Stacked Garbling Gadget

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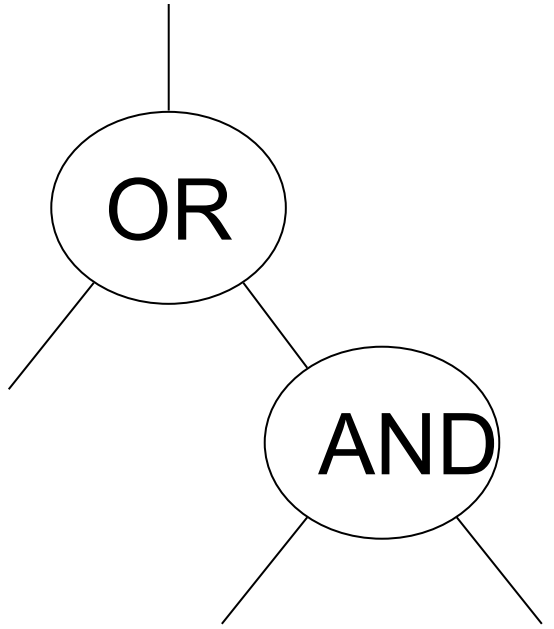
Outline

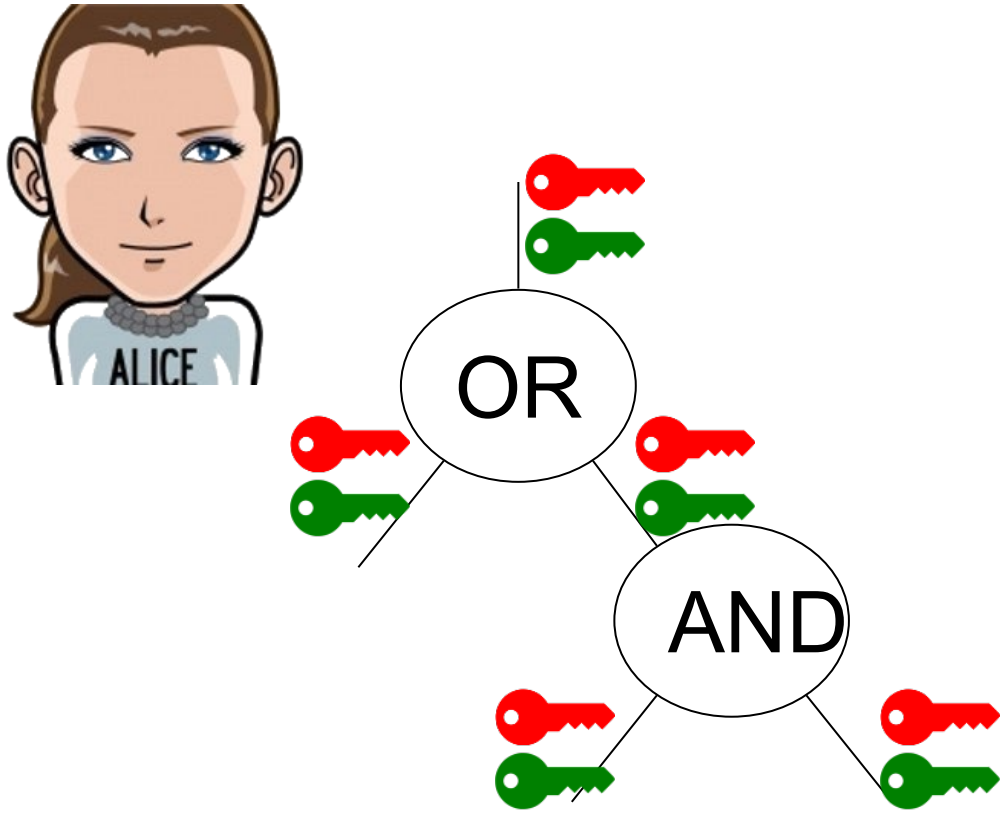
- Garbled Circuits (GC)
- Gadget: Stacked Garbling (SGC)
- Applications and standardization comments

Functions are circuits

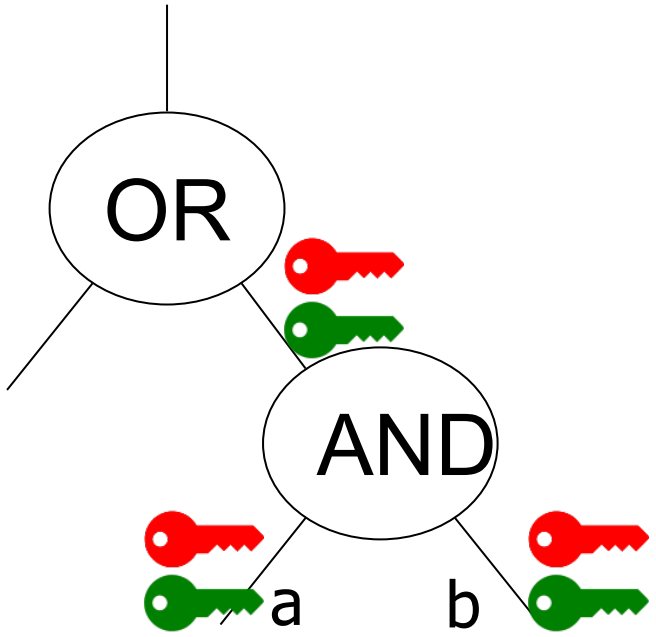
$$F(x, y) \quad \rightarrow$$

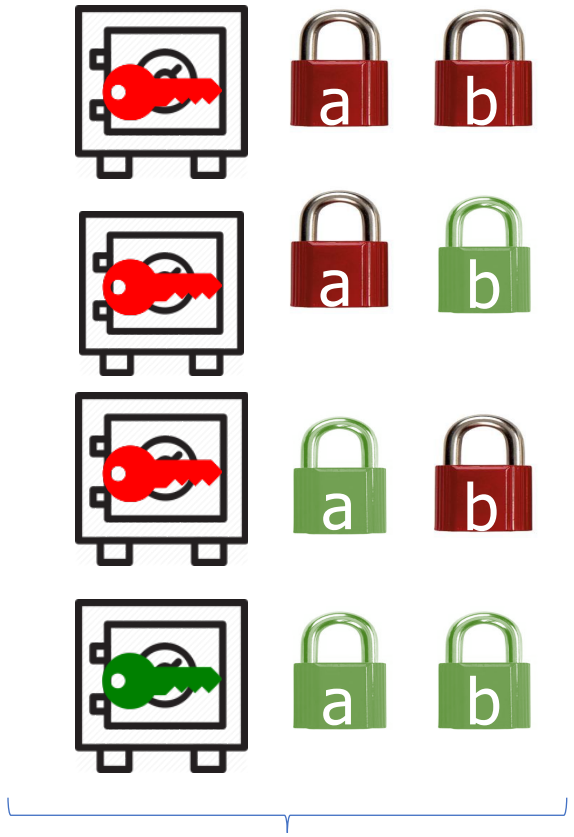


GC intuition: computing on encrypted values



GC intuition: computing on encrypted values

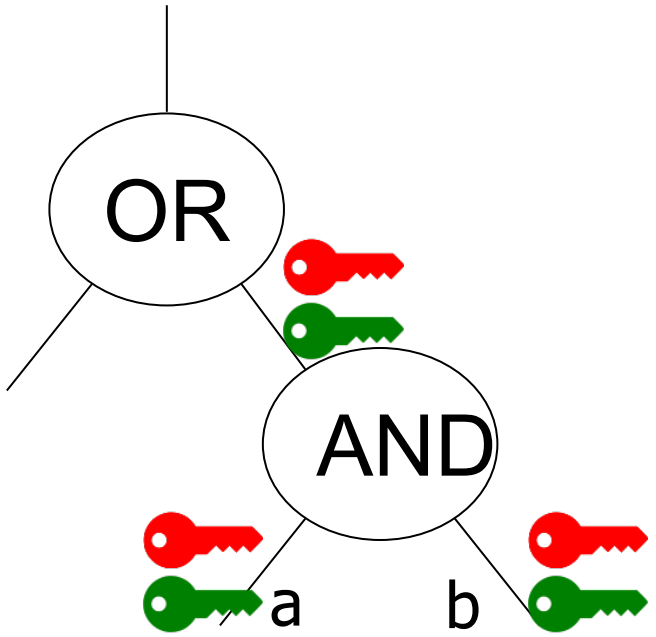


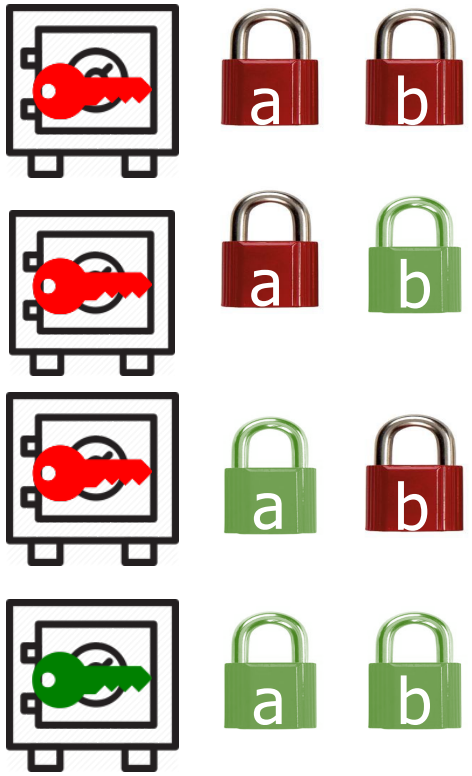


GC Material

a	b	a^b
0	0	0
0	1	0
1	0	0
1	1	1

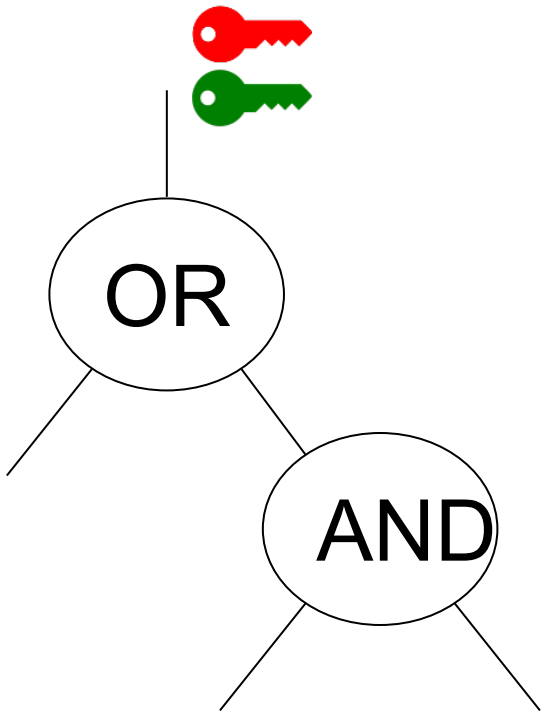
GC intuition: computing on encrypted values





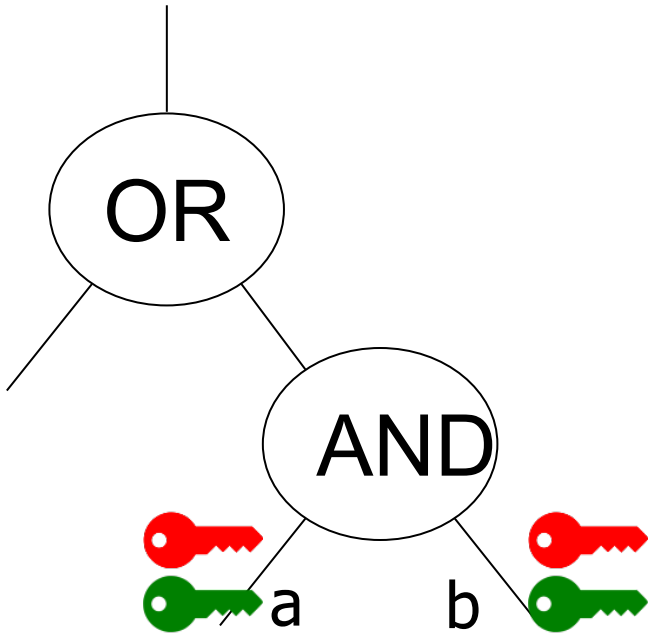
a	b	a^b
0	0	0
0	1	0
1	0	0
1	1	1

GC intuition: decoding encrypted output



	0
	1

GC intuition: OT for transferring input labels



Stacked garbling [HK20]

- Sequence of works [K18,HK20a,HK20b,HK21]
- Let's question the circuit model of computation.
- But not too much..
- Just consider circuits with conditionals

Let C0, C1 be two arbitrary circuits. The space of circuits is defined as follows:

$$C ::= \text{Netlist}(\cdot) \mid \text{Cond}(C0, C1) \mid \text{Seq}(C0, C1)$$

Stacked garbling [HK20]

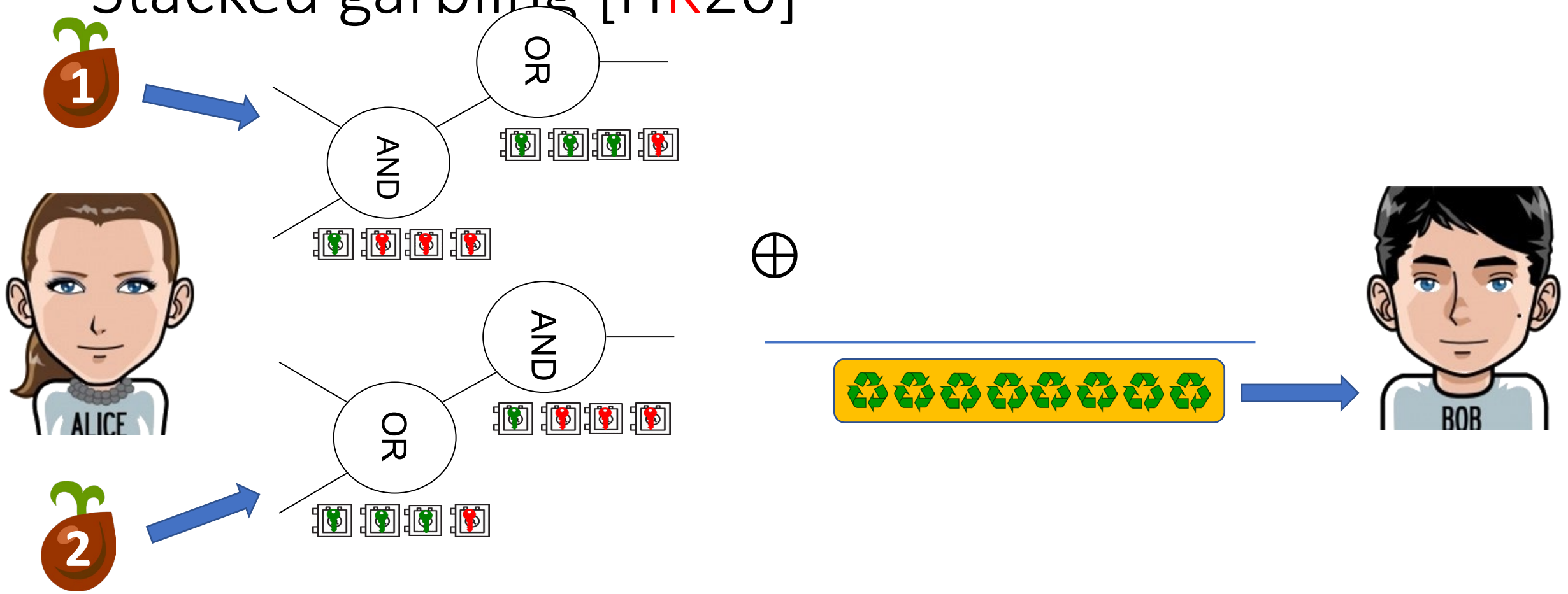
$$C ::= \text{Netlist}(\cdot) \mid \text{Cond}(C0, C1) \mid \text{Seq}(C0, C1)$$

HK20: Can evaluate $\text{Cond}(C0, C1)$ while transmitting only one branch

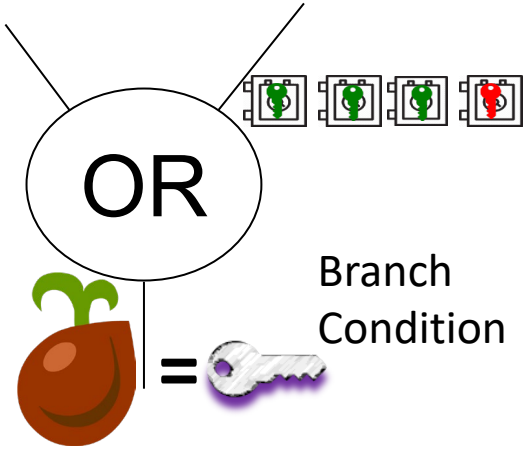
Idea:

* the same GC *material* M is used for evaluation of $C0$ and $C1$.
* GC outputs a key to Eval which converts material M to a valid GC when evaluated on the active branch or or to a random-looking string otherwise (Eval can't distinguish)
* Eval evaluates both $C0, C1$. One of them will produce garbage labels. They are canceled (garbage-collected) by gadgets constructed by Garbler.
* Material reuse (novel general idea; works for other protocols as well)

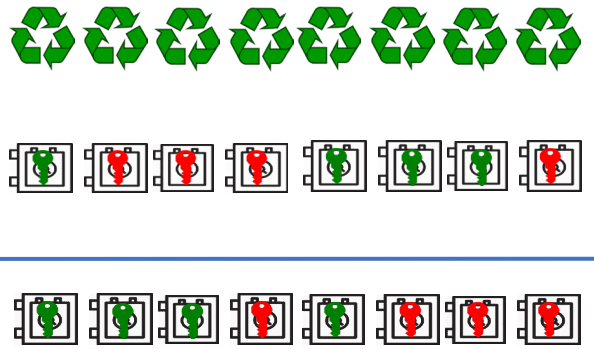
Stacked garbling [HK20]

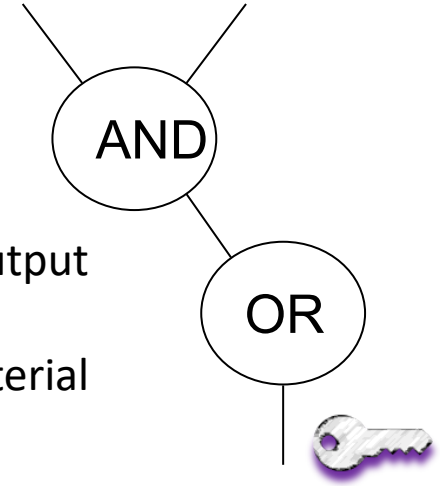


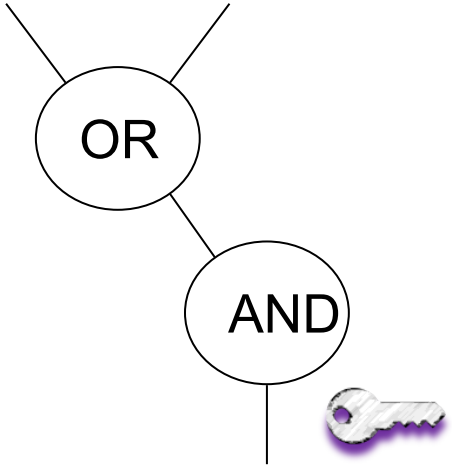
Stacked garbling [HK20]



Guess active branch 1
Expand seed as branch 2
Guess active branch 2
Expand seed as branch 1



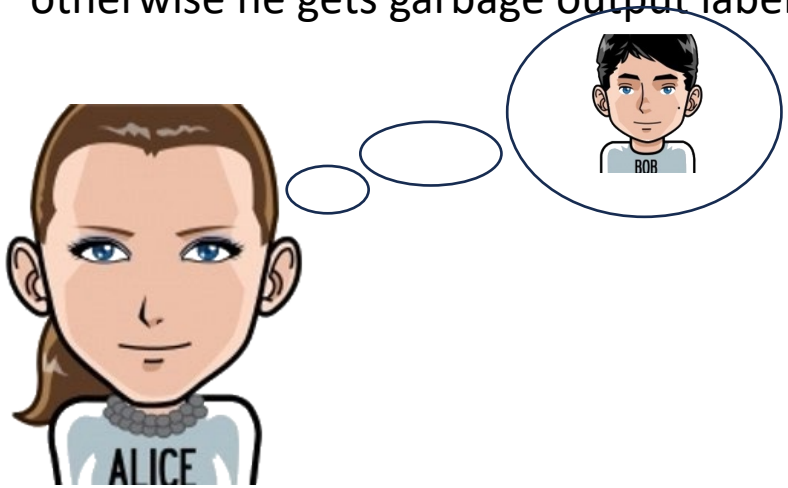


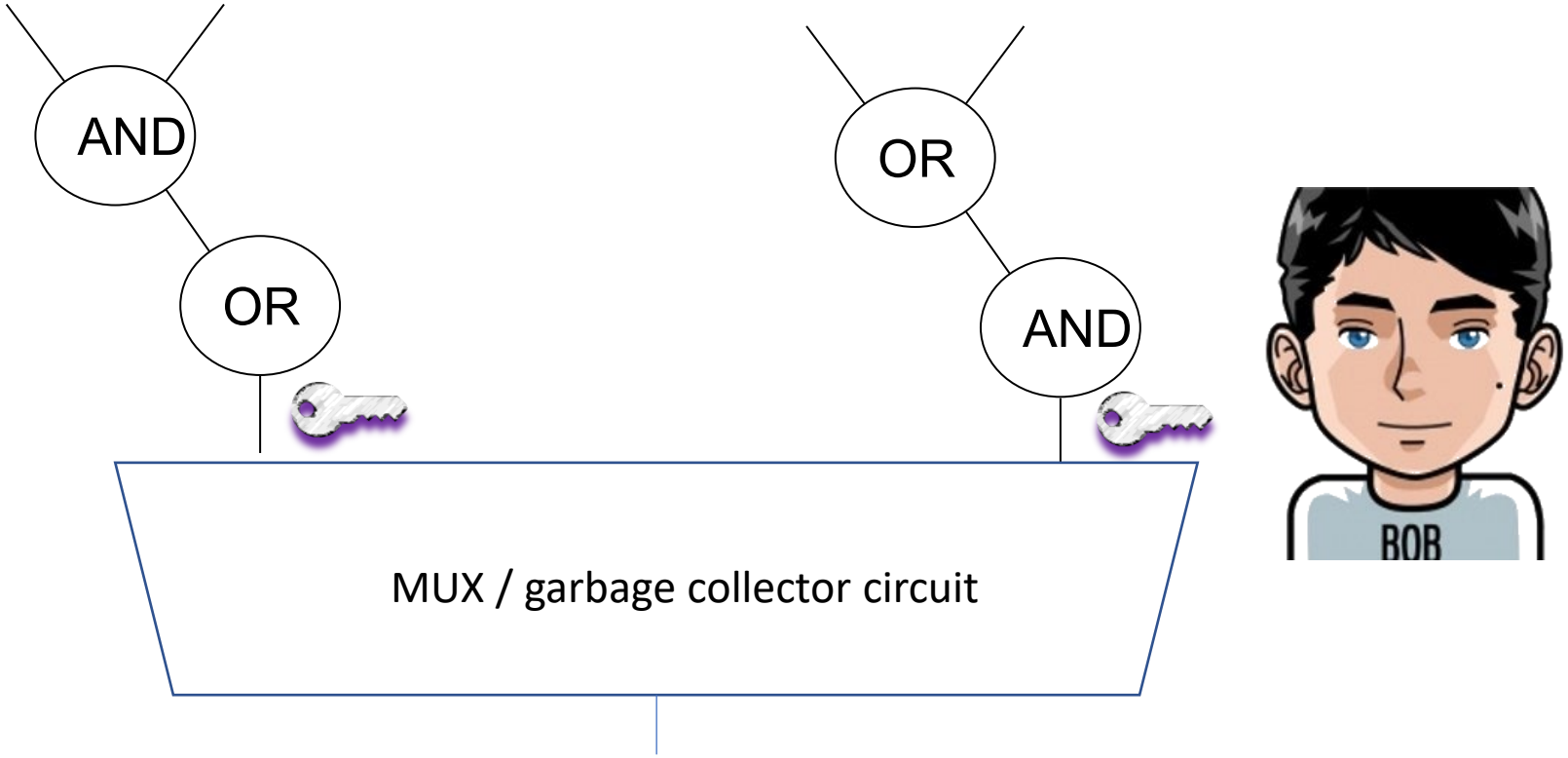


For each branch, if it is active, Bob gets a good output label, otherwise he gets garbage output label.
He can't tell which is which (requires that GC material and labels look random – achieved by half-gates scheme)

Stacked garbling [HK20]

For active branch, Bob gets a valid label, otherwise he gets garbage output label.





We need to obliviously discard garbage.
Key idea: Bob is deterministic and Alice can emulate him and *predict* the possible garbage keys
Then Alice constructs a MUX gadget which collects garbage.

Logstack

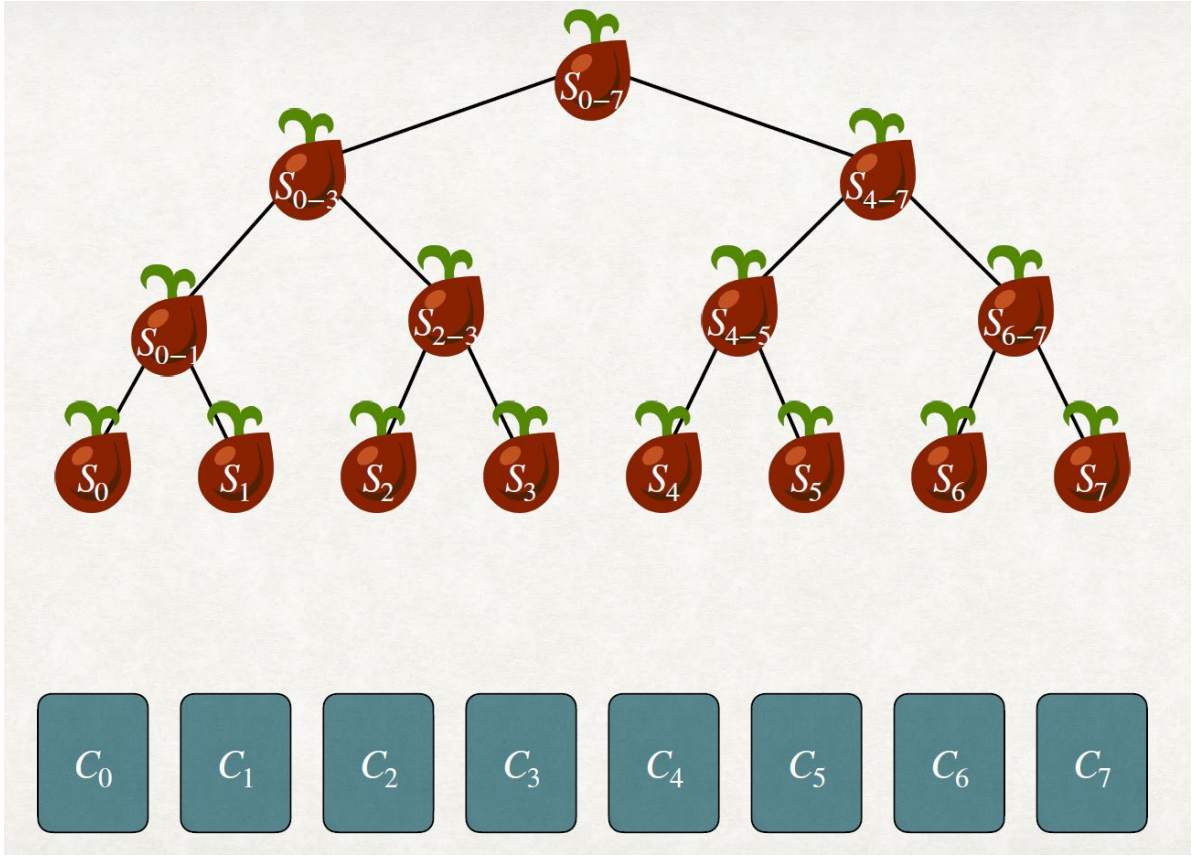
SGC

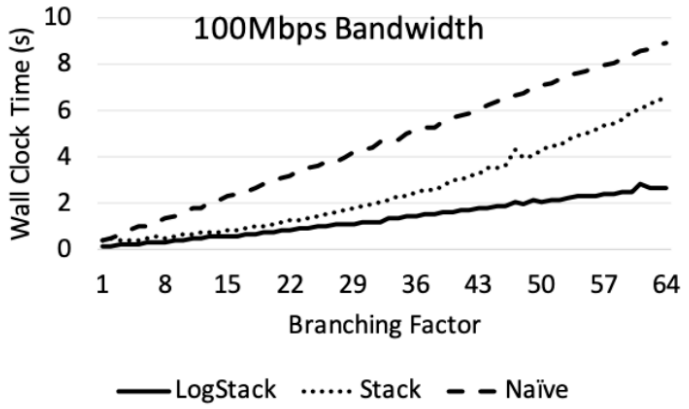
- For each of b branches, E will attempt b guesses
- Each produces different output wire keys (total $O(b^2)$)
- To proceed past the conditional we must collect **garbage outputs** that result from each possible bad evaluation
- Garbler's work is $O(b^2)$

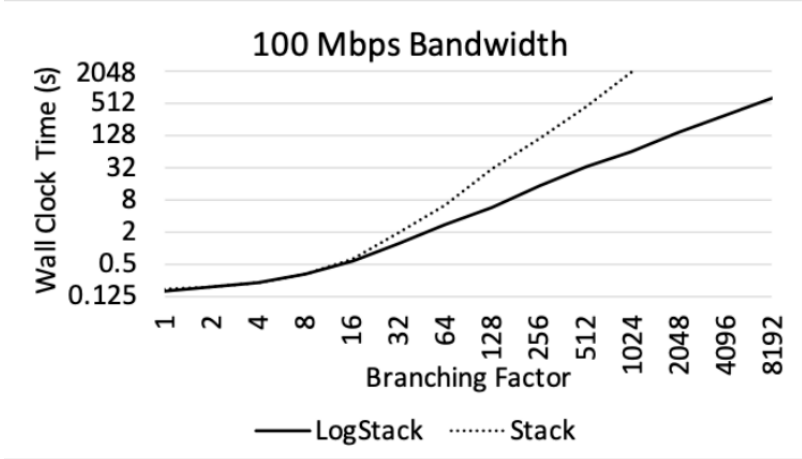
Logstack (HK21)

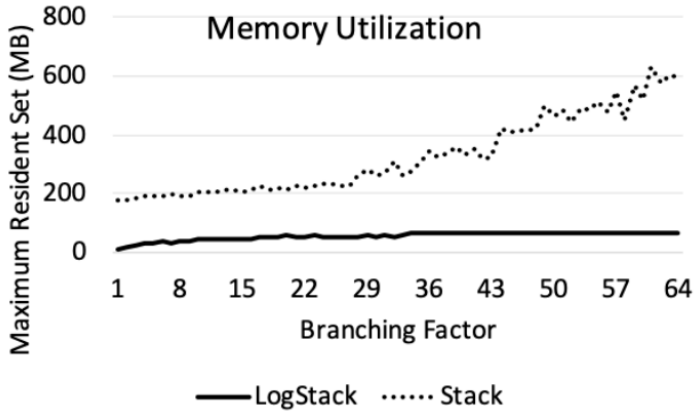
Idea

- Consolidate garbage collection by setting branches/seeds into a binary tree
- Each branch's garbage depends on *which sibling subtree holds the active branch.* Hence, each branch has $\lceil \log b \rceil$ possible garbage labels
- G can precompute all garbage in $O(\lceil \log b \rceil)$ time and build a garbage-collecting multiplexer



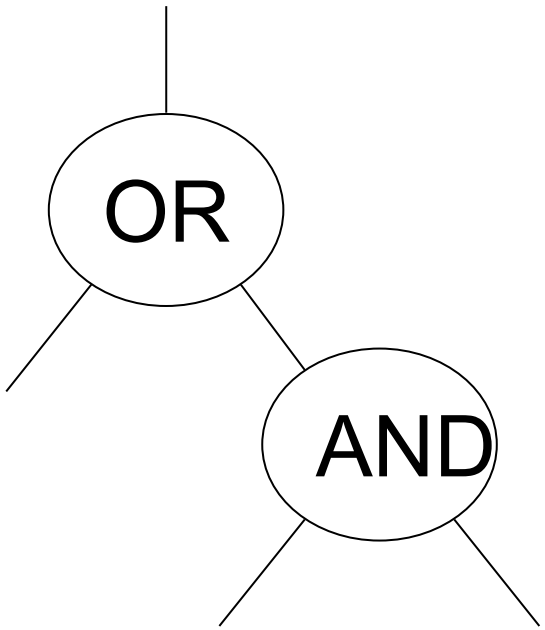


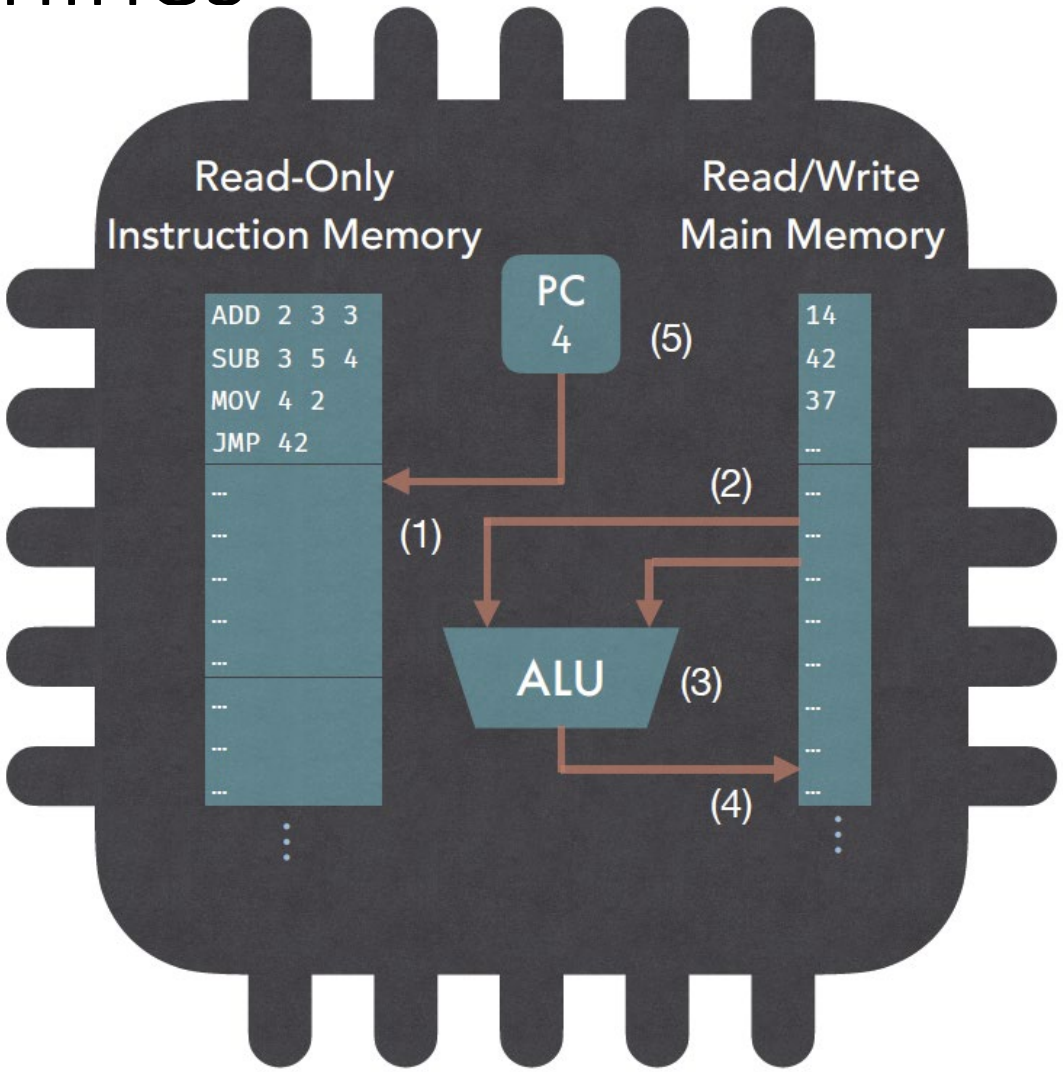




Applications to threshold cryptography

From circuits to RAM machines





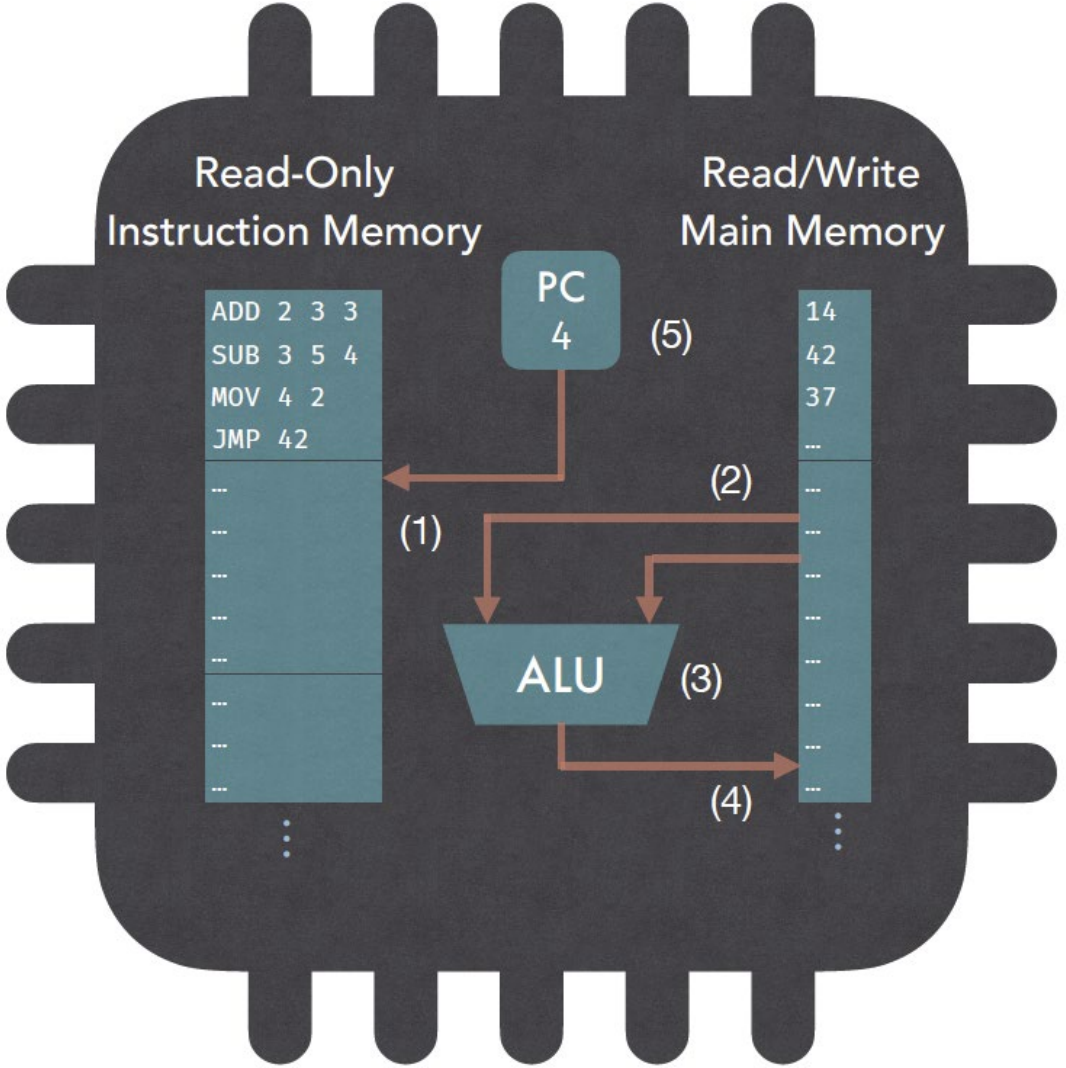
Need: compute F
Represent F as circuit vs as a C program

Implementing CPU with Stacked garbling

[HK20] For circuit $C = Cond(C_0, C_1, \ldots C_{n-1})$
Performance improvement factor n

CPU is such a conditional circuit!

Implement N CPU steps as sequence of N circuits. Each circuit ALU is now as large as a single instruction!



GC is basic

- It is a simple object; it is not a protocol
- Standardizing just GC gives cryptographic object with clean security properties.
- Optional OT/GC usage standardization makes is a secure MPC standard
- In MPTS’20, GC world was relatively simple. Since then there were some nice developments.

GC standardization

Basic GC is very stable.

Standardizing basic GC

- Not likely to hinder future algorithmic enhancements
- Encourage development and standardization of gadgets
- Will aid in Threshold crypto (mandate of this group),
  - and be a catalyst for MPC development *and adoption.*

So let's go!