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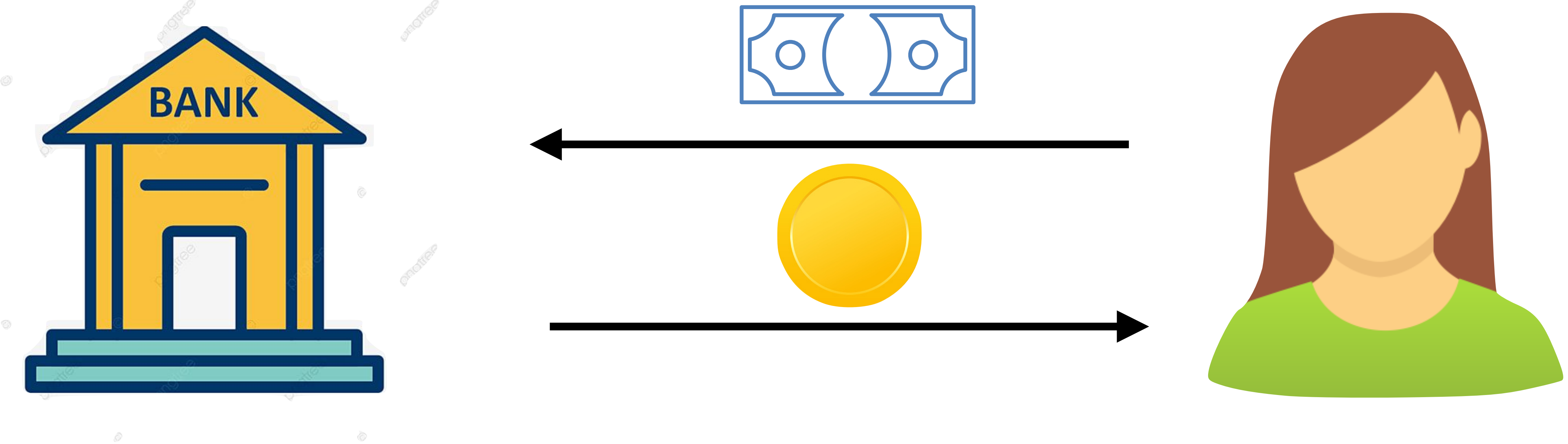
Blind Signatures: Past, Present, and Future

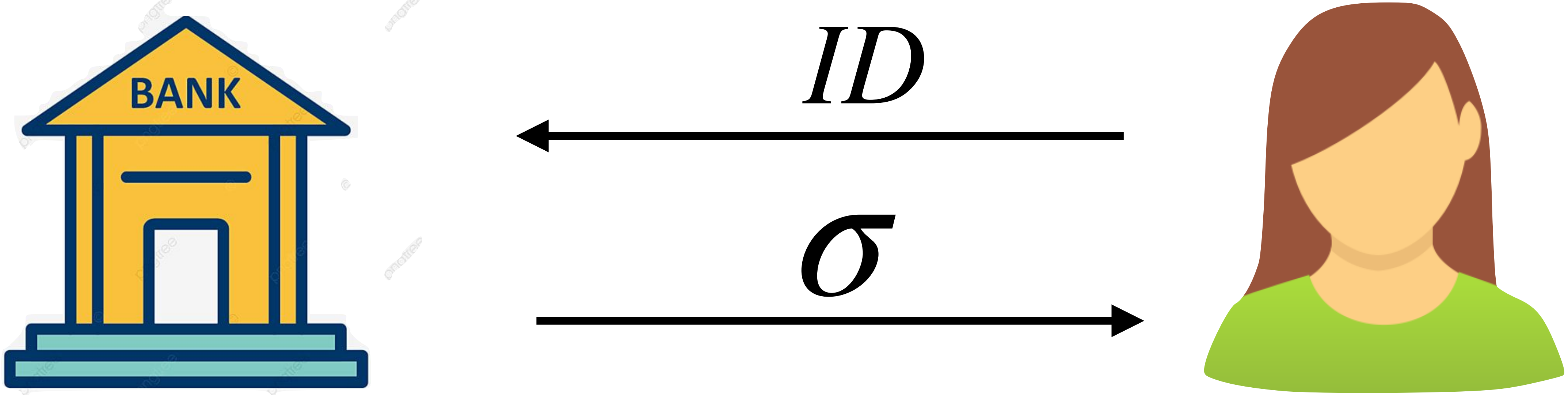
Julian Loss

ECash Systems (Chaum 1983 [C83])



- User wants to trade physical for digital currency
- Challenge: retain the **physical** attributes of cash
  - Unforgeable (no double spending)
  - Untraceable (pecunia non olet)

Solution: Blind Signatures

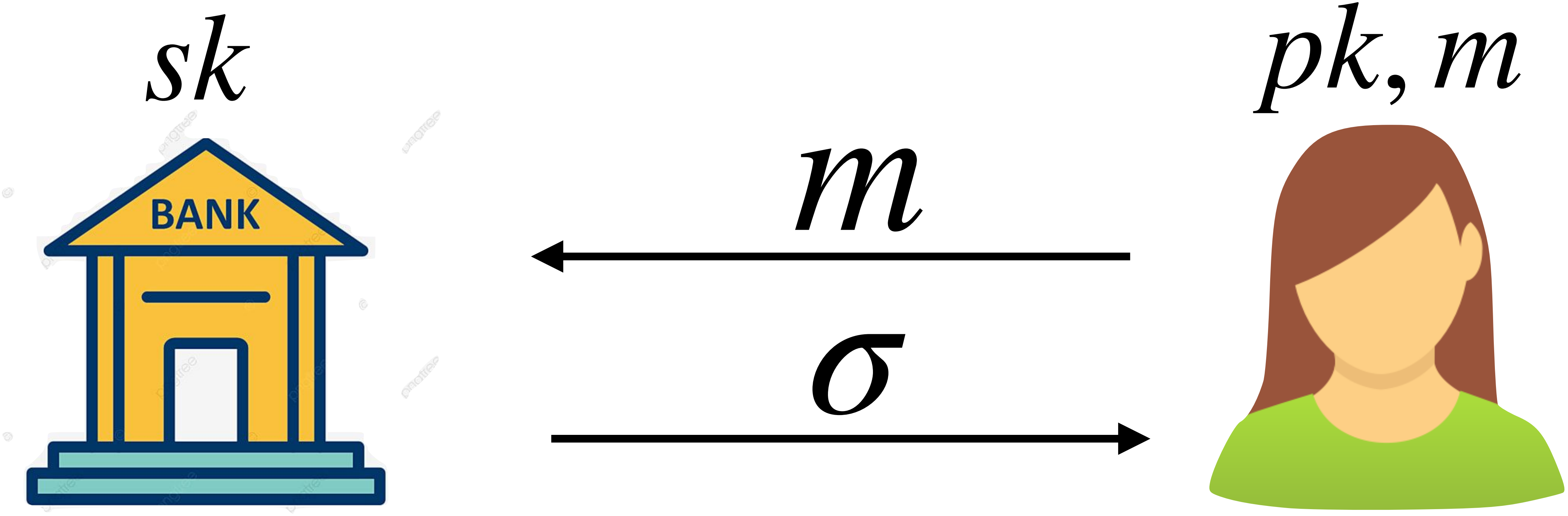


- User: create identifier ID
- Bank: sign ID with signature σ
- User: derive coin from σ
- **Blindness:** Bank does not see ID
  - $\Longrightarrow$ Coin looks “unrecognizable” to bank

More Applications of Blind Signatures

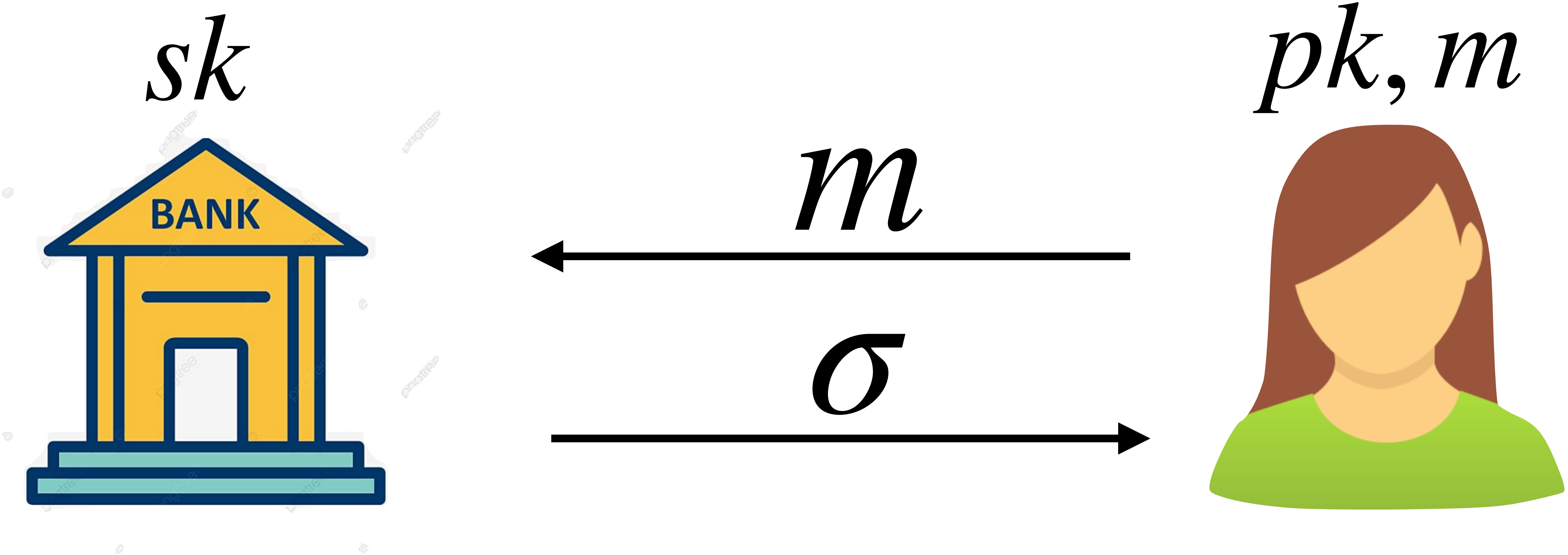
- EVoting: Voting authority blindly signs ballot for approval
- Anonymous Credentials: Credential authority blindly signs credential for validation
- More recently: Blockchain Applications, e.g. Coin Shuffling/Mixing

Blind Signatures: Syntax



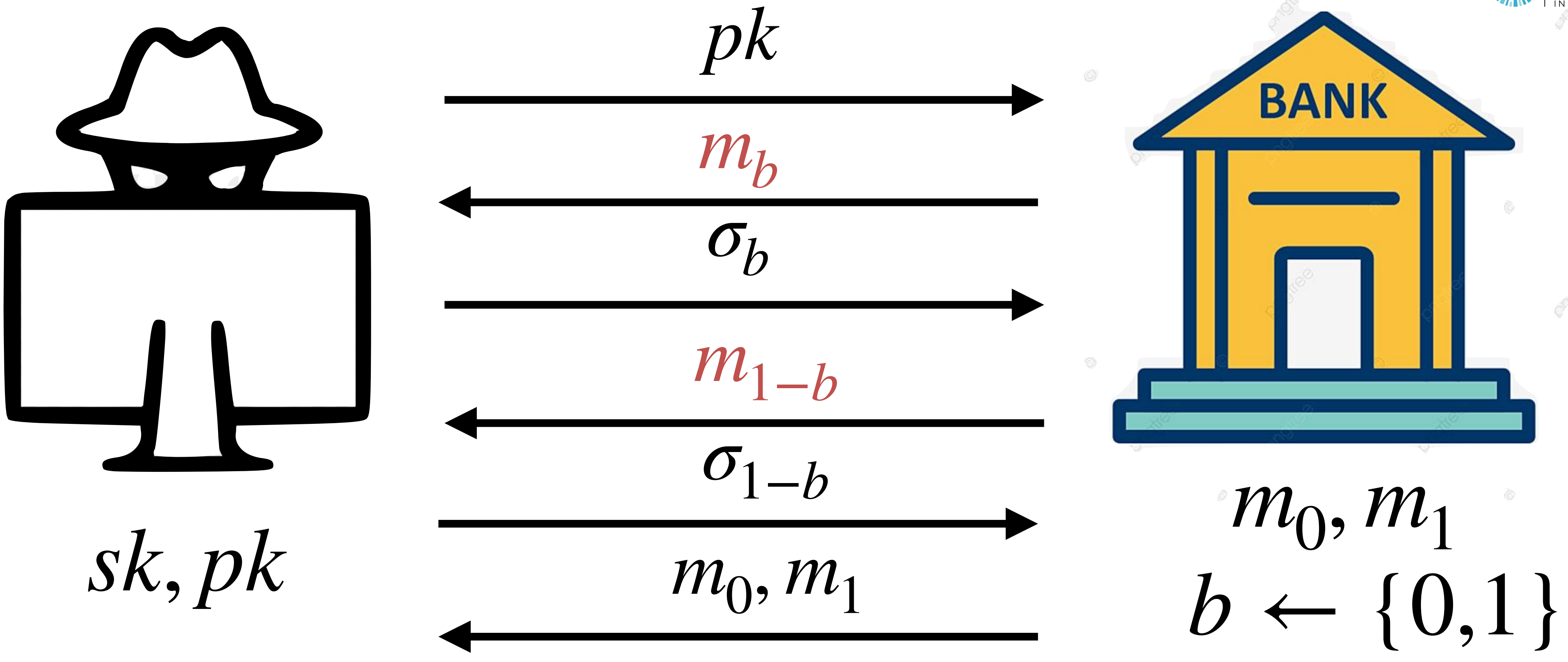
- Signer holds secret key sk
- User holds public key pk, message m
- Signer generates signature σ on m

Blind Signatures: Security Properties



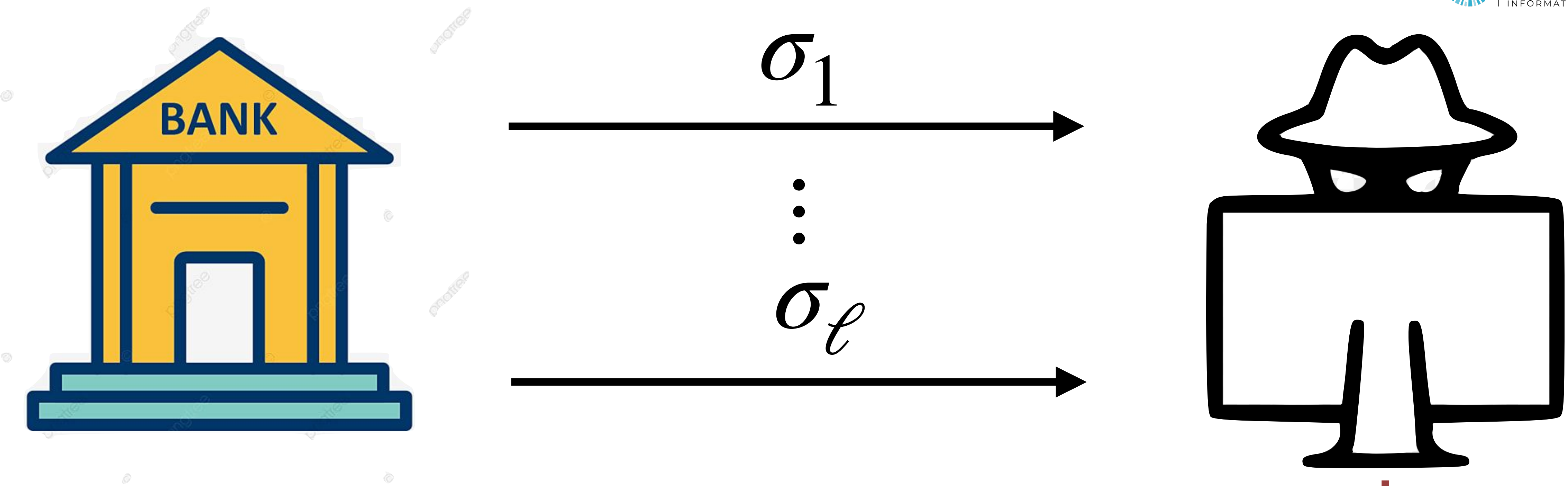
- **Blindness:** Signer does not learn m
  - Should hold even if Signer generates keys
- **Unforgeability:** User cannot create σ by itself
- How to formalize these properties?

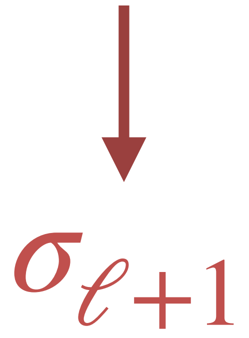
Blindness



- Signer cannot determine b
- $\Longrightarrow$ Signatures cannot be linked to signing sessions

One-More Unforgeability





- Signer and User engage in ℓ sessions
  - $\Longrightarrow$User obtains ℓ signatures
- User cannot generate $\ell + 1$ signatures
- **Very strong security property**
  - $\Longrightarrow$ **Difficult to construct**

The History of Blind Signatures

- This talk: overview of the state of the field; what question are open?
- Complicated history: Bugs, attacks, forgotten papers, and more
- Active field of research with major improvements still being made

Simple Analogy: Speedrunning

- **Speedrunning**: beating a video game as fast as possible
- What constitutes a valid run? Rules?
- For blind signatures, we care about signature sizes and communication, model assumptions

Glitchless	No Major Glitches (a.k.a. Memory Corruption)	Any %
1 : 45 : 05	0 : 11 : 33	0 : 01 : 18.893
σ	σ	σ
Plain model	Random Oracle Model, Conservative assumptions	Generic group model, strong assumptions

Conservative Assumptions?

- Cryptography relies heavily on **number theory** for problem with average-case hardness
- Example: Factoring large numbers into prime components
- Conjectured hardness $\Longrightarrow$ security of scheme
- Ideally want conservative conjectures:
  - Simplest version of problem (DLOG < CDH < DDH < …, FAC < RSA < …)
  - Non-interactive, non-parametrized problems (non-examples: One-More DLOG/RSA, LRSW,…)
  - Should have stood test of time (DLOG, CDH, DDH, FAC, RSA, QR, SIS, LWE,…)

Past I: 1983-2002

1982: Chaum's Blind Signature Scheme [C83]

d, N

$$H(m) \cdot r^e \pmod{N}$$

$$\longleftarrow$$

$$u = (H(m) \cdot r^e)^d \pmod{N}$$

$$\longrightarrow$$

$(N, e), m$

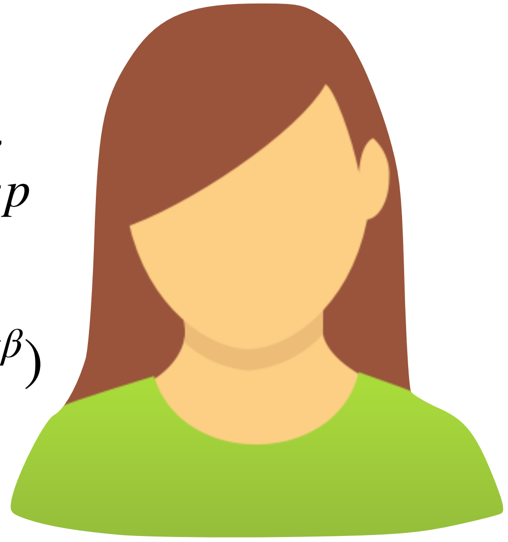
- Based on the RSA Full-Domain Hash Signature Scheme
- $N = P \cdot Q$ for primes P and Q,
- e is an integer s.t. gcd$((P-1)\cdot(Q-1), e) = 1$
- $pk = (N, e)$ and $sk = d$ s.t. $d \cdot e = 1 \pmod{(P-1)\cdot(Q-1)}$
- Problem: unforgeability relies on very strong hardness assumption

$$\sigma = u \cdot r^{-1} = H(m)^d \pmod{N}$$

1990: Schnorr's Blind Signature Scheme

x

X



$R = g^{r+r'}$ →

$\alpha, \beta \leftarrow \mathbb{Z}_p$

$c = c' + \beta$ ←

$c' = H(R \cdot g^{\alpha} \cdot X^{\beta})$

$s = c \cdot x + r + r'$ →

- Based on the Schnorr Signature Scheme
- Uses group $\mathbb{G}$ of prime order p with generator g
- $x \in \mathbb{Z}_p, X = g^x$
- $sk = x, pk = X$
- Inherently "linear"

$s' = s + \alpha, \sigma = (c', s')$

1996: Toward Provable Security

- Breakthrough Papers of Pointcheval and Stern [PS96,PS977,PS00]
- First formal definition of One-More-Unforgeability
- First provably secure blind signatures,
- Introduces rewinding in the Random Oracle Model
- Double generator variant of Schnorr's Scheme (Okamoto-Schnorr)
- Caveat: only **logarithmically** many signatures per public key

Limitations of PS: A Closer Look

- ℓ = number of signatures
- Q = number of hash queries
- p is the group order
- Then, PS is secure as long as $Q^{\ell+1}/p \ll 1$
- For 256 bit prime p and $Q = 2^{128}$, $\ell \geq 1$ makes PS theorem meaningless
- Explicit Question in PS: Is this limitation inherent?

Communication and Signature Sizes

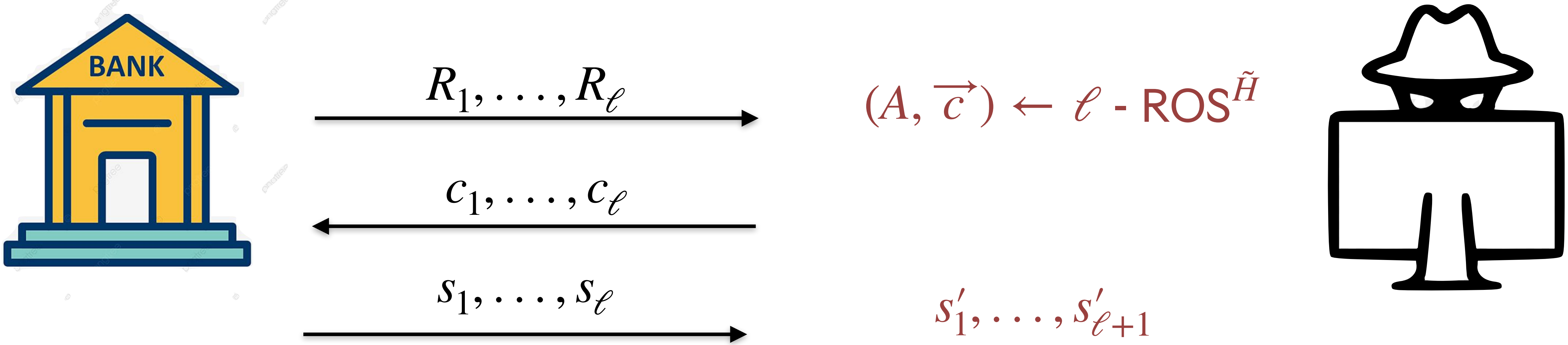
- Assuming 2^{30} signatures, 2^{128} hash queries
- 2000:
  - Roughly 2^{37} bits (17.18GB)
  - RSA, FAC, DLOG

Schnorr's ROS Problem [S01]

$$A := \begin{bmatrix} A_{1,1} & \cdots & A_{1,\ell}, & H(\mathbf{A_1}) \\ \vdots & \vdots & \vdots & \vdots \\ A_{\ell+1,1} & \cdots & A_{\ell+1,\ell}, & H(\mathbf{A}_{\ell+1}) \end{bmatrix} \qquad \mathbf{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_\ell \\ -1 \end{bmatrix}$$

- ℓ-ROS: Find $A \in \mathbb{F}_p^{(\ell+1)\times(\ell+1)}$ and $\overrightarrow{c} \in \mathbb{F}_p^{\ell+1}$ s.t. $A \cdot \overrightarrow{c} = 0$
- Conditions on last column of A and vector $\overrightarrow{c}$:
  - Last column of is generated via random oracle H
  - Depends on the first ℓ columns of A
  - Last column is a linear combination of ℓ first columns of A
- Solution exists iff $Q^{\ell+1}/p \geq 1$

Schnorr's Attack



- Attacker opens $\ell = \log(p)$ **concurrent** sessions, gets $R_1, \ldots, R_\ell$
- Solves ℓ - ROS problem relative to $\tilde{H}$, where $\tilde{H}(\vec{a}, u) = H(\Pi_{i=1}^{\ell} R_i^{a_i}, u)$
- Constructs s'_i from $s_1, \ldots, s_\ell$ via ROS solution as $s'_i = \Sigma_{j=1}^{\ell} A_{i,j} \cdot s_j$, $c'_i = H(\Pi_{j=1}^{\ell} R_j^{A_{i,j}}, m_i)$
- Applies to Schnorr and OS Blind Signature Schemes
- Schnorr does **not** give an algorithm for ROS

The k-List Problem (Wagner 2002)

- Given k random lists $L_1, \ldots, L_k$ find $x_1 \in L_1, \ldots, x_k \in L_k$ s.t. $\Sigma_i x_i \pmod{p} = 0$
- Generalization of Birthday Problem
- Wagner proposes **heuristic** algorithm with subexponential runtime [W02]
- Can be used to solve ROS in subexponential time, given there exists a solution
- Shows that PS-analysis is optimal!

Past II: 2003-2018

Interest in Blind Signatures Fades

- ECash and EVoting mostly thought of as theoretical concepts
- Some papers on anonymous credentials and blind signatures in plain model
  - Mostly of theoretical interest
  - Only sequential security (hard to use in practice)
  - Use “unreasonable” hardness assumption
- Exception: Rückert gives first (efficient) Lattice-based construction
  - Later shown to be flawed
- Result:
  - 15-year period of stagnation (relative to the explosion of the field)
  - Much of the literature is forgotten, people are not aware of ROS by 2018

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Present: 2019-2022

2019: Blockchains?

- Blockchains lead to renewed interest in blind signatures
- At this point, almost no one:
  - Remembers the ROS attack
  - Understands the proofs of PS
- Two important papers 2019:
  - Modular formulations of PS papers [HKL19]
  - Applying ROS attack to various multisignature schemes [DEFKLNS19]
- Rekindles interest in blind signatures and raises awareness of ROS attack

2020: Lattices, mROS

- New Version of ROS: mROS [FPS20]
  - Potentially harder, easy to work with
  - No lower bounds (currently)
- Revisiting and correcting Rückert’s Lattice-based construction [HKLN20]
  - Still logarithmically bounded number of signatures
  - Still no progress toward efficient schemes from simple assumptions

2021: ROS, Boosting

- EUROCRYPT 2021: new algorithm for ROS [BLLOR21]
  - Polynomial time when $\ell \geq \log_2(p)$
  - Major improvements even for smaller ℓ
  - Makes ROS attack actually practical
- ASIACRYPT 2021: revisits "boosting" construction of Pointcheval [KLR21]
  - Transforms any of the EUROCRYPT 2019 schemes into polynomially secure ones
  - Large communication overhead (linear in number of signatures)
  - Does not work for lattice constructions

Communication and Signature Sizes

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- 2021:
  - Roughly 12000 bits (1.5KB)
  - RSA, FAC, DLOG

2022: Efficient Boosting, More Lattice Constructions

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- Boosting:
  - Exponentially improvements in communication overhead for boosting construction
  - Practical schemes from pairings
  - Computation still linear in number of issued signatures
  - Manuscript: reduces **computation** [HLW22]
- Lattices:
  - First **practical** lattice-based blind signature schemes [PK22]

Communication and Signature Sizes

- Assuming 2^{30} signatures, 2^{128} hash queries
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- 2021:
  - Roughly 12000 bits (1.5KB)
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- 2022
  - 3KB size, 120KB communication (CDH + pairings)
  - 9KB size, 8KB communication (RSA)
  - 100KB size, 850KB communication (NTRU)

Future: 2023-

Important Open Questions

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- Constructions:
  - Pairing-free constructions
  - Blind signatures with strong compatibility (e.g., Bitcoin, Ethereum, etc.)
  - Lattices: communication still around 1MB per signature
- Cryptanalysis:
  - Polynomial-time ROS attack for lower dimensions?
  - Extend ROS attack to lattices
  - Prove (or disprove) attack on mROS [FPS20]

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THANK YOU!

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