- PERK -

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NIST Sixth PQC Standardization Conference (09/25)

Overview

PERK is a digital signature scheme named after the **PER**muted **K**ernel problem

- Fiat-Shamir (FS) based signature along with a Zero-Knowledge Proof of Knowledge (PoK)
- PoK built using the Multi-Party Computation in the Head (MPCitH) paradigm
- PoK relies on the hardness of the Permuted Kernel Problem (PKP)

https://pqc-perk.org

Agenda

1. Round 2 Updates
2. Permuted Kernel Problem
3. Scheme Overview
4. Sizes & Performances
5. Advantages & Limitations

Round 2 Updates

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New results since Round 1

- New modeling for PKP [BBGK24]
- New MPCitH frameworks - **TCitH** [FR25] & **VOLEitH** [BBD+23]

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Modifications for Round 2

- v2.0.0 - No modification
- v2.1.0 - Design update using the new modeling along with new MPCitH frameworks
  Implementation update

Round 2 Updates

PERK Instance	Modeling	Proof System	Size (pk + sig.)
Round 1	Shared Permutation	MPCitH	5.8 - 8.1 kB
Round 2	**Permutation Matrix & Elementary Vectors**	**VOLEitH**	**3.5 - 4.4 kB**

Table 1: Modifications for PERK (sizes are given for NIST-1 security level)

Permuted Kernel Problem

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Input

- Secret permutation $\pi \in \mathcal{S}_n$
- Public matrix $\mathbf{H} \in \mathbb{F}_q^{m \times n}$ and vector $\mathbf{x} \in \mathbb{F}_q^n$ such that $\mathbf{H}(\pi[\mathbf{x}]) = \mathbf{0}$

Goal

- Find $\tilde{\pi} \in \mathcal{S}_n$ such that $\mathbf{H}(\tilde{\pi}[\mathbf{x}]) = \mathbf{0}$

PERK & PKP

PERK Security

- PERK security **no longer** relies on a **relaxed** PKP variant (r-IPKP)
- PERK now relies on standard PKP but defined over $\mathbb{F}_q$ an extension field of $\mathbb{F}_2$

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PKP Assumption

- No known attacks exploiting the structure of the extension field $\mathbb{F}_q$
- New result on PKP cryptanalysis to appear soon [BDP25]

  $\Rightarrow$ ***No impact*** on PERK parameters

Scheme Overview

Modeling

PERK v2.1 relies on a modeling based on **permutation matrix** & **elementary vectors** [BBGK24]

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Permutation matrix

- Let $\mathbf{P}$ be the permutation matrix representation of the secret permutation π
- $\mathbf{P}$ is a permutation if (1) Each of its rows is an elementary vector
  (2) For each of its columns, the sum of all its coefficients is equal to 1
- $\mathbf{P}$ is a solution to a PKP instance $(\mathbf{H}, \mathbf{x})$ if $\mathbf{HPx} = \mathbf{0}$

Modeling

Elementary Vectors (EV)

- Vectors of $\mathbb{F}_2^n$ with one coordinate equal to 1 and the remaining ones equal to 0

$$\mathbf{v} = (0, \cdots, 0, 1, 0, \cdots, 0)$$

- EV of size n can be computed from tensor products of EV of smaller sizes

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- [BBGK24] suggests EV of size 2 as input $\rightarrow$ modeling degree $d = \lceil \log_2(n) \rceil$
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$\mathbf{v} = (v_1, v_2, v_3, v_4)$ is an EV $\Leftrightarrow v_1 \oplus v_2 \oplus v_3 \oplus v_4 = 1$ and $v_1 \cdot v_2 = 0$ and $v_3 \cdot v_4 = 0$

Modeling

Protocol Overview

Public Input

- An instance $(\mathbf{H}, \mathbf{x})$ of the PKP problem

Private Input

- Representation of π using n sets of EV $(\mathbf{v}_1^{(i)}, \cdots, \mathbf{v}_d^{(i)})$ for all $i \in [n]$

Protocol

1. Verify that $\mathbf{v}_k^{(i)}$ is an EV for all $i \in [n]$ and $k \in [d]$
2. Compute $\mathbf{P} = (p_{ij})_{i \in [n], j \in [n]}$ where $(p_{i1}, \cdots, p_{in}) = \bigotimes_{k \in [d]} \mathbf{v}_k^{(i)}$ for all $i \in [n]$
3. Verify that $\sum_{i \in [n]} p_{ij} = 1$ for all $j \in [n]$
4. Verify that $\mathbf{HPx} = \mathbf{0}$

Proof System

MPCitH Frameworks

- Two recent improvements to the MPCitH paradigm - **TCitH** [FR25] & **VOLEitH** [BBD+23]
- TCitH and VOLEitH can be described using the PIOP formalism [Fen24]

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TCitH

- **5**-round protocol
- Computation over a small field
- Several protocol repetitions
- *Arguably* simpler

VOLEitH

- **7**-round protocol
- Computation over a large field
- One protocol execution
- Smaller signatures

Proof System

PERK & TCitH vs VOLEitH

- VOLEitH lead to smaller sizes (in particular for modeling with degree $d > 2$)
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PERK Instantiation

- PERK is instantiated with the **VOLEitH** framework
- PERK uses the *one tree* optimization for GGM trees [BBM+24]

Sizes & Performances

Implementation

Implementation Updates

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Benchmark & Ongoing Work

- Numbers reported for the fastest variant of the optimized implementation (avx2 & aes-ni)
- Ongoing work targeting additional performance improvements

Sizes & Performances

PERK-1 Instance		$\|sk\|$	$\|pk\|$	$\|sig.\|$	Keygen	Sign	Verify
Round 1	Short	32 B	241 B	5.8 kB	92 K	35 M	25 M
Round 2 (v2.1.0)	Short	32 B	104 B	3.5 kB	34 K	16 M	13 M
Round 1	Fast	32 B	241 B	8.1 kB	89 K	7.0 M	4.9 M
Round 2 (v2.1.0)	Fast	32 B	104 B	4.4 kB	33 K	5.0 M	3.4 M

Table 2: Sizes and performances (CPU cycles) of PERK for NIST-1 security level

Sizes & Performances

PERK-5 Instance		$\lvert sk \rvert$	$\lvert pk \rvert$	$\lvert sig. \rvert$	Keygen	Sign	Verify
Round 1	Short	64 B	507 B	23.0 kB	333 K	168 M	131 M
Round 2 (v2.1.0)	Short	64 B	195 B	14.9 kB	102 K	191 M	172 M
Round 1	Fast	64 B	507 B	31.7 kB	324 K	33 M	26 M
Round 2 (v2.1.0)	Fast	64 B	195 B	18.3 kB	100 K	53 M	37 M

Table 3: Sizes and performances (CPU cycles) of PERK for NIST-5 security level

Comparison to other schemes

- Stay tuned till the end of the session -

Overview of MPCitH based Signatures using the ***PQ-SORT*** *benchmarking tool*

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Limitations

- *Size* - Quadratic growth of signature sizes with respect to security level
- *Performances* - Slower than lattice-based signature schemes
  But competitive with many other post-quantum signatures

Thank you for your attention.

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- Overview of MPCitH based Signatures -

L. Bidoux

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PQ-SORT: Post Quantum Signatures On-Ramp Tests

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Benchmarking Tool

- *Comparison* - All round 2 candidates within NIST Standardization of Additional Signatures
- *Visualization* - Interactive tool providing scatter plots, tables, graphs & Filtering capabilities

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- *Fairness* - All algorithms are tested under the same controlled environment
- *Transparency* - Detailled methodology available on the project website
- *Platform* - Intel CPU (i7 - 13 700K @ 3.4 GHz), Ubuntu 24.04.1 & gcc version 13.3.0

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- **pqsort.tii.ae** -

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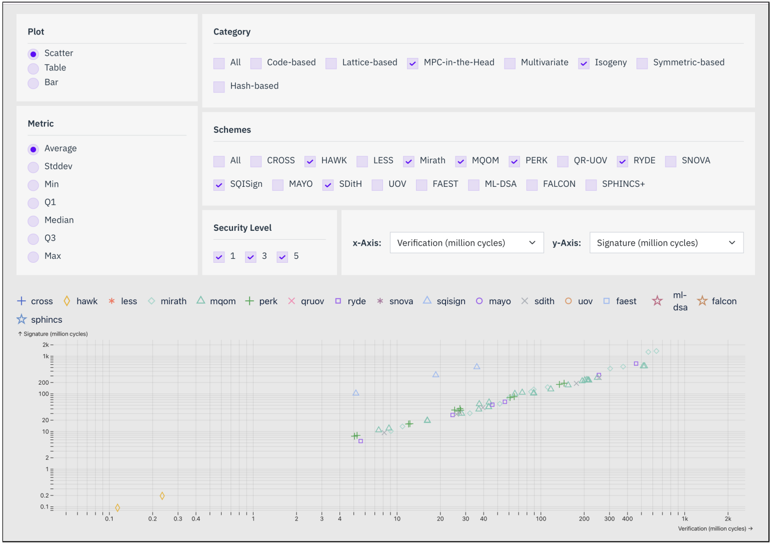
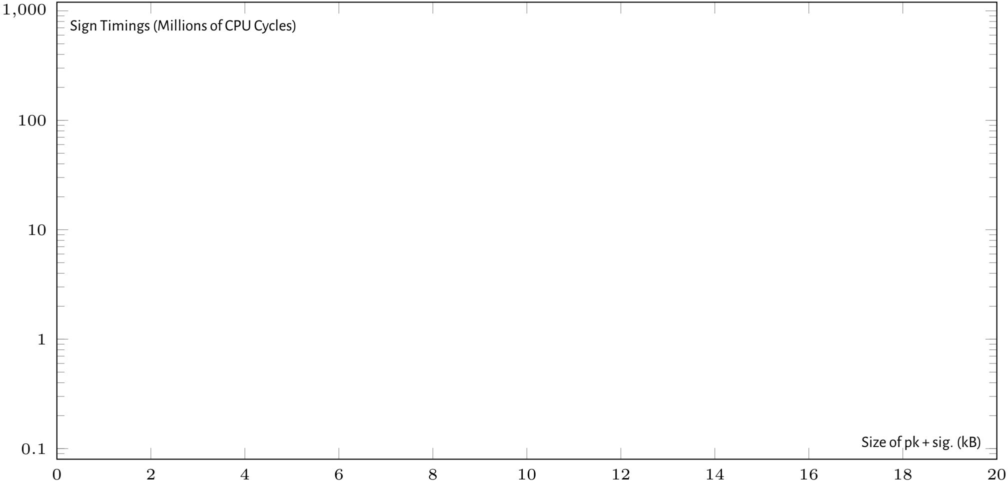


Comparaison between MPCitH and other Signatures



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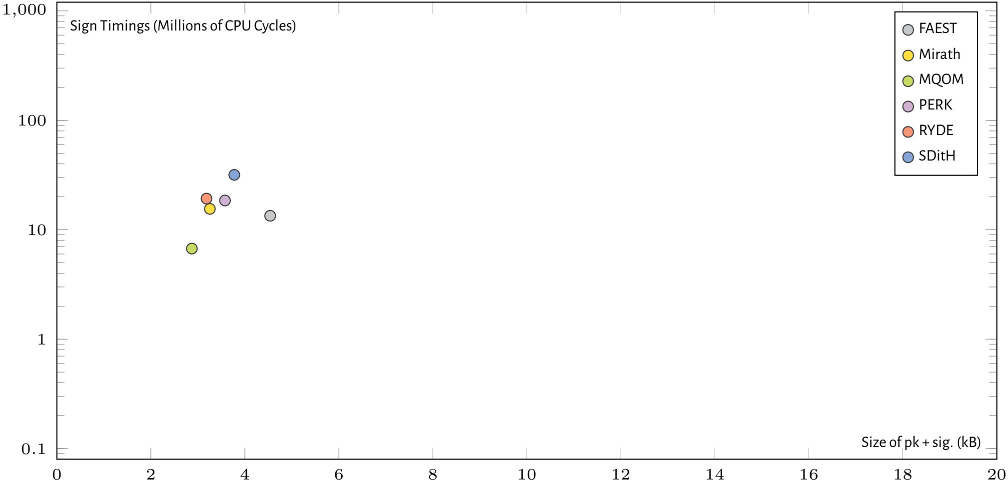

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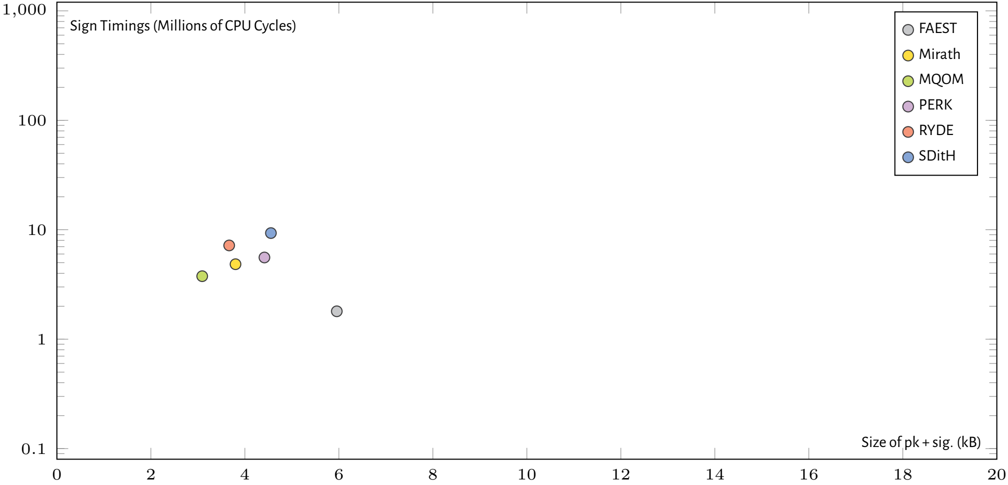

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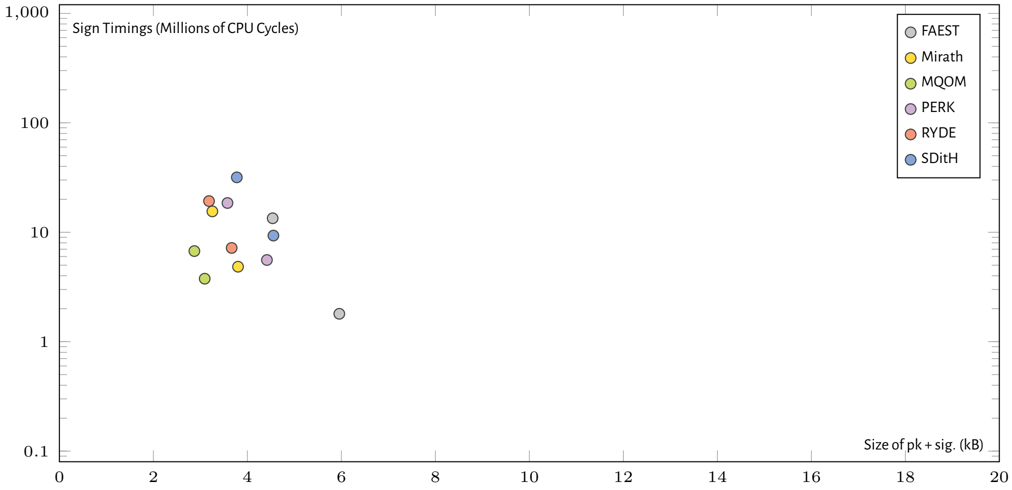

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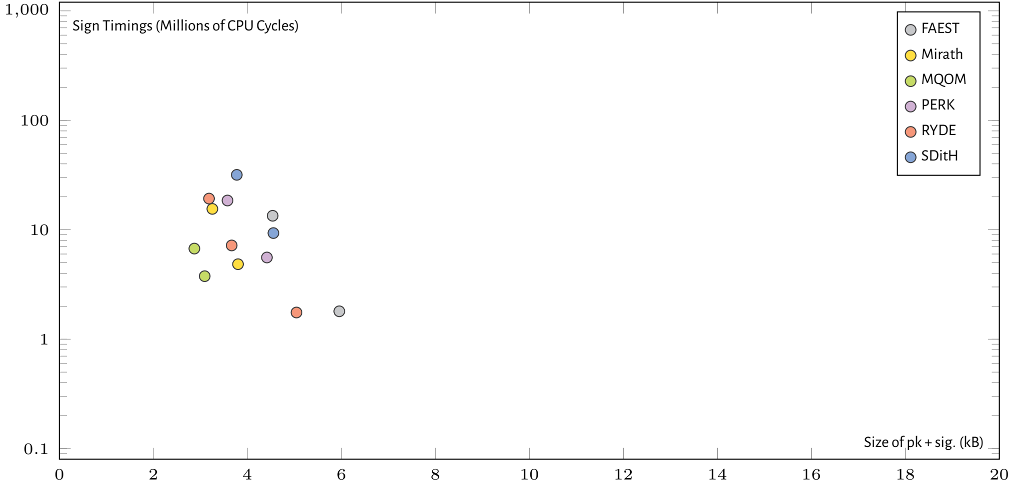

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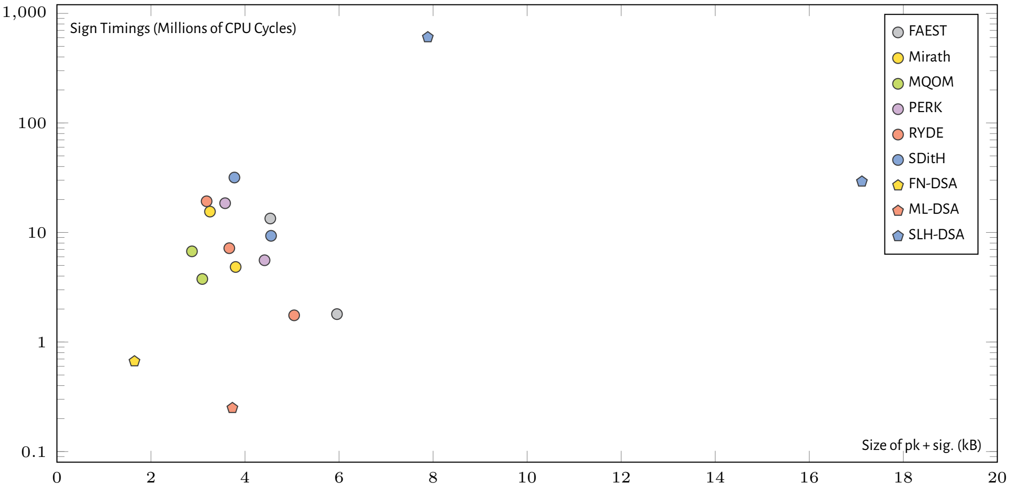

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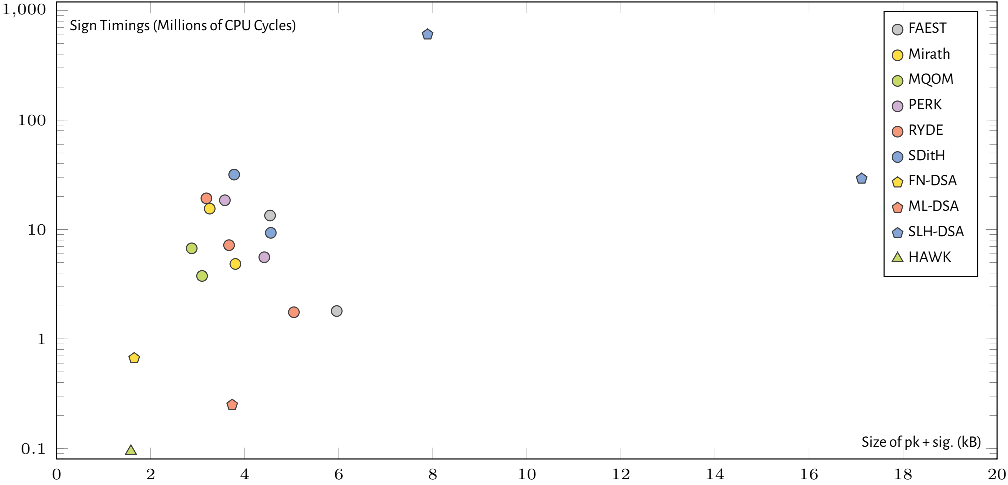

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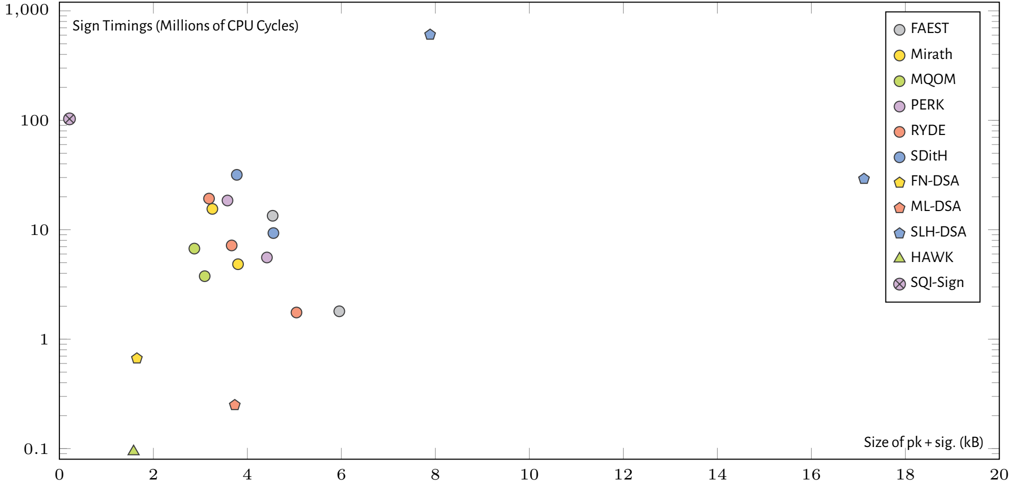

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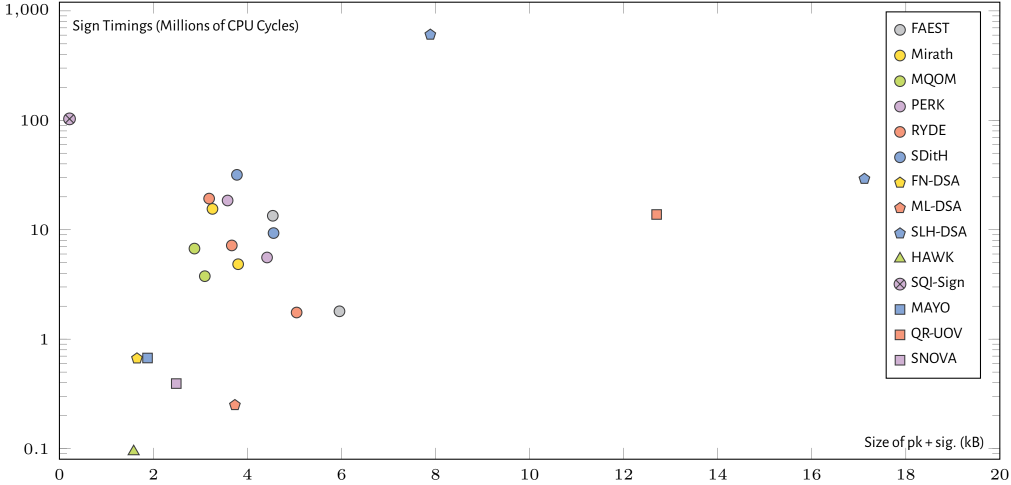

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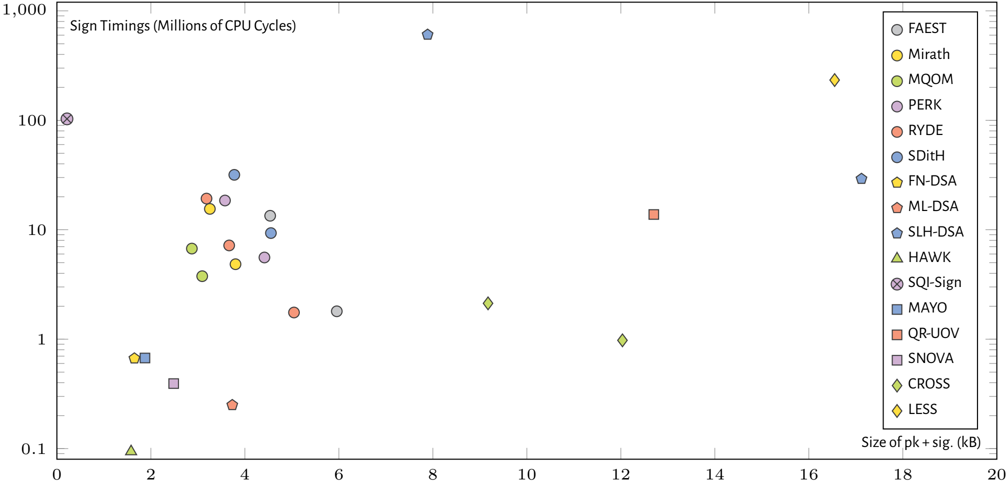

Concluding Remarks

Security - Conservative schemes based on the hardness of ***unstructured*** problems

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- FAEST - Based on **AES** one-wayness assumption
- Mirath - Based on **MinRank** problem
- MQOM - Based on Multivariate Quadratic (**MQ**) problem
- PERK - Based on the Permuted Kernel Problem (**PKP**)
- RYDE - Based on the Rank Syndrome Decoding (**RSD**) problem
- SDitH - Based on the Syndrome Decoding (**SD**) problem

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Security - Conservative schemes based on the hardness of ***unstructured*** problems

Design - Many improvements between round 1 & 2, design now stabilizing

- *Modeling* - All the schemes now employ "natural and short" witness representation
- *MPCitH* - Efficient proof systems with small overhead (scaling with modeling degree)
- *GGM Trees* - No consensus yet, different choices & instantiations across schemes

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- *Future Improvements* - Performance gap between schemes may decrease over time

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- MPCitH schemes are interesting candidates to complement NIST PQC portofolio -

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