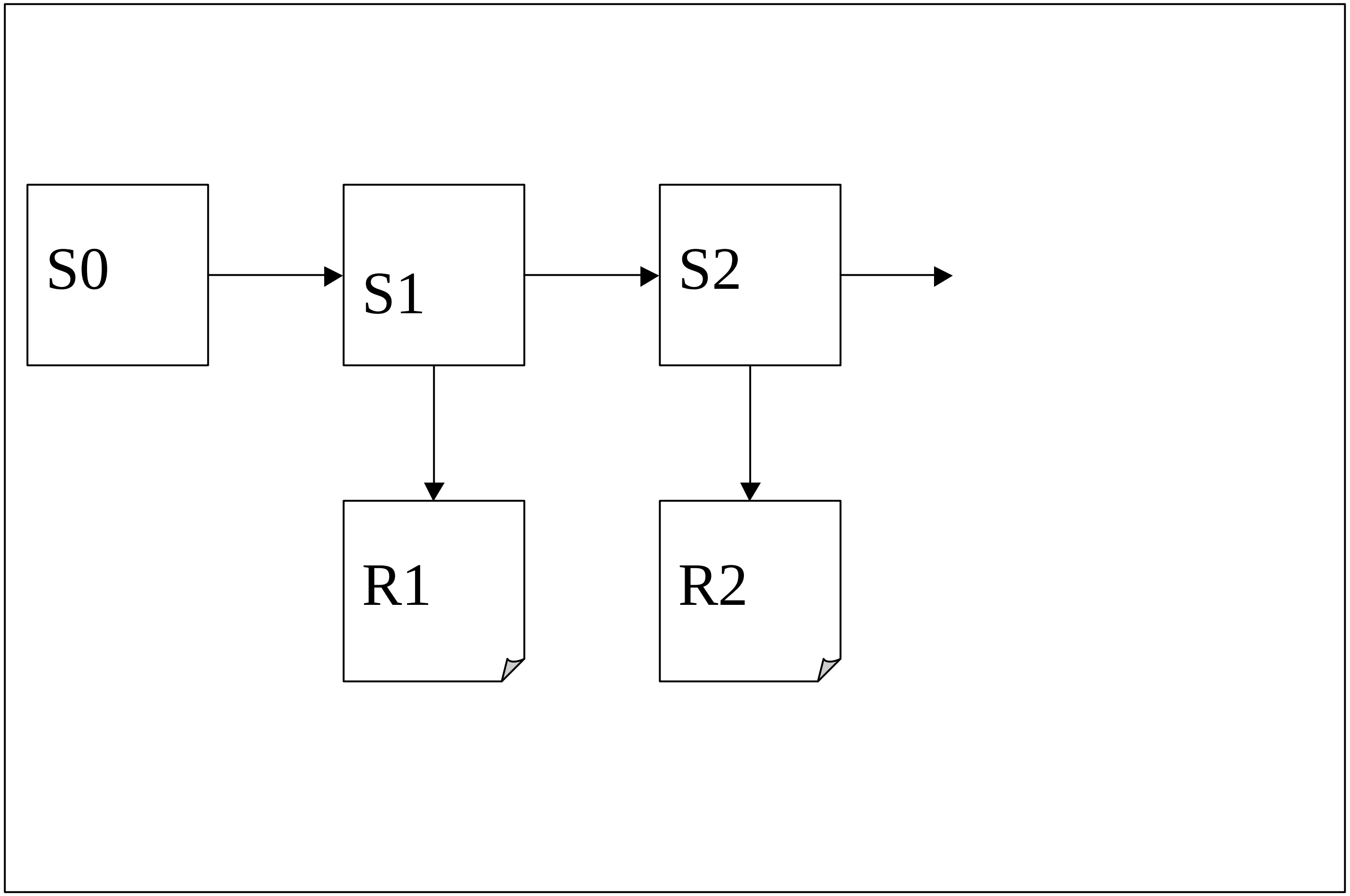
Quick RSA Review

- Choose odd public exponent e and primes p and q such that e has no common factor with p or q, set n = pq
- Find d such ed = 1 mod (p-1)(q-1)
- Public key is (e, n), private key is (d, n)
- Hard to find d from (e, n) if n >= 1024 bits
- (M^e mod n) is **hard to invert** for most M

Micali-Schnorr DRBG



Unlooped Flowchart



Micali-Schnorr Truncation

- For MS truncation, we only use the RSA **hard core bits** as random bits
- This has high assurance that the number theory problem to be solved is as hard as possible!
- Reseed after 50,000 iterations

NIST/ANSI X9
Security Levels Table

Security Levels (in bits)	ECC (order of G in bits)	MS (RSA) (modulus in bits)
80	160	1024, 10 hardcore bits
112	224	2048, 11 hardcore bits
128	256	3072, 11 hardcore bits
192	384	Not specified

Number Theory DRBGs Summary

- 2 Number Theory DRBGs are specified based on **ECC and RSA**
- Use one for **increased assurance**, but do not expect it to be the fastest one possible
- No one has yet asked for an FFC DRBG, straightforward to design from ECC DRBG, but specifying algorithm and validation method has a cost